Thermo-Mechanical and Thermo-Chemical Processes
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Lecture-11
Stereographic Projection

Hello friends we will start with a new module today on crystallographic texture. So, the idea here is to make you understand the crystallographic texture and suppose if you see it anywhere then you should not be leaving it maybe because you are not able to understand it completely. So, it will not be a very exhaustive course on how to do the texture analysis, the idea here is just to make you acquaint with the crystallographic texture. So, that if you see a particular representation of texture, you should be able to understand and correlate with the mechanical properties if there are any mechanical properties or any other properties are related with the texture. So, the first basic of the understanding starts from the stereographic projection. So, that is what I am taking today as the first lecture in this particular module.

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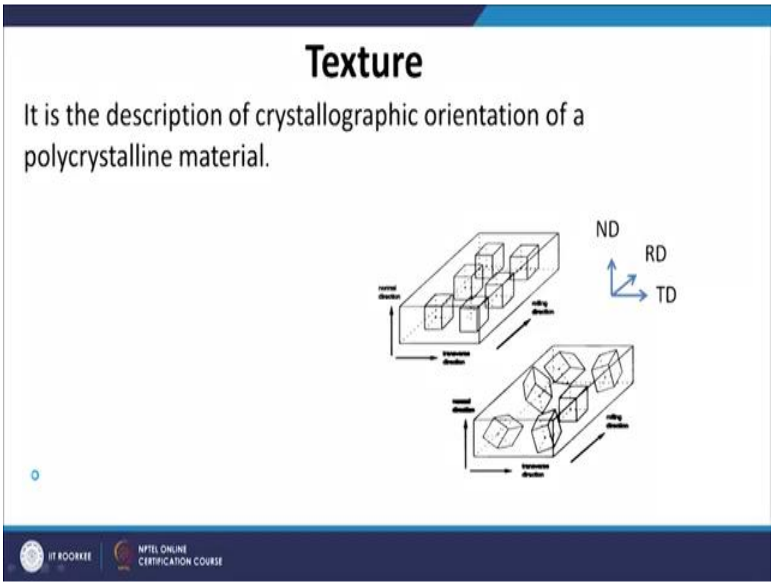

What do we mean by texture? Basically, if you see an individual grain in a poly crystalline material, the identity of the grain is that how the unit cell is oriented in the grain for example, in one grain the unit cell is like this (refer to above figure), in another grain the unit cell maybe like this, in another grain the unit cell may be like this.

Suppose all these unit cells in individual grain are randomly oriented then we would call it as a weak texture or random texture. There is no particular way the unit cells are arranged in individual grain. But if you find that some of the grain or good number of grains have a particular orientation of unit cell and these are not required to be together, these grains spatially can be anywhere throughout the material. But if we find that lets say out of 100 grains suppose 30 grains have a particular orientation for example like this (refer to above figure), then I would call that the material has a certain crystallographic texture. There is a preference for a certain type of arrangement. So, that is what is shown here in the schematic (refer to above figure). As you can see in this first instance, all the unit cells are arranged in a particular fashion.

So, because in this case all the unit cells are arranged in a particular fashion, I would call it is highly textured. Whereas in this case the second case, you can see that all the unit cells are randomly oriented, all they have different orientation. So, I would be calling it has a random texture. There is no particular preference. In three dimensions, if I want to define that what is the orientation of this unit cell, I have to have some reference from which I can take or I can tell you that what is the orientation of this unit cell, if there is no reference in the space I cannot define that what is the orientation of this unit cell. So, to define reference frame which I can use to define the orientation of this unit cell, because rolling is one of the most popular processing techniques, the symmetry is taken from there. And it has orthogonal symmetry as you can see (refer to above figure).

So, this is your rolling direction RD, this is your transverse direction TD and this is your normal direction ND, all perpendicular to each other (refer to above figure). So, this is a reference frame, which I can use to define the orientation of the unit cell. How we are going to do that we will see. So, in terms of texture representation, you have three methods either you can show it in terms of pole figure or you can tell it in terms of {hkl} <uvw> or you can tell it in terms of what we call as orientation distribution function method.

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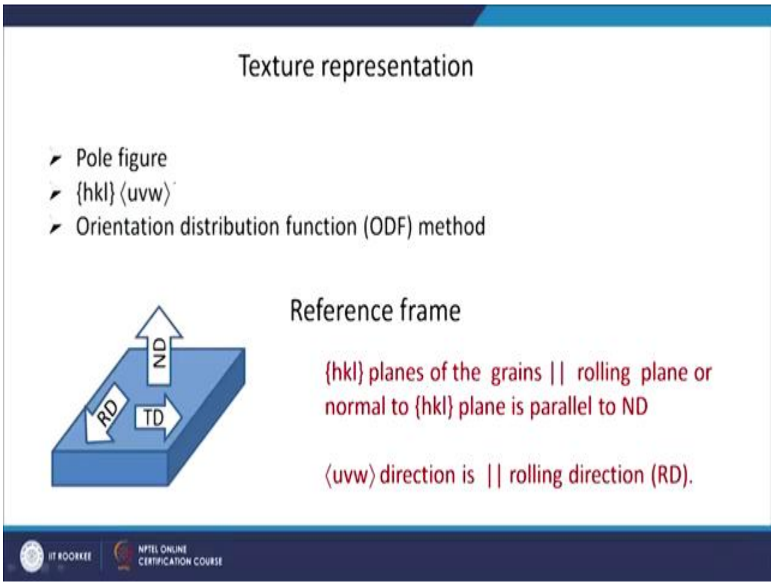

For example, this hkl value, the example is shown here (refer to above figure), as we have already seen that my rolling direction, transverse direction and normal direction are the reference frame. So, basically, we are trying to say that which {hkl} plane of the grain is parallel the rolling plane and which <uvw> direction is parallel to the rolling direction. So, I can use this {hkl} <uvw> method to express the orientation of the unit cell.

Because pole figure is one of the most used and important way of representing texture so, before coming to pole figure I will first introduce you to the stereographic projection because using stereographic projection only we can go to the pole figure, how to create pole figure for a particular orientation. So, what is stereographic projection? So, it is a very convenient way to depict any, three-dimensional information.

So, as you can see that when I am talking about a unit cell, I am talking about a three dimensional information you have different planes in three dimensions, you have different directions, some plane can be at angle, but I cannot do any analysis in three dimension. So, I have to depict all this information on two dimensions. And when we are discussing about crystallography, our main purpose is about what are the angles between the different planes.

I am not concerned with the dimension of the plane because it can be of infinite size, but what are the angles between different planes and direction that is of importance to us. So, we when we are doing this projection, we want to preserve the angular information. Angular information should not be distorted. So, when we are doing this projection, we are preserving the angular relationship.

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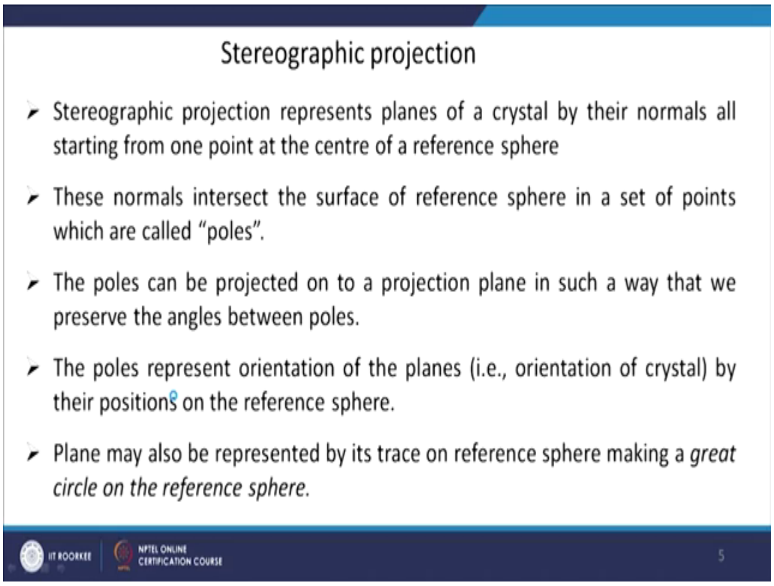

Now, continuing with this stereographic projection, every plane is defined by its normal. And these normal of different planes are starting from one we call as sphere. I will tell you that what do we mean by this reference sphere. Actually, we are saying that all the normals to the planes are starting from the centre of a reference sphere. And what these normal are doing, they are intersecting the surface of reference sphere in a set of points which are called poles.

I will explain all these things, to you in an animation. So, basically if I take a unit cell, so you have a plane and there is a normal. So, this normal will go and cut a sphere. I will discuss what is sphere means. And then we project these poles on a projection plane in such a way that we preserve the angles. And the poles represent the orientation of the planes, orientation of crystal by their position on the reference sphere.

And plane may also be represented by its trace on reference sphere making a great circle on the references sphere. I will show you what do we mean by this.

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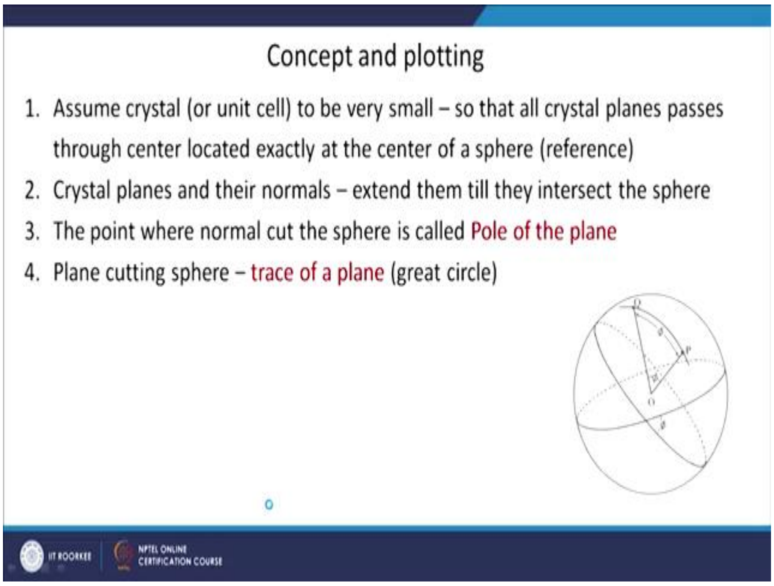

So, basically this is the idea of crystallographic projection you can take a unit cell. Take a very tiny unit cell so that all the normal from the plane, should be seen as they are coming out from the centre of the sphere. And what is this sphere? A sphere is an imaginary sphere around this particular unit cell. This unit cell is at the centre of the sphere and you can take the imaginary sphere as big as you want, maybe equal to the size of the universe if you like it does not matter.

And then we are looking at the normal which are coming out of these planes and where they are cutting this sphere that will be called as pole. And if I extend a particular plane, this plane will also go and cut the sphere at some place. So that will be called as the trace of the plane. So, for every plane in the stereographic projection you have a trace and you have a pole, but it is still in three dimensions till now. We have to bring it to two dimension that is where the projection will come.

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So, I will explain that with a animation here (refer to amination). So basically, you have a unit cells like this, I am taking it a very tiny unit cell. And there is a sphere around the unit cell and I am taking a plane which is cutting the centre of the sphere and passing through the centre of the sphere. And this plane I am calling it as a equatorial plane which is dividing this sphere into two equal parts, two hemispheres basically.

And lets call one is top hemisphere another as bottom hemisphere and then I am taking the normal. So, in this case I am interested in 100 planes (refer to animation). So, I am taking the normal of 100. So, one normal is coming out from this particular plane here and cutting the sphere at some point. So, this is the pole of this particular plane. So, if I call this plane P_1. So, this is the pole of P_1 lets call it as PP_1.

Then there is another normal going from another 100 type of plane, then there is a third normal which is also coming out of another 100 type of plane. So, you can see that I have taken three normals here from three different plane I can take from other planes also just for simplicity I am taking it from three 100 type of planes.

There will be also normals to 100 plane which will go in the southern hemisphere like this, but I am not interested in the normal which are going in the southern hemisphere or the bottom hemisphere I am only interested in the normal which are going in the top hemisphere. And I am also kind of creating a reference frame on this plane equatorial plane and that I am calling as north, south and east, west (refer to animation).

So, right now it is in three dimensions. When we see from the top then all these four will be projected on the screen. So, now still all the information is in three dimension. So now I am connecting this pole with the pole of the bottom hemisphere, when I am doing this, projection of this particular pole is projected on the equatorial plane like this (refer to animation).

Similarly, I will join this pole with the bottom hemisphere pole, it is also cutting the equatorial plain at this point there are two already. Then the third one projection this will also cut the equatorial plain at one point. So, basically now three-dimensional information we have reduce into two dimensions by projection of poles. So, now I am just changing the view here. Now I will start seeing from the top. So, from here I will see that what is the condition of different projections.

Now I am looking from the top so T should come here (refer to animation) at the centre and north will come here, south will come here, east will come here and west will come here. So, I have removed the other part of the equatorial plane. I am only taking the trace of the equatorial plane which was cutting the sphere.

So this is my trace of the equatorial plane. So this is my north, south, east and west. So, these three points the normals which we projected, will look something like this (refer to animation).
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So only 100 poles we were taking. So those three poles are projected here, the other three poles will be in the bottom hemisphere that we are not projecting here. Because then it will be more confusing to do analysis and because of symmetry it doesn't make any difference. So only we are projecting the top part of the hemisphere and these three are the projected poles on the equatorial plane.

Now, just to give you a more understanding of this, so as I told you that we started with the unit cell and suppose this unit cell is at the centre of this particular sphere. So, suppose that it is oriented something like this, so this one Pole will go maybe hit somewhere here. This one will go and hit somewhere here. And lets say take slightly here. So, the third one maybe hit somewhere here. And then we are taking projection on the; this particular plane here which is going through this and that is projected.

If I want to find the trace of the planes, I am just showing in your in this particular slide here.

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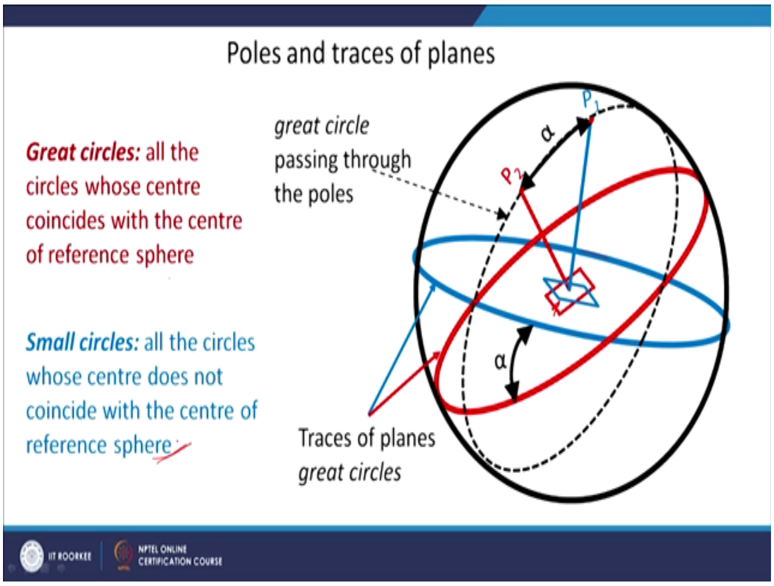

So, this (refer to above figure) is a plane for which this is my pole P_1. And this is the plane which is extended and when it is cutting the sphere it will form a projection like this. There is another plane for which this is the pole P_2 and this is another trace for that pole and then this is the angle between the two P_1 and P_2, I will show you how to find this angle between two planes. So, this is how you can find Pole and trace of the plane.

So, when you have this kind of stereographic projection we defined two types of circle and this is very close to what you see on the on the Atlas. There you have some what we call it great circles and these circles are the one for which the centre coincides with the centre of the reference sphere and some are small circle for which the centre does not coincide with the centre of the reference sphere.

And as you can see all these (refer to above figure) are actually great circle because their centre and the centre of the sphere are same. So, all the angular measurements we do on the great circle. I will show you that how we can do that in the in the next lecture. First let's see that how the circular projection is can be divided into multiple circles.

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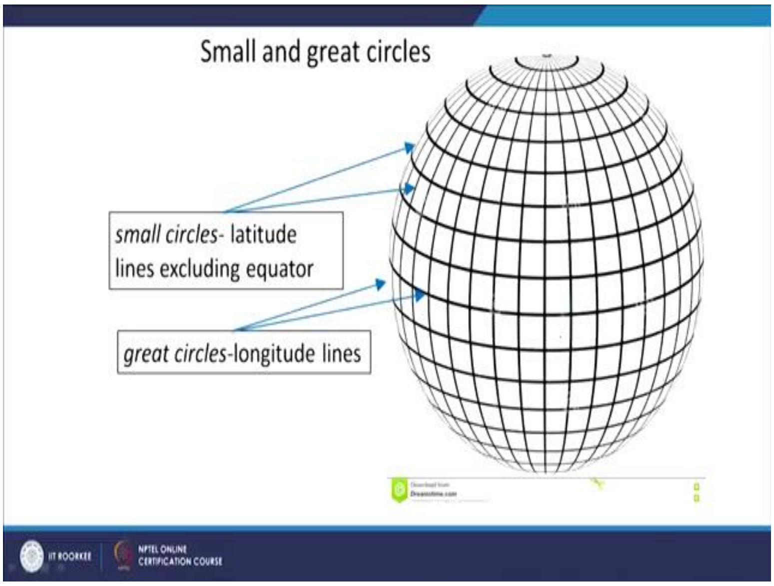

For example, this (refer to above figure) is a right now a three-dimensional globe kind of sphere and you can see that it is divided by different longitudes and latitudes. So, all these longitudes are great circle because their centre and the centre of the sphere same whereas, all these latitudes are small circles their centre will be somewhere here which is not going to match with the centre of the sphere.

Only one latitude is such that that its centre matches with the centre of the sphere and that is what we call as equator. Very similar to what you see in the in the Atlas.

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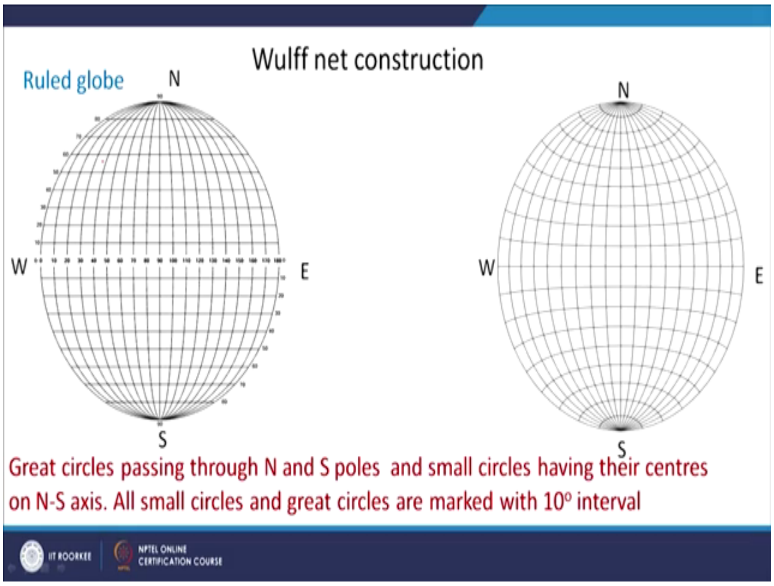

So, all these traces which we have plotted that can also be projected on the equatorial plane. So, if you do that, they will form this (refer to above figure) kind of great circles. So, you can do a graduation here, you can do a marking and that is how you can get a construction which we call as Wulff net. So, this Wulff net we are going to use for doing all our calculation for the stereographic projection. So, it has north, south, east, west and so on. And this (refer to above figure) particular one is graduated with 10° interval. So, every two graduations have a distance of 10°. And when you completely do these projections, basically what you get at the end of it, is a standard stereographic projection.

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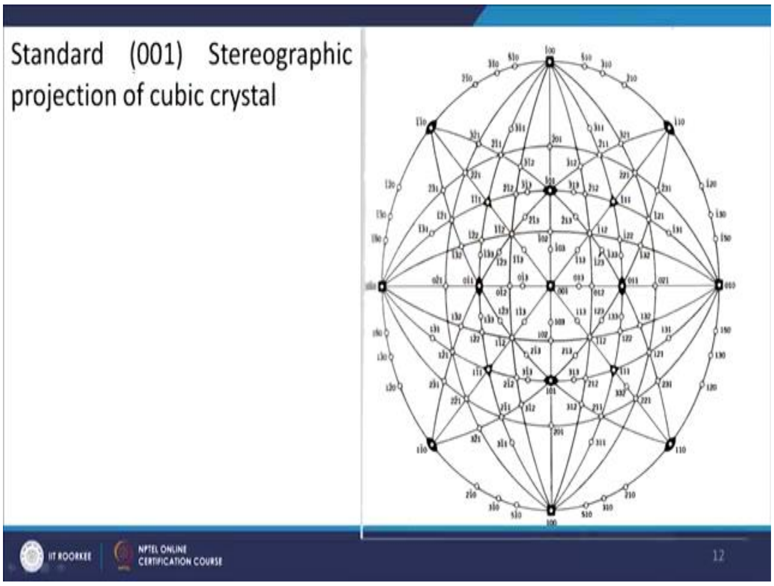

So, what does it mean? For example, in this particular case (refer to above figure), we are saying that it is a standard 001 stereographic projection of cubic crystal. So, when I am saying standard 001 stereographic projection, I am keeping the unit cell like this at the centre of the sphere, so, what will happen, the 001 will go and hit directly the top pole and it will be projected exactly at the centre because whatever is hitting right there and when I am connecting it to the bottom pole that will go through the centre of the sphere itself.

So, that will be obviously coming at the centre and that will be 001. Now, you can see that other normals. Suppose this is your 100 plane and this is your 010 plane. So, 100 will go and it is going in the equator itself and hitting the sphere at the end and when I take the projection from there the projection will be exactly at the equatorial plane itself. Similarly, 010 will go like this and hit the sphere. And if I take the projection of that it will be directly at that particular location itself. So, you can see these 2 conditions here very nicely. So, you have 100 at one end, and you have 010 at the other end, and there is a 90° relationship between the two. The other poles I am not showing right now, because in the next lecture, we will see that how I can use this stereographic projection, to plot a standard stereographic projection like what you are seeing here (refer to above figure).

So, basically what you are seeing here is that all the poles or all the planes are plotted on a two dimension and their angular relationship is preserved. And you can easily see that using Wulff net what will be the angle between different planes. So all the poles like 011 type, will be plotted here, you can see all these poles of 100 type, 110 type, then you have all the poles of 111 type, these are the important planes for us (refer to above figure).

So, all these are plotted on a stereographic projection. So, in the next lecture, I will show you that how I can use the Wulff net or how I can use this stereographic projection to draw a standard stereographic projection of a particular orientation of the unit cell and that once we understand that, that will help us in creating the pole figure for a particular orientation of the unit cell.

There is actually some kind of imagination is required in three dimensions. So, I hope you might have able to understand that how I can plot a standard stereographic projection. And now I will tell you how you can use this to create the standard stereographic projections. Thank you.

Keywords- Stereographic projection, Wulff net.