Non - Metallic Materials
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Module – 04
Mechanical properties of non – metallic and composite materials
Lecture – 19
Brittle and ductile materials, introduction to fracture mechanics, strength of brittle materials

Welcome to my course Non-Metallic Materials. Now we are in module number 4 Mechanical properties of non-metallic and composite materials and this is lecture number 19 Brittle and ductile materials, introduction to fracture mechanics, and strength of brittle materials I will cover.

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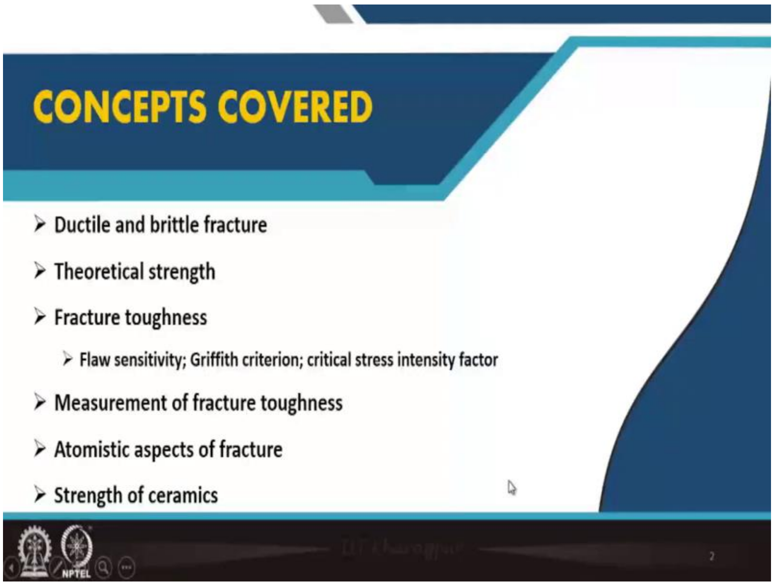

Now, first I will again relook at the ductile and brittle fracture which already I discussed in my last class then we will introduce the concept of theoretical strength, what is the fracture stress that you can estimate from its Young’s modulus.

So, what are the theoretical basis to know the theoretical strength actual strength of the ceramic never follows the theoretical strength value it is much much less than that. Why it is so, we will talk about it in terms of the fracture mechanics, fracture toughness, flaw sensitivity measurement of the fracture toughness and atomistic behaviour or aspects of the fracture that I will cover and finally, strength of the ceramic the concepts will be developed part of this lecture.

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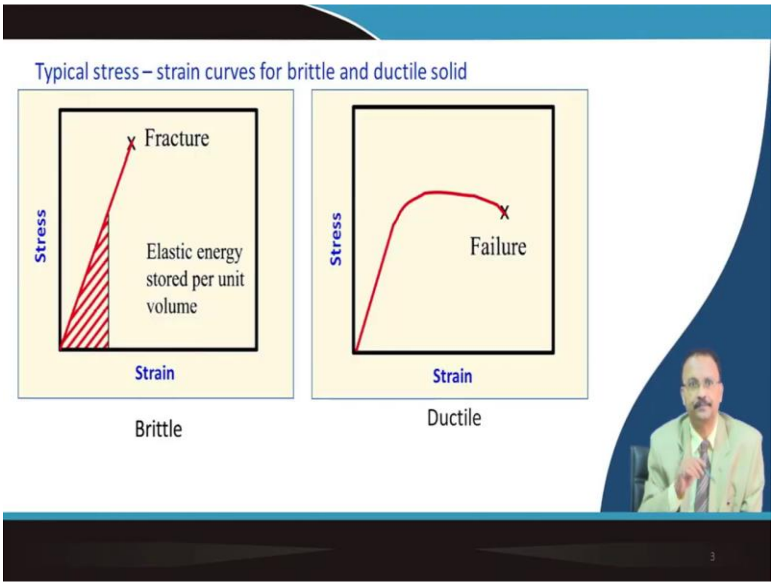

Now, again I come back to the same view graph which I showed in the last lecture as well in case of a brittle material this is this breaks quite suddenly and the slope is reasonably high. So, Young's modulus is high and this shaded area it is the elastic energy that is stored per unit volume in the material.

So, the material is has not yet been fractured because you have not applied this much stress into it or it attained this much strain. But in between whatever energy we are giving that energy is stored as elastic energy and at any point of the stress you can estimate from this the shaded area the area what is the elastic energy stored per unit volume of the material.

In case of ductile material, the slope can be either similar to it or it can be little bit less, but there is a yield point. So, the elastic behaviour is taken over by the plastic behaviour and eventually it fractures. So, this is very preliminary stress strain diagram for a brittle material like ceramics and for a metallic material like any metal.

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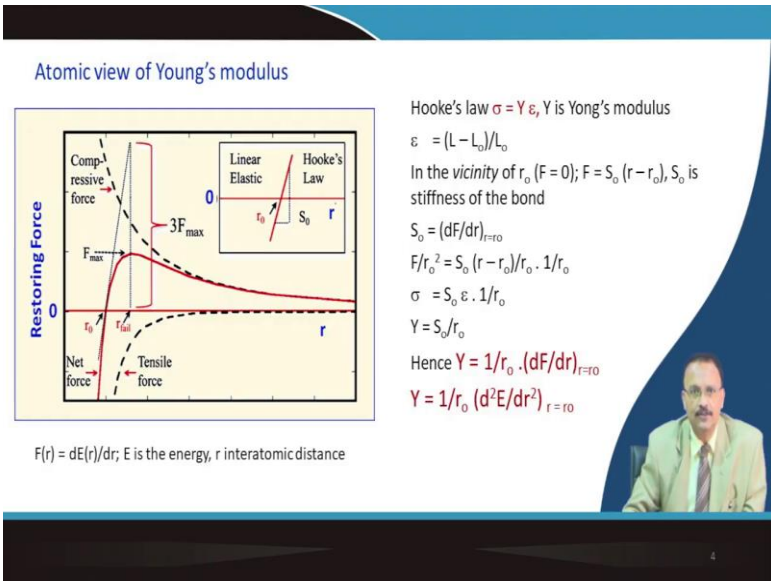

Now, if I want to go little bit more details and redraw the restoring force versus inter atomic distance this plot. As already I have mentioned that something similar to your energy potential energy diagram, you have a attractive force and you have a repulsive force and then you add it up you get the force curve. And there is a equilibrium distance between the two atom where the force is actually 0 so, that it is in a stable condition.

So, first let us apply Hooke's law which relates stress with strain and this is the Young modulus and strain is we have considered it is an engineering strain. So, L is the original length is L_0 and this is the final length. So, you can estimate the strain value. So, in the vicinity of this equilibrium r_0 position equilibrium separation distance between these two atoms the force is 0, but in this vicinity it is linear in nature right.

So, you can apply that F is equal to some kind of constant and the difference r minus r 0 r r is at any point and this is valid right. So, this is nothing, but the slope of this plot. So, your S_0 is differential of the force and with this distance and in the vicinity of r equal to r 0.

Now, in this particular relation if you divide it by equilibrium distance square at both the end then you get this relation and force and r 0 is having a dimension of area. So, this is actually stress and this value is S 0 and this is nothing, but the strain and 1 by r 0 remains here.

So, elastic modulus you know that this is S 0 by r 0 because the ratio of this and this is elastic modulus. So, actually elastic modulus is nothing, but the S 0 value which is the stiffness of the bond between these two particles. So, you can just put it back in this relation S 0 is nothing, but the slope F versus r curve at r 0.

So, you just put it back. So, you can have a relation Young modulus is given as 1 by r 0 d F by d r at r 0 r 0 1 and as you know that this potential energy curve if you differentiate then you get the force curve. So, eventually you can also write the potential energy curve as d 2 E by d r square and this is valid at this equilibrium distance at that position.

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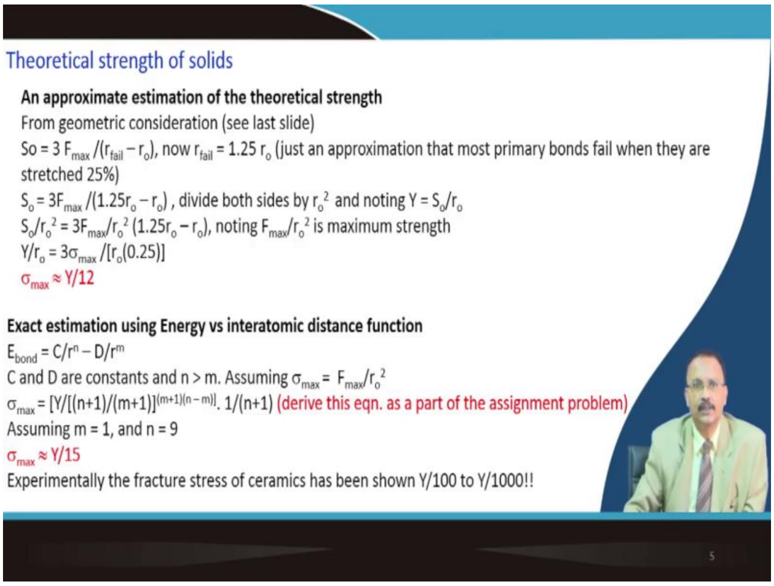

So, if you have a very straightforward estimate that if you can relate the fracture stress which is required to separate these two atom so, that it breaks down. So, you can do it something in this fashion that in the last slide if you see the this thing roughly you have F max here somewhere and this straight line you extend it. So, you get the restoring force here and this value is r 0 minus r failure minus r 0 because at the maximum force it will eventually fail.

So, if this geometry is valid then you can work it out generally when the force is 3 times as F max from that particular graph and another approximation let us take that most primary bond will fail when they are stretched 25 percent. So, in ceramic that is a very large number, but if 25 percent you can stretch then we are assuming that it will fail. So, just put these two value here. So, roughly your S 0 is 3 times of F max divided by the difference between r failures where it fails the separation minus r 0.

Now, we are assuming this failure is 25 percent. So, it is 1.25 into r 0 it is just an approximation. So, you put it back and divide the both side by r 0 square and we know we just derived that this elastic modulus is stiffness divided by r 0. So, you put it back here and do this calculation. So, the maximum sigma that you get the fracture stress that you get is equivalent to the Young modulus divided by 12.

So, this is a very broad approximation it tells that if your Young modulus is some 20 Giga Pascal. So, your fracture stress will be Y by 12. So, it is a quite large value you can do an exact relationship by taking the attractive potential and repulsive potential the way I showed in my last lecture.

And here C and D as you know they are constant and you have two exponent n and m usually n is much larger than m and one can put m is equal to 1. So, again you assume that the force divided by this equilibrium distance square that give you the maximum sigma F value.

Then you come up with this relation this relation and you can just work it out you can work it out by differentiating this one equating it with 0. So, that you can get the force the way we described in the last lecture and you assume here m value is typically 1 and n value say it is 9. You will come up with the maximum stress required for this fracture to occur this two atom will get separated is something similar to Y by 12. So, in that case I am getting Y by 15.

But actually in ceramic if you want to measure the fracture stress you will see the value is much much smaller it is not Y by 12 or Y by 15, but it is as low as Y by 100 and or in some case Y by 1000.

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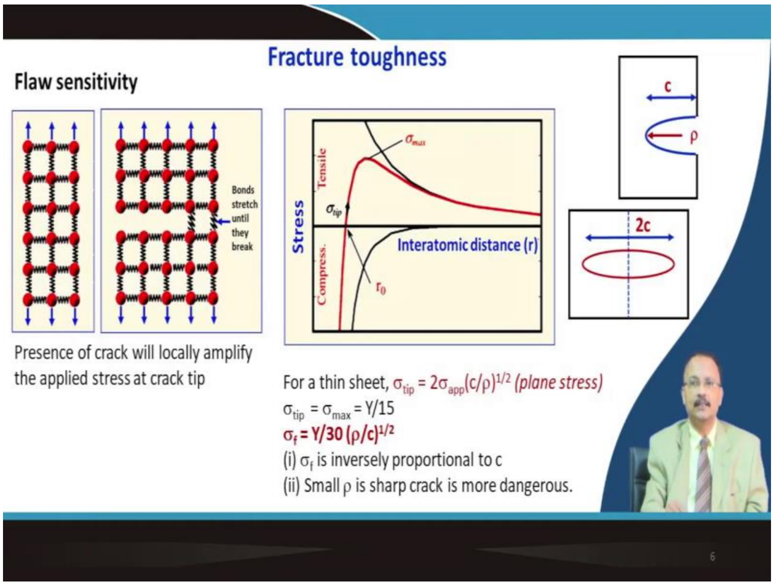

And the reason is that the material is not something like this. So, the atom is we are considering as if it is bounded by this spring. So, these are the actual ionic or covalent bond.

When you are applying a force here and you see that so, many springs are there. So, these are all distributed. So, the number of n here is quite large. So, eventually F by n is a small number. So, it is experiencing a small force, but think of this kind of situation where you have a defect and this is a atomistically sharp defect I am assuming that some of the bonds are missing. So, you have a defect here.

So, all these force they will eventually be on top of this two spring. So, there is a intensification of the force that is taking place. So, whatever you are applying that is multiplied because number of n number of spring is far less at that place.

So, for a thin sheet if you consider and if it is a crack at the surface or crack in the middle right. So, it is in a macroscopic scale if you consider this, then under plane stress condition you can have that at the tip of this curve I mean at the tip of this defect here the stress is equivalent to 2 times of the stress that you are applying.

So, your applied stress 2 times of that applied stress and it is a function of the half length of this crack which is denoted by c and the radius of curvature of this crack which is rho to the power half and this is can be derived in a plane stress condition.

So, at the tip here you see that if tip stress once it reaches a maximum stress value then we are still assuming that this fracture stress value is Y by 15 Y is the elastic modulus then you put it this value of fracture stress here then you get this relation. Although this will be further lowered, but let us assume that it is Y by 30 into radius of curvature divided by half crack length to the power half.

So, this relation tells you that this fracture stress is inversely proportional to c; that means, if c increases then fracture stress will reduce. So, you have a larger crack in your system your fracture stress is reduced and second one how sharp is this crack. So, it is related to rho.

So, if rho is less; that means, it is a sharp crack then it is more dangerous because that will reduce this value further. So, the fracture stress will be further reduced. So, that gives you some kind of idea that if you have crack or void in your sample then this fractured stress is very much reduced and this is the reason that why the theoretical strength is never achieved.

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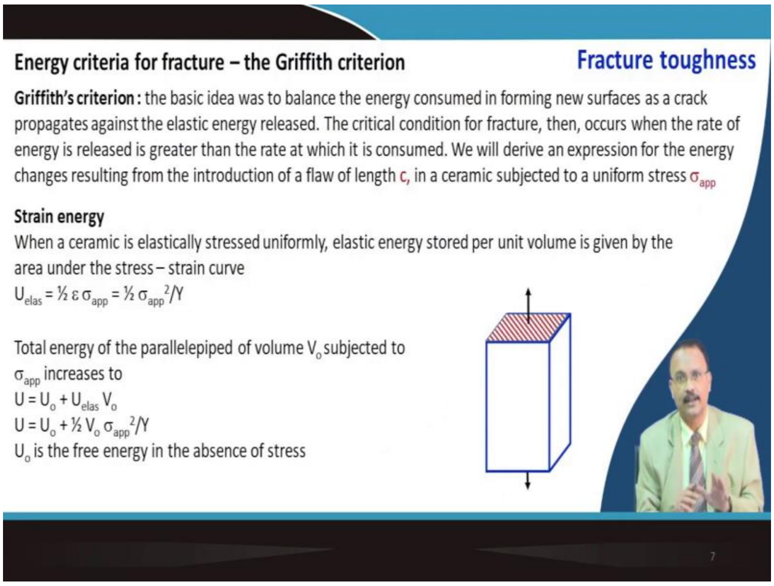

So, theoretical strength whatever you can get from the calculation like Y by 12 or Y by 15 in practical case you your fracture stress is much much lower than that and this is due to the reason of having crack and void inside the sample. So, when I will talk about the sintering you will see why sintering is so, important for ceramic material in particular and also in powder metallurgy because it annihilate the cracks and voids from the material itself to make it more denser.

So, the criteria of the fracture that can be better understood from a theory by Griffith and it says that the basic idea was to balance the energy consumed for in forming two new surfaces as the crack propagates against the elastic energy released. So, some balance will be there I will clarify this point you know that when we are applying a tensile load, then the spring that I showed they are extended right they are not broken. So, the energy is stored as elastic energy inside the material.

Now, it has a withstanding capacity, it cannot hold it for long. So, at one point what will happen? This energy will try to release and once it releases then it release in the form of creating two surface the surface energy is balanced with the elastic energy that is released. And this surface creation means that the crack propagates and the material is fractured and getting two surface open. So, that is the whole concept.

So, first we will calculate the strain energy we are taking a parallelepiped kind of sample like this and this is straightforward calculation. So, elastic energy as I have shown from my first curve that it is a triangle half into base into its height. So, you put the value of the Young's modulus here you get this relation.

And the total energy of the parallelepiped which is having a volume V 0 which is subjected to a applied stress that increases. So, first it is its internal energy plus the value of elastic energy into the associated volume. So, elastic energy you put it back half V 0 square sigma app square by Y and this U 0 as I said it is the free energy in the absence of any stress involved in it.

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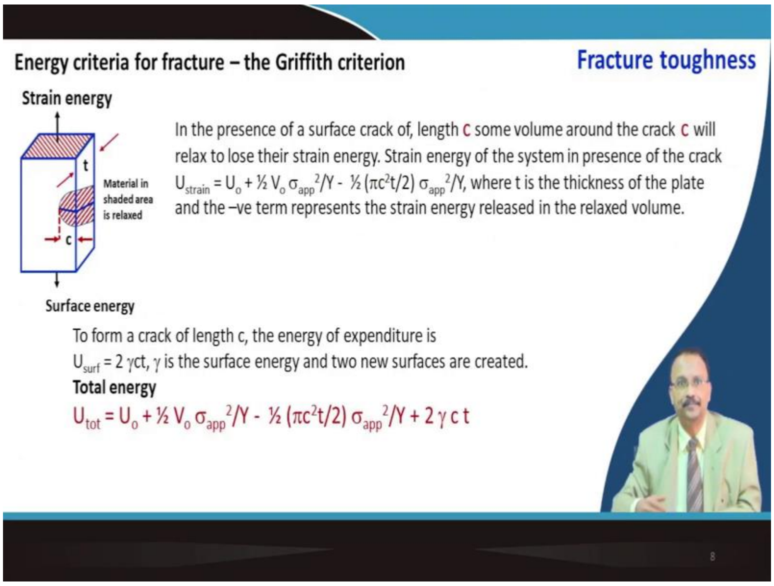

Now, we are talking about the strain energy continuing now you imagine that in that case you introduce a small crack at the surface. So, the presence of the surface crack the length I have assumed it is C and it has some kind of volume around it and it will start to propagate because at one point it cannot withstand that energy. So, the strain energy this part will be there and then depending on the shaded area this volume this will start to release this energy will start to release right.

So, if you have once it releases then there will be opening and it will create two surface. Now you will have to consider the surface energy which is having a positive term. So, to form a crack of length c the energy of expenditure is this much 2 into gamma, gamma is the surface energy c is the half crack length and t is the thickness of this plate.

So, that is giving you the area, area term and gamma into that area; that means, the total energy is now is this elastic energy term and the surface energy term. So, that is giving you the total energy.

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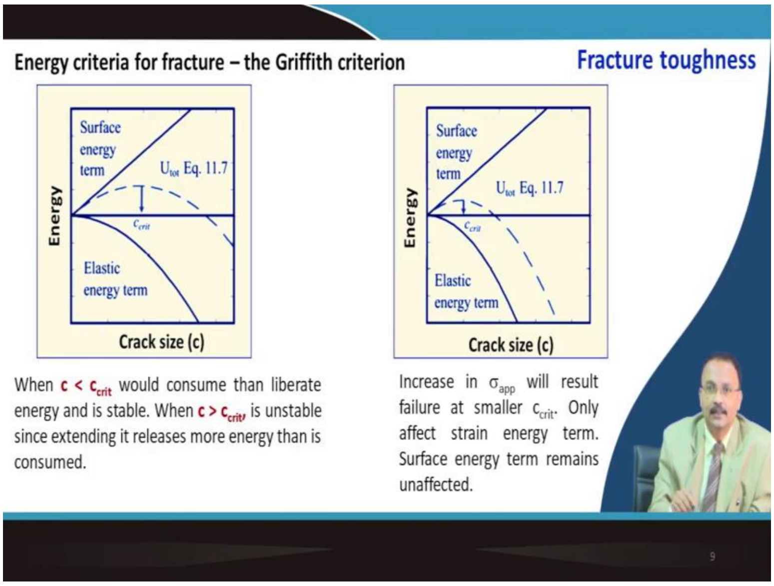

Now, if you plot these two relation one relation which is positive which is the surface energy part and another relation which is the negative part this is the elastic energy term and then you add it up and you get this total energy. And you can see that you get a maxima and corresponding to this maxima this size is the crack length which is a critical crack length.

So, below this critical crack length whatever energy you are putting by applying the stress that will be restored inside the material. But once you cross this critical crack length then it will start to release by creation of two surface and the material will get fractured. So, that is the whole idea.

If you apply more applied stress then the failure will occur at relatively smaller critical length because it is having different types of crack inside the material. If it is a large crack and the small distance it is fine because this will withstand elastic energy will be stored, but if suddenly the applied stress is manifold increased then you will see that this barrier will reduce and this critical length will also be smaller.

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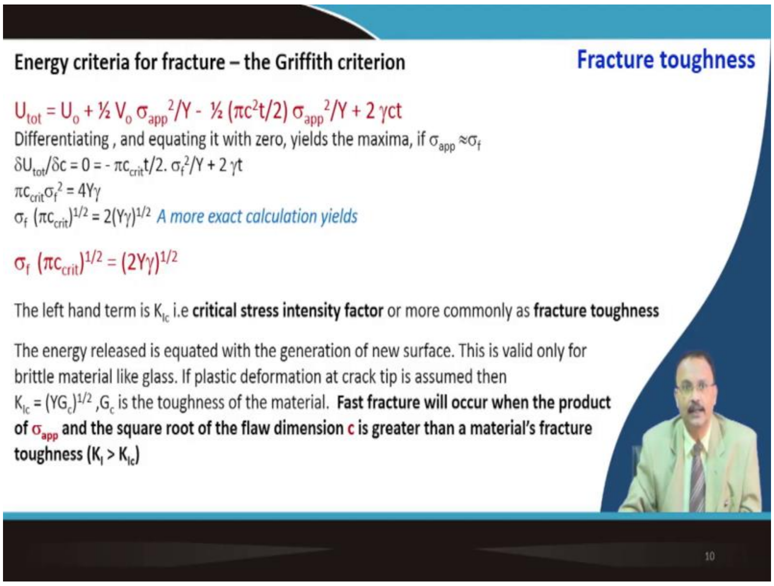

So, the crack dynamics inside the material is also very important to give you a good fracture stress. So, we can have this maxima by differentiating this equation and equating this total energy with the crack length with 0 with standard mathematics and you come up with this relation.

And more exact calculation it gives that this fracture stress and pi is a constant c critical is the critical length beyond which the crack will start to propagate and that is equating with the Young's modulus and the surface energy term the new surface that is created.

So, this left part when the c is c critical, we call it is a critical stress intensity factor or this is known as fracture toughness. So, the energy released is equated with the generation of new surface that I have explained. This is only valid for brittle material like glass, but in normal ceramic material little bit plastic deformation cannot be avoided at the crack tip.

So, if you assume that small plastic deformation occurs, then the relation is slightly modified here you see that this is critical stress intensity factor. The elastic modulus remains here and this value this gamma is change with the toughness of the material.

So, eventually the fast fracture will occur when the product of this applied stress and the core square root of the flow dimension that is greater than the material fracture toughness. So, this critical stress intensity factor if this one is less than the intensity factor which is K 1, then the fracture will be there. So; that means, in a nutshell that when you are applying a stress then if the material is defect free then to some extent this stress material can withstand.

But if there is a crack void etcetera inside the material then this stress intensity at the crack tip they will try to release through the formation of two substrates. So, it is a balance between these two force one is the surface energy creation and another one is the strain energy. And once this stress intensity factor is more than critical stress intensity factor, then the material will fail. So, that is the idea we have derived from Griffith criterion.

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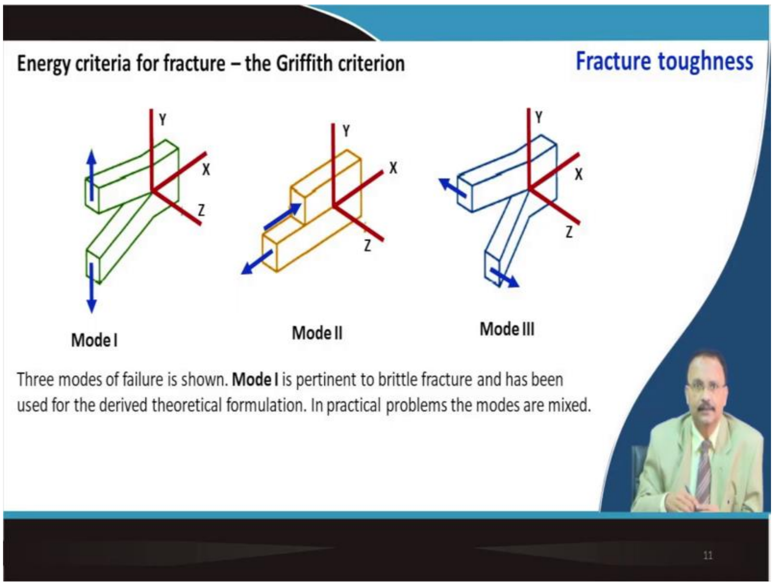

Now, actually we are interested in this type of mode where the crack propagates in perpendicular to the applied stress. Apart from that you have mode two also possible where you are applying a shear stress and the crack is propagating in parallel to this or you can have a shear stress something like that we call it is a tearing kind of force and the crack propagates in this direction.

So, depending on what type of situation you are in we can call it; it is a Mode I or Mode II or Mode III type of fracture. For brittle material we are particularly interested with Mode I type of fracture that therefore, we have explained that this is critical stress intensity factor of Mode I as fracture toughness of this material.

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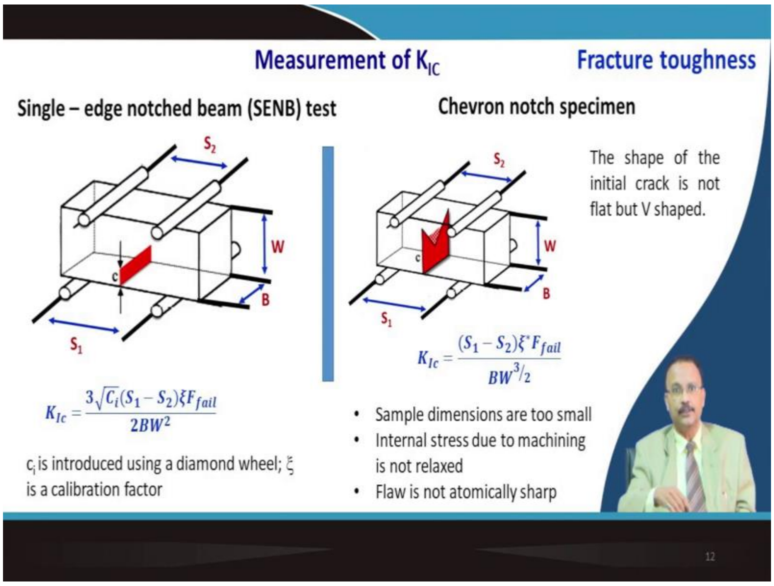

Now, how this fracture toughness you exactly measure? It is measured by a 4-point bending test I already talked about 3 point bending. So, it is something similar to this now this material is well polished this beam and then a small notch of c length.

It is externally from a diamond blade it is introduced here and this is called a single edge notch beam test. So, it is nothing, but a 4-point bending test. So, it will get bent like this and you can estimate from the load that you are giving from the top when it start fracturing when it fractures spontaneously it will fracture at once it is a brittle fracture

So, this $K_{1\ c}$ is given by this relation. So, you know what is the dimension of c i because you yourself has put it through a diamond cutter and all these dimensions are as far ASTM standard S 1 is the span this width of the sample breadth of the sample you put it back and then you get the fracture toughness value.

Slightly different thing is called Chevron notch specimen test where a small kind of a notch a V shaped notch is already put in the material. And then with using this relation you can get the fracture toughness value experimentally verified.

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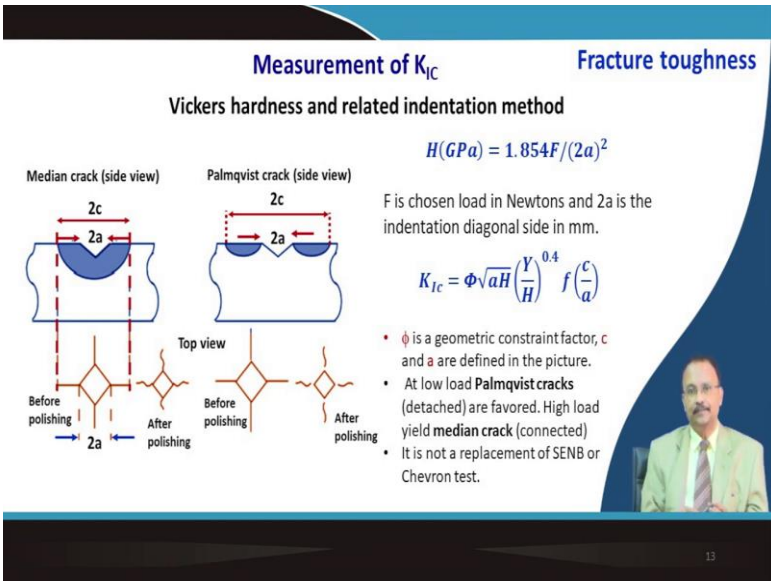

Another way to measure the fracture toughness is through hardness measurement. Already I introduced the hardness part and we talked about the micro hardness. So, hardness is given by these dimensions this 2 a this dimension is given the anvil dimension and this dimension is 2 c.

So, depending on the type of crack that is propagating from this diamond tip whether it is attached to this anvil this impression or it is separated we have two different types of name one is Palmqvist crack and another one is median crack.

So, first you derive the hardness from this micro hardness test and then you just put the hardness value noting these dimensions to get the fracture toughness of a particular material. Here phi is a geometric constant and as I said that just by differentiating these two types of crack dimension in a micro microscope you can identify the type of the crack. And this value is not very reliable as compared to the SENB and Chevron test, but this give you a fairly good idea about the fracture toughness.

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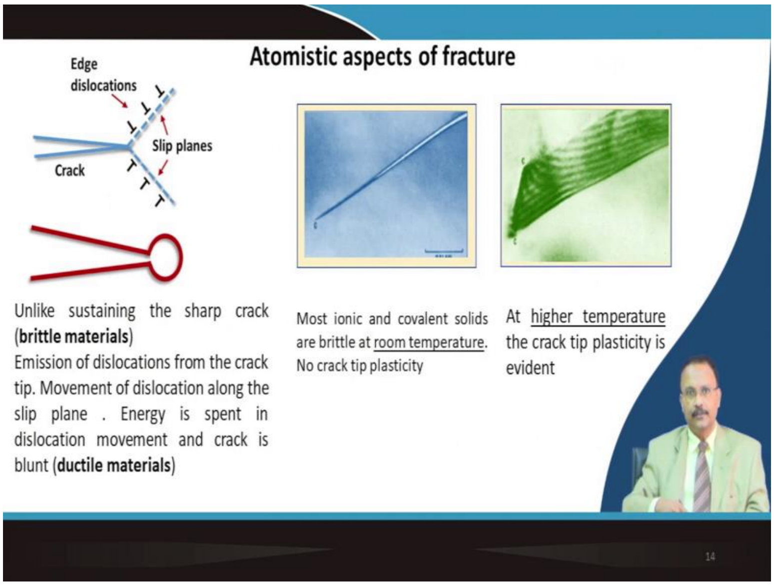

Atomistically, actually the brittle material you do not have this plastic deformation, but there are small plasticity is there. And in case of metal it is very prominent when the crack propensity at the tip is very high then there are lot of dislocation and slip plane available. So, the dislocation starts to travel. So, the part of this energy is spent in the movement of the dislocation. So, the fracture toughness is manifold high in metal as compared to ceramics; ceramics fracture toughness is very small.

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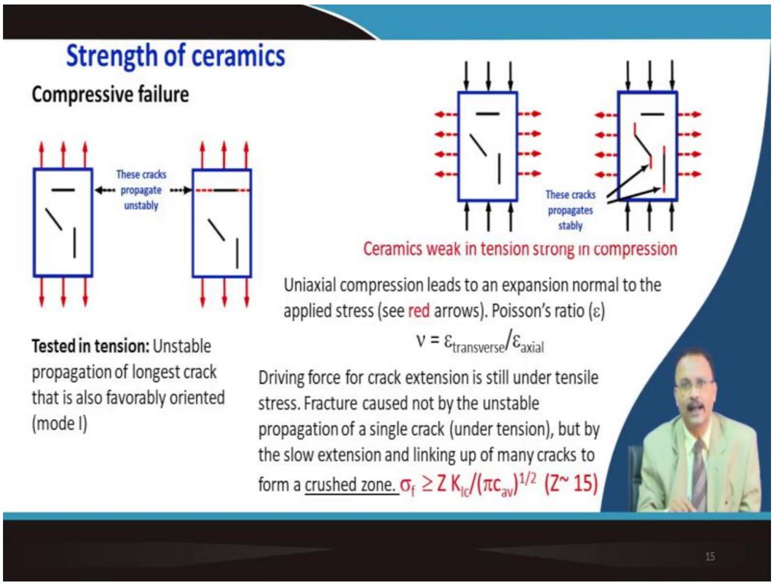

In designing of ceramics we will discuss that having that small fracture toughness how we can manage the situation. Now the strength of the ceramics usually they are very strong in compression because as you know in case of tension your crack if it is favourably oriented like this it will start to propagate right. Because as I told that tensile force it propagates to the this direction.

Now, under compression you see that once you are loading this one actually you have a tensile load in this dimension and when the crack is favourably oriented to this tensile load then only it will propagate.

Usually, this value is little bit higher and much more higher as compared to the normal fracture toughness in the tensile load. So, you will see that this is not a very unstable crack like the tensile loading, but actually this crack will propagate to some extent it will mix with the other crack propagation. So, there is a zone of failure.

So, therefore, when it crashes it is not a catastrophic failure, but to some extent it is catastrophic failure, but it is not as detrimental as the tensile load. So, it is usually strong in compression because of this reason that it is anyway tensile load that is propagating the crack.

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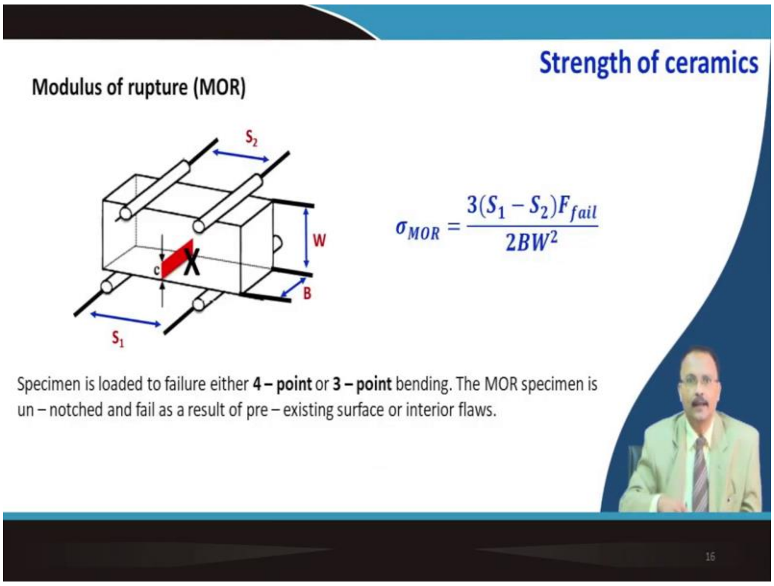

Modulus of rupture is another thing that also I mentioned in my last lecture that it is again 4-point bending test, but now unlike the fracture toughness test you do not have this small crack here. So, modulus of rupture is actually estimated from this relation according to the ASTM specimen and this will give you a fairly good idea of a mixed type of strength that this material that you have.

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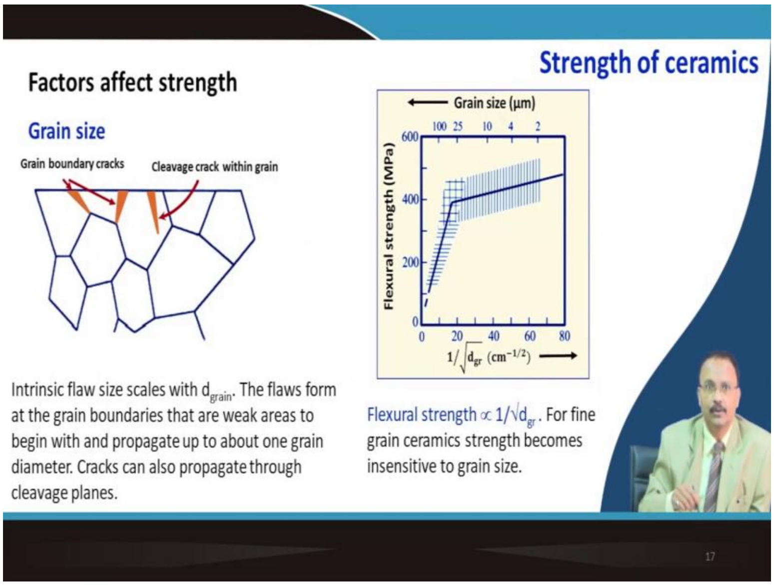

The strength is affected by grain size because of the fact as you can see that in one grain this crack length is defined and actually the crack propagates by this grain boundary. So, you have a relation I have shown you that applied stress is multiplied with your crack length to the power root. So, now, it scales with the grain size.

So, here I have plotted the fracture strength with the grain size and lower the grain size, grain size is higher in this dimension. So, lower the grain size you can see the fracture stress increases because it scales with the crack, but beyond that it is having not much effect on it.

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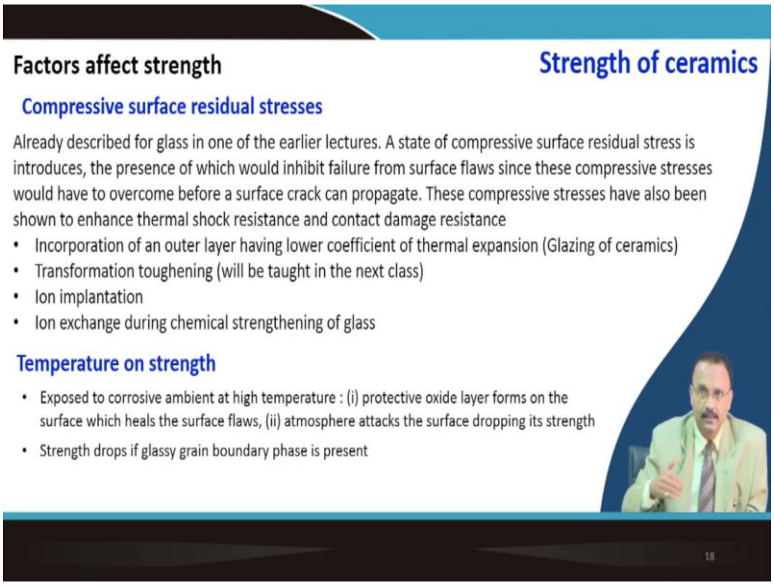

So, compressive surface residual stress I have already described while I was talking with the glass at the surface you can have impart a compressive load and that that if you apply a tensile load, it will have to overcome that compressive stress before the fracture takes place. So, that is one-way that will affect the strength. Temperature on strength it is having an effect and sometimes if the material is in a corrosive environment it forms a some kind of oxide layer which heal part of this crack.

So, it is good in that situation in some situation actually the strength drops down when the atmosphere attacks the ceramic sample and also at higher temperature the grain boundary phase sometimes the glassy phase is there. So, the glassy phase fails because at high temperature it crosses it is dizzy. So, it is a viscous liquid. So, the crack can propagate much faster.

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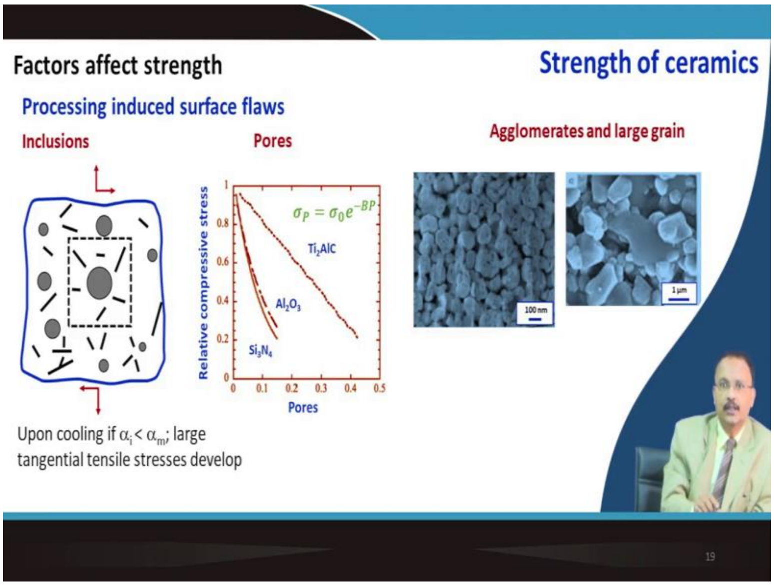

So, processing is important any kind of inclusion if they have different thermal expansion coefficient then there will be micro crack formation which is not good for your material. So, if the pores are there then of course, the strength will be reduced and if there are agglomerates in the material to start with or there is exaggerated grain growth during sintering then the strength will get reduced. This factor I will elaborately discuss when I will talk sintering aspect of ceramic material.

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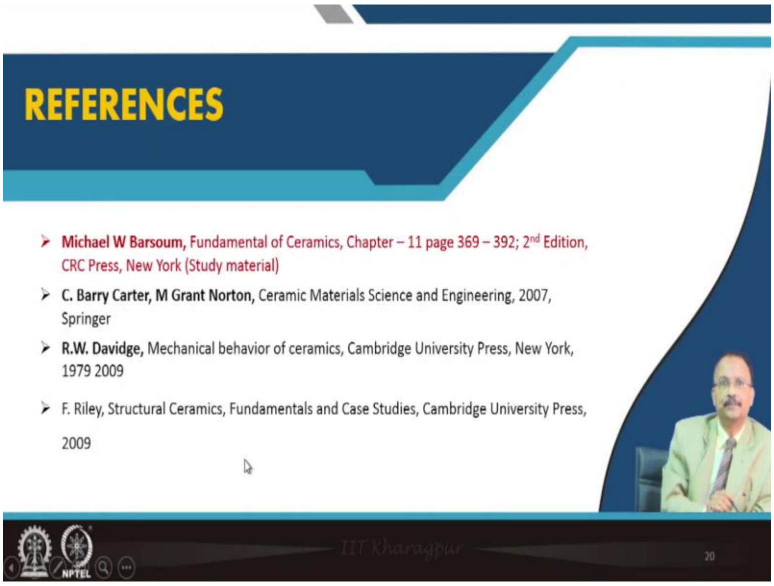

So, the study material is the book by Barsoum and Barry Carter’s book that also you can use to read this concept along with the other thing that other the reference I have talked about.

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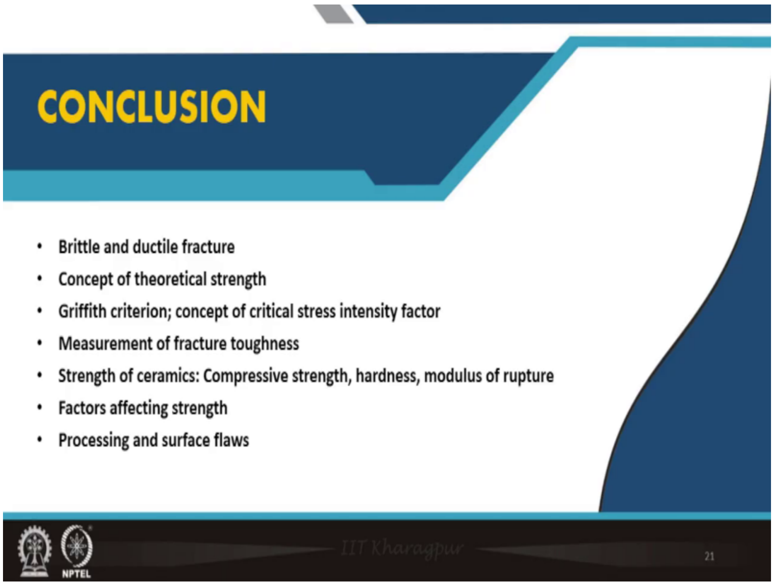

So, I talked about the Brittle and ductile fracture, the concept of the tensile strength was introduced, Griffith criterion was elucidated. Then measurement of the fracture toughness what are the different ways.

Then the strength of the ceramics particularly compressive strength, hardness, modulus of rupture was described. Then factors affecting the strength I have just touched upon I will again come back in my forthcoming lectures to elucidate this concept and processing and surface flow they are important to be considered.

Thank you so, much for your attention.