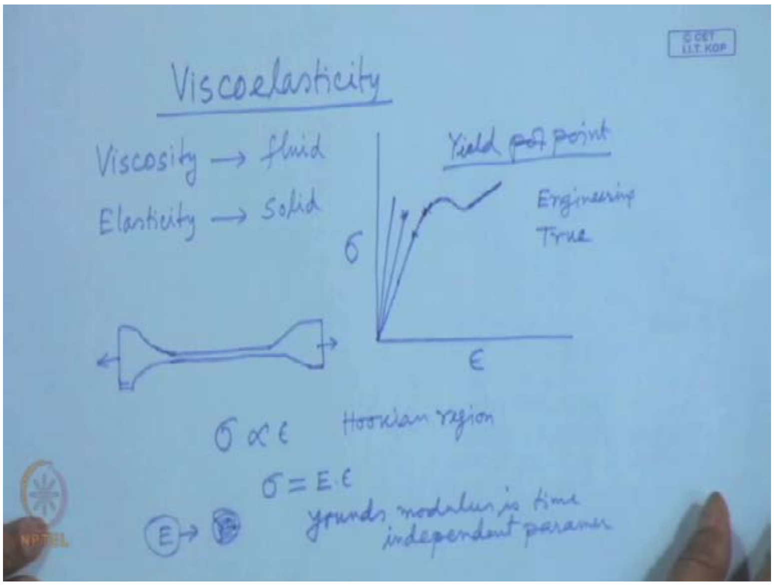
Science and Technology of Polymers
Prof. Basudam Adhikari
Materials Science Centre
Indian Institute of Technology, Kharagpur

Lecture -38
Viscoelasticity

Let us start discussion on another topic that is viscoelastic phenomenon or Viscoelasticity.

(Refer Slide Time: 00:27)



You now that viscosity is a property of fluid and elasticity is a property of solid, if we take a stress temper profile, stress represented by sigma, strength represented by epsilon that means, if you take an example specimen. You have to measure the densile properties of a material, on application of densile load, densile force on the specimen, that keeps a profile in stress strength curve like this.

On application of stress the formation increases and gradually, it process through some stage where, this linear variation of stress it is strength mobs it was non-linearity and it process through some regions come down again it increases or it may fell. So, this is a property of solid on application of stress the material deforms, the deformation can be increase in strength for densile load there can deformation, compressive deformation, if the load is compressive.

Here the stress is proportional to strength up to certain regions, which is known as hooks region hooken regions, means stress is equals to some constant into deformation epsilon. This constant is proportionatelly constant, this is known as the youngs modules, E is the youngs modules youngs modules E is the youngs modulus.

And this youngs modules indicates the elasticity or elastic behaviour or elastic response of the solid, which is time, young modules is a time independent parameter, on release of this stress the specimen reverse back to its original dimension without any permanent change in its dimensional seg.

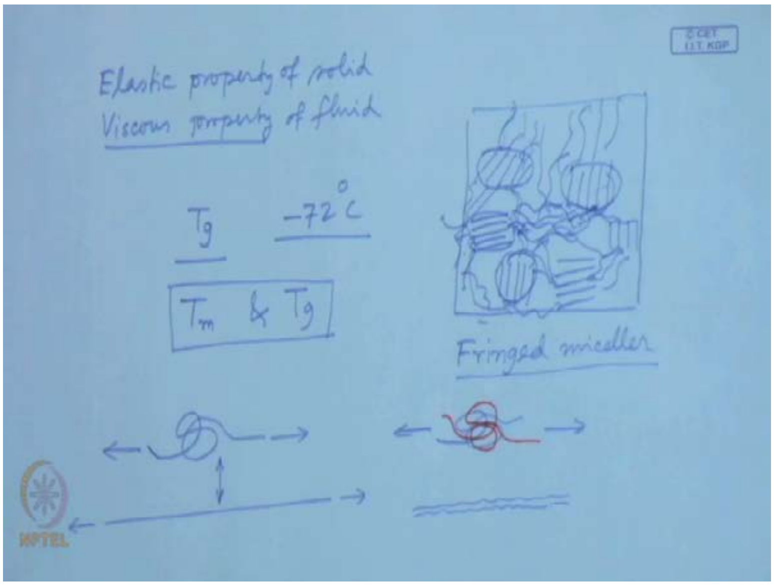
The beyond such point beyond certain limit, it deviates from linearity and deformation occurs in such a way that on release of the stress from any of these points on the stress and curve, it cannot revert back to the original dimension instantaneously and it may contain some residual deformation in it, which is called as permanent seg. So, this is a behaviour of a polymer, polymeric material, it shows the deformation after certain region linearly proportionately and beyond to it changes.

Such change over point is known as yield point, deformation yield point deformation at yield, it shows a plastic deformation. There can be other solids where, the stress curves are increasing the length that will this is perfectly elastic body and we can say, this is a strong material, but it can be brittle also, if the deformation is quite less sleep like this is a characteristic properties of ceramic materials. Metal can behave like this, either it can be necking and it can fail anywhere after necking and in that case this portion is not there.

This is due to this is due to molecular alignment, which is called strain hardly molecule alignment or that is due to decrease of the dimension of this part where, necking occurs and it is a kind of engineering. It can be explained further better with the help of by comparing the engineering stress engineering stress with this its true stress behaviour engineering and true test behaviour.

Anyway what I like to point out that, this elastic region is constantans and proportionate to the stress, this stress is proportionate to the strength and beyond the elastic region it shows plastic deformation of flow. Now, let us explain why this kind of phenomenon occurs in materials like polymers, polymer is called a visco elastic material.

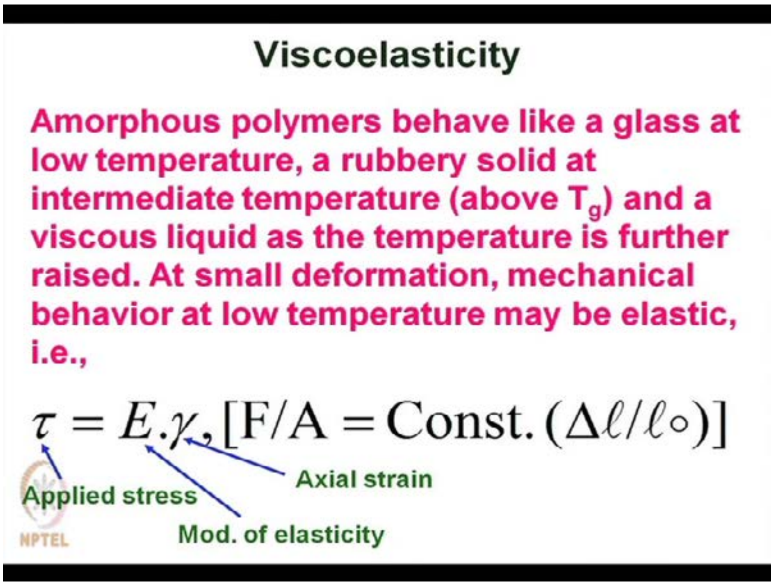
(Refer Slide Time: 07:40)



Because in polymer we find elastic property of solid and viscous property of fluid, due to the presence of viscous property of fluid, we find good processivity in polymer materials. When you compare polymer materials with other ceramic and metals, we talk about this parameter in favour of polymers as good processivity, because of the presence of viscous component in the polymer molecule along with the elastic component. We can get very good strength properties as well as good processivility as compared to metals and ceramics, let us discuss about that.

Now, having the presence of elastic components and viscous components, if there is viscous components, we call that polymer is amorphous. So, amorphous polymers behave like a glass, at low temperature means, below certain temperature say, below glass transition temperature Tg, it is rigid glass like material, it can break like a glass. For example, say a vulcanized or a cross linked boll made of rubber say rubber boll rubber boll, which shows the bouncing property, which is flexible.

(Refer Slide Time: 09:37)



Viscoelasticity

Amorphous polymers behave like a glass at low temperature, a rubbery solid at intermediate temperature (above T_g) and a viscous liquid as the temperature is further raised. At small deformation, mechanical behavior at low temperature may be elastic, i.e.,

$$\tau = E.\gamma, [\mathrm{F/A} = \mathrm{Const.}\,(\Delta\ell/\ell\circ)]$$

And it shows bouncing characteristic, if that rubber boll is emastine liquid nitrogen, it becomes very heard. And if it is dropped from hand on to a rigid floor, it breaks into pieces, set us into pieces, that happen due to the reason that, this polymer the rubber or the ball weight of rubber, that goes below its glass type of temperature. Tg of natural rubber is around minus 72 degree Celsius and liquid nitrogen temperature is much below that shows that at that temperature, it becomes little like a glass.

So, since natural rubber is a amorphous that amorphous rubber boll in the amorphous rubber amorphous rubber in the boll, that becomes glass like and breaks like a glass below this glass type of temperature. And if a rubber is solid at intermediate temperature above Tg and viscous liquid as the temperature is further raised.

So, between Tm melting temperature and Tg at this intermediate temperature, it becomes behaves like a flexible material, soft flexible material and its property can be explained in this fashion. As small deformation at this mechanical behaviour at low temperature of that kind of material may show some kind of elastic behaviour and which can be explained with the help of this equation tau is equals to this tau is equals to, this E into gamma where, tau is applied stress and E is the modulus of elasticity or young modulus and gamma is the axial strain if it strength size stress strain applied on the material. And this stress E can be defined as the load divided by the area, force devided by area.

So, if this applied stress is a by a on the constant, which is the modulus of elasticity and this elongation, which is is the dimension less quantity, the change in the length beyond original length devided by the original length delta l by lgo is the strain this is a dimension less quantity. So, stress is proportionate to the strength up to the elastic limit and which is time intermittent, this is this is the definition of elasticity. So, this component, if polymer is solid this elasticity elastic component is present over there and that component is contributed by a crystalline part of the polymer.

(Refer Slide Time: 12:55)

Viscoelasticity

At intermediate temperature a rubbery solid exhibits the combined mechanical characteristics of these two extremes; the condition is termed viscoelasticity.

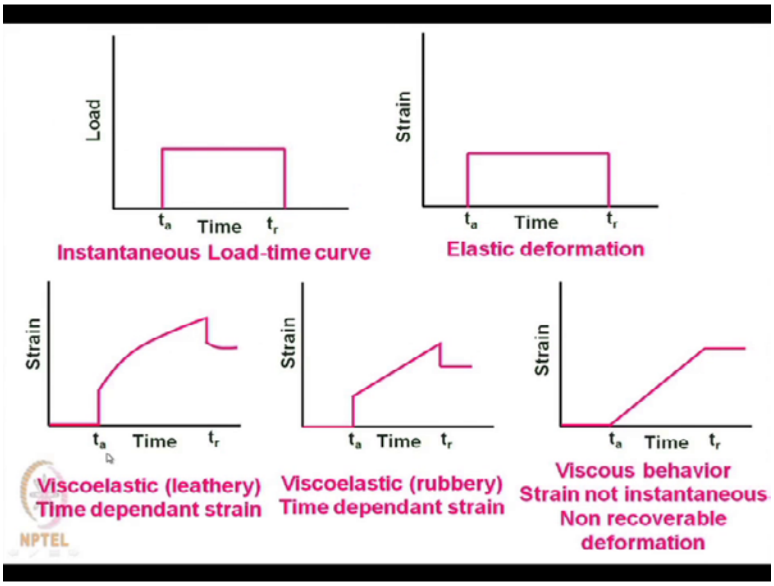
Elastic deformation is instantaneous (independent of time) and is totally recovered.

If the polymer contains amorphous regions, because a polymer may be considered a semi crystalline material having such domains of crystallites dispersed in dispersed in a flexible matrix a flexible matrix like this. So, it can be built like this, I mean to say that a polymer chain can starts from here having flexible linkage. Certain segment of that polymer molecules may forms some crystalline regions or domains that means, a polymer can pass trough flexible region to crystalline region then the flexible region can crystalline region then the flexible region can crystalline region this way.

So, in fact, we can have distribution of some crystallites in an amorphous state, this kind of morphology is known as fringed myscular morphology or frindged morphology concept. Now here, this crystalline behaviour that means, certain segments of the polymer chain enter into form long range order long range order of that segmental part of the polymer chain, which is crystalline connected through amorphous regions. So, that crystalline regions dispersed in amorphous region, constitute as a whole the visco elastic material.

The viscosity property is contributed by the flexible zones and whereas, the elastic property is contributed by the crystalline zones crystalline domains. So, this viscosity viscosity property, elasticity property is contributed by the can be defined and can be explained with the help of hooks law whereas, the amorphous property can be explained with the help of viscosity phenomenon. Now, at intermediate temperature a rubbery solid exhibits the combined mechanical characteristics of the of these two extremes properties of solid and properties of fluids properties of liquids. These such type of conditions is termed as viscoelasticity elasticity, elastic deformation is instantaneous as I mentioned, which is independent of time and is totally recalled.

(Refer Slide Time: 16:38)



Instantaneous Load-time curve

Elastic deformation

Viscoelastic (leathery)
Time dependant strain

Viscoelastic (rubbery)
Time dependant strain

Viscous behavior
Strain not instantaneous
Non recoverable
deformation

Now, let us look into these flow curves, so this is the first one shows an instanteous load time curve, here this represents the time of application of load and this represents the time of release of load. So, on application of this load is applied at this time and load is released at this time and the strength time profile here say, the strength is increased on application of load, that strain is maintained, so long the load is kept over the specimen it is kept on the specimen, then on release of load the strength is recovered.

So, this kind of strain time profile is found in case of elastic body elastic material and this is known as elastic deformation. And there is total recovery of the strength, which was present at the time of application that means the dimension, which was initially there in the body dimension of the body, that dimension is totally recovered on release of the stress, this kind of phenomenon strain time profile is used to explain the elastic deformation behaviour of materials.

Now, if the strength behaves in a different fashion say, this is the application of timer application of load, on application of load, there can be some instantaneous deformation, that means some strain is found, deformation some deformation is found on the body. Then slowly with time, on increase of load strain goes on increasing, now when, that load is removed from such body, what happens, there can be certain recovery of the strength.

Now, here you see this is the instantaneous strain, which is parallel to the strain axis without any time lag, this strain was available then gradually, strain increases with time as the load increases, at on release of the stress. Now, this body is supposed to recover its original dimension; that means, you supposed to come back to this stage, but it is not, but here it is shows some amount of recovery.

Then this is the permanent strength, which remains or permanent deformation, which remained within the body. Why this happens, that can be explained with the help of crystalline and amorphous morphology of polymers or this is a property of polymer having more elasticity property of solid as you see in this case and viscous property of fluid as you see in this case.

So, this kind of deformation is known as plastic deformation and this happens due to molecules sleepage, sleepage of the molecules one over the other during such deformation. And which cannot be regained that means, such molecules sleepage leads to some permanent change in the materials morphology whereas, in case of elastic body that permanent sleepage occurs sleepage occurs, but the permanent sleep age does not occurs.

This can be explained, if you look into this configuration in a simple fashion suppose, this is a molecule having such entangled morphology. Suppose, we consider that decision an example of elastic material, now if we apply some densile stress, what happens, this will be elongated to like a thread, straight thread an application of densile stress.

Now such elongations that means, up to this deformation, it is we can considered that, in case of elastic body, elastic system, this is instantaneous, it does not is not time dependant, now stress is released that means, it can it should it come back to this original configuration. So, this is a reverse of phenomenon on application of stress this entanglement is opened uncoiling on folding can happen and it is straightened on release of the stress it reverse back to this original configuration.

This kind of phenomenon is known as elastic deformation or elastic behaviour. Now, here many other things, may be explained, which is found in case of polymer material, if we find there is some entanglement, intra molecular entanglement, intra molecular coiling as well as, there can be intermolecular entanglement.

So, this one molecule and this is another molecule, so this is, so this shows an example of entanglement of 2 molecules two molecules when such densile load is applied what can happens, in this case this folded segments can be unfolded. During such unfolding what happens sleepage of this segments one over the other occurs, that depends on to the extent of deformation extent of deformation to what extent this deformed to what elongation, it is stressed.

On release of the stress, it may not come back to its original configurations, if there is perment sleepage of say this red molecule over this blue molecule, it cannot come back to this original or initial morphology. So, it cannot be reversible as we have seen in this case it is reversible, in this case it is irreversible, that happens due to permanent sleepage of the molecule as a whole.

Now, here in this case such small deformation to a small extent, can unfold the unfold a folder segments, but if the amplitude of such deformation is less, it can revert back in this case it cannot, that leads to permanent deformation and here is the phenomena, here is the flow curve this strain time profile it leads behind this permanent deformation it cannot come back to the original come back to the 0 strength situation. This kind of material is known as leathery material and which is vescoelasticity.

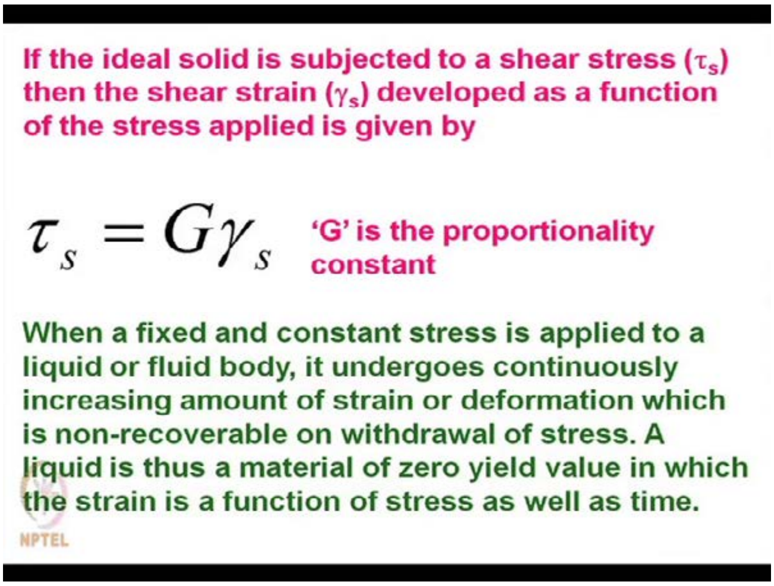
The contribution from the elastic part of the material and this contribution from the fluid part or liquid part of the material or viscous part of the material, so this material is a characteristic of visco elastic material. And find such type of behaviour, in case of leathery material, so this phenomena is known as time dependant strain phenomenon.

There can be another situation, where again it can have this instantaneous deformation on application of stress, here in the later part, this deformation with time can be linear with type, in this case deformation is non-linear. Now this is not a purely fluid behaviour, but due to the presence of this elastic part behaves like non-linear deformation, but here this part shows complete property of viscous plate on application of stress strain increase with time in a linear fashion.

This is a kind of behaviour obtained from rubbery material say viscoelastic rubbery material and this also a case of time dependant strain. If we look into the viscous behaviour or viscosity phenomenon of a fluid say liquid, here the strain is not instantaneous, in this case the instantness strain was present, in this case the instantness strain was present, but in this case no such instantness strain is visible. Here is a purely liquid is a liquid purely liquid without having any properties of solid and it shows the deformation on application of stress and this deformation increases with time and when with that, stress is released this same deformation maintains without any recovery.

In this case you see, due to the presence of the elastic components, there can be certain amount of recovery due to the presence of the elastic components and the strain continues, strain maintained due to the presence of the viscous components. And this is a case of permanent deformation this is a case of permanent deformation and this is case of instantaneous deformation. So, these are these 2 at the cases of viscoelastic material, one is leathery type, one is rubbery type, in this case it is a purely a viscous material purely liquid material, which shows non recoverable deformation.

(Refer Slide Time: 28:23)



If the ideal solid is subjected to a shear stress (τ_s) then the shear strain (γ_s) developed as a function of the stress applied is given by

$$\tau_s = G\gamma_s$$

'G' is the proportionality constant

When a fixed and constant stress is applied to a liquid or fluid body, it undergoes continuously increasing amount of strain or deformation which is non-recoverable on withdrawal of stress. A liquid is thus a material of zero yield value in which the strain is a function of stress as well as time.

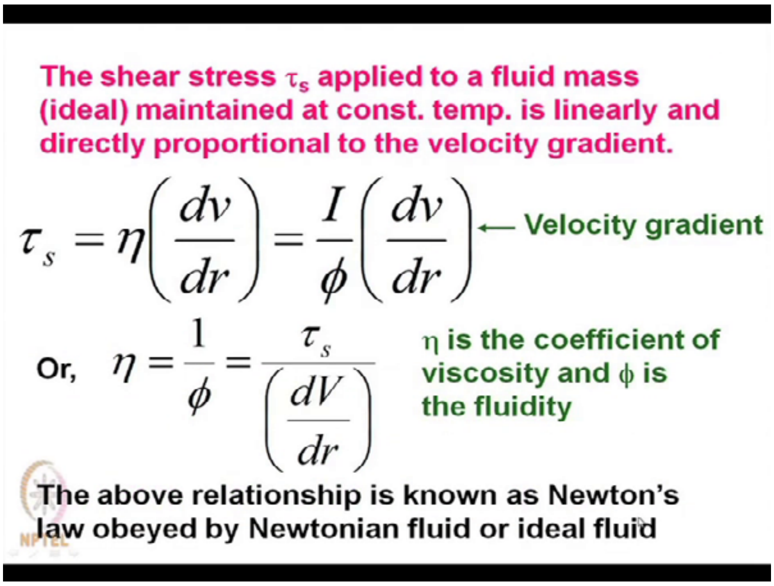
So, if a solid is ideal and is that solid ideal solid is subjected to a shear stress, now s then the shear strain gamma s, developed due to such shear stress application, as a function of the stress applied that can be represented by tau s is equals to G gamma s. Shear stress is looks like this suppose we can have a track of playing card track, we can place the pack of playing card on this board on the table, suppose this is a track of playing card, you put your finger fingers like this like this and apply some stress in this direction.

So, this way if you do, what will happen the top most card will move the further distance or maximum distance and the card below that, will move little less distance, than that of the top most distance cards. So, that is a kind of deformation, if you considered a material of some imaginary layers, the top most layers to which if the stress is applied shear stress is applied in this way, that shear stress helps in some that causes some deformations on the top most layer and the amount of deformation on this layer below that.

This happens due to the fractional force and the resistance offered by the bottom layer on the top most layer, that kind of deformation is known as shear deformation and the load the kind of load, which is applied stress applied on that body is known as the shear stress. So, this shear stress and shear strain, that is also a proportional and the proportionately constant G cr module or the modulus of rigidity.

When a fixed and constant stress is applied to a liquid or fluid body, it undergoes continuously increasing amount of strain or deformation, which is non recoverable on withdrawal of stress, as I have explained just before this. A liquid is thus a material of 0 yield value whereas, a solid shows a finite yield value, yield means deformation value in which strain is a function of stress as well as time.

(Refer Slide Time: 31:19)



The shear stress τ_s applied to a fluid mass (ideal) maintained at const. temp. is linearly and directly proportional to the velocity gradient.

$$\tau_s = \eta\left(\frac{dv}{dr}\right) = \frac{I}{\phi}\left(\frac{dv}{dr}\right) \longleftarrow \text{Velocity gradient}$$

Or, $$\eta = \frac{1}{\phi} = \frac{\tau_s}{\left(\frac{dV}{dr}\right)}$$

η is the coefficient of viscosity and ϕ is the fluidity

The above relationship is known as Newton's law obeyed by Newtonian fluid or ideal fluid

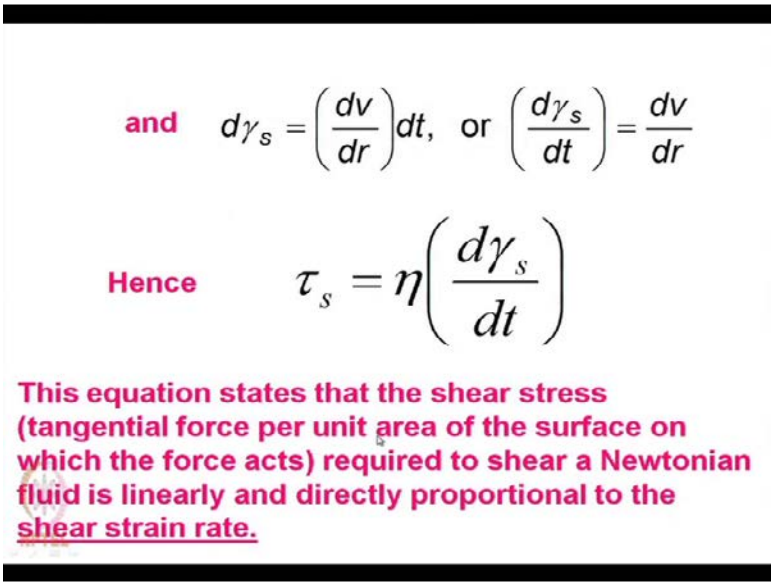
So, this shear stress tau s applied to a fluid mask, ideal fluid mask, that maintained at constant temperature is linearly and directly proportionate to the velocity gradient. In case of a fluid mask, if some shear stress is applied, that is proportionate to the velocity gradient. As you have seen, in case of that shear stress versus shear strength the proportionity of shear stress versus strength and the proportionity constant for shear modulus, in case of pure ideal fluid, if some shear stress is applied it is proportion to velocity gradient dv dr.

That means, the velocity gradient, the velocity profile, in case of a fluid flowing in a pipe can be also explained where, the velocity of the layer at the centre velocity of the layer at the centre of the pipe is maximum whereas, the velocity at the edge is adjacent to the valve of the pipe is minimum. So, that gives a concept of velocity gradient and the proportionately constant in this case is the viscosity eta is dignated by eta is a coefficient viscosity.

And coefficient viscosity is again related to the another parameter, which is pie is the reciprocal of viscosity or the vice versa. So, eta is 1 by 1 upon pie and that is equals to tau s pie dv by dr and eta is coefficient of viscosity pie is the fluidity. They have a known as Newtons law obeyed by Newtons fluid from this you understand that an ideal fluid or ideal liquid, which follows or obeys, this Newton law where, the cs stress is directly proportionate to the velocity gradient.

And the proportionately constant is the coefficients viscosity, that kind of fluid mass is known as the ideal fluid or Newtonian fluid.

(Refer Slide Time: 33:50)



and $d\gamma_s = \left(\frac{dv}{dr}\right)dt$, or $\left(\frac{d\gamma_s}{dt}\right) = \frac{dv}{dr}$

Hence $\tau_s = \eta\left(\frac{d\gamma_s}{dt}\right)$

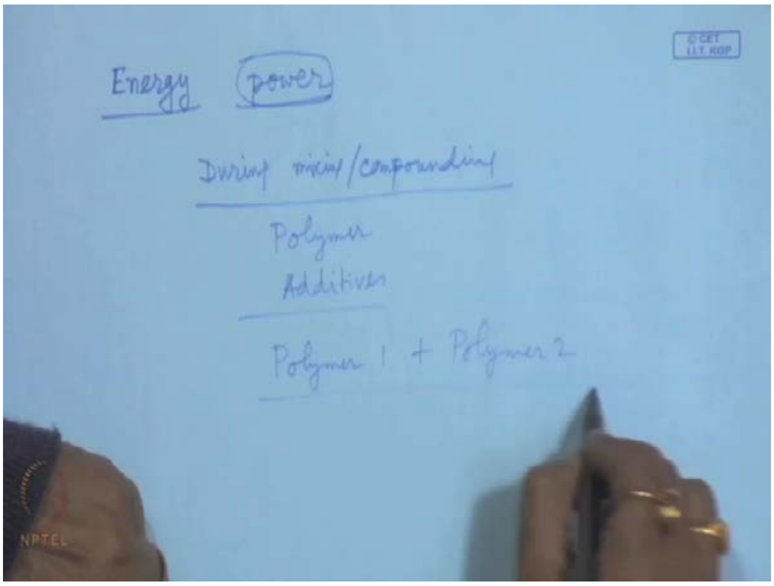
This equation states that the shear stress (tangential force per unit area of the surface on which the force acts) required to shear a Newtonian fluid is linearly and directly proportional to the shear strain rate.

Also there is a that this d gamma s d gamma s shear strength is equals to velocity gradient dividity, we can also write this way, shear stress rate, we can have say shear strength rate becomes having can be is equals to dv by dr velocity gradient. So, we can replace the this shear strength rate with the velocity gradient dv dr, if we replace that fellow in that in that equation of flow equation of newtonian fluid here dv dr, if we represents the dv dr with this t gamma dt, the equation becomes tau is equals to eta into dt.

This equation states, that the shear stress, which is the tangential force per unit area of the surface on which the force acts, that shear stress is required to shear a Newtonian fluid and that is linearly and directly portion to the shear strain rate. So, how do we, I have got that shear stress is directly proportional to the shear strain rate that means, if we want to increase the deformation, rate of deformation rate of deformation of the fluid.

So, we have to increase the shear stress apply the shear stress to increase the stress proportionate proportionally, otherwise we cannot make, that material deformed to a particular extend what we recur. This equation is very must useful or can be used to explain the behaviour of the polymers.

(Refer Slide Time: 36:21)



Energy (power)

During mixing/compounding

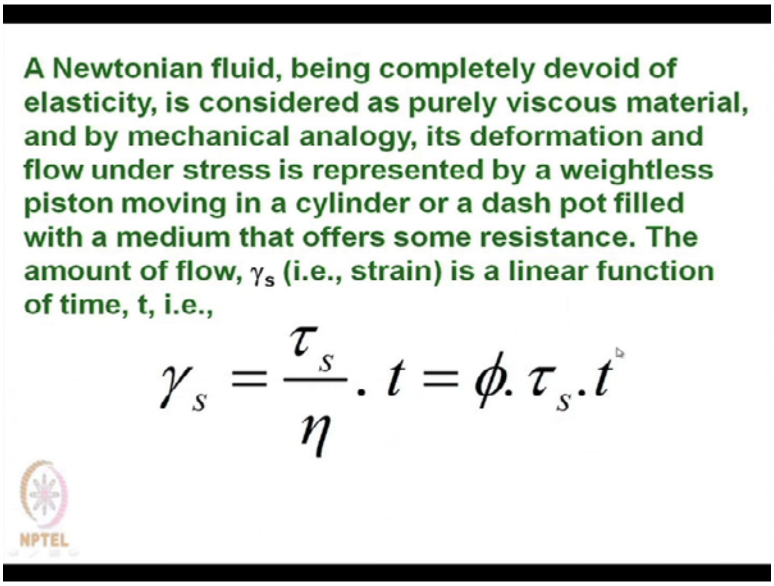
Polymer
Additives

Polymer 1 + Polymer 2

During processing and fabrication during compounding, that means one can have estimate of energy or mixing power mixing power power of the mixing machine in power of the motor. That is related to this tau s equal to eta into the stress d gamma s dt, that is how much shear stress is required to have proper flow characteristic proper fluidity of the material say during mixing or compounding of a polymer or formulation having polymer and different additives.

As well as in case of making blend up polymer 1 with polymer 2 etcetera etcetera, how much shear stress is required to obtain a proper fluidity. So, that proper mixing is there, proper compound is formed properly these additives, functionalities during compounding how their disburse or to make a uniform dispersion of the additives to make a proper mixing of the components to have a comfortable or miscible blend. This information is necessary and that can be available that means, how much stress to be applied that can be calculated with the help of this equation.

(Refer Slide Time: 38:03)

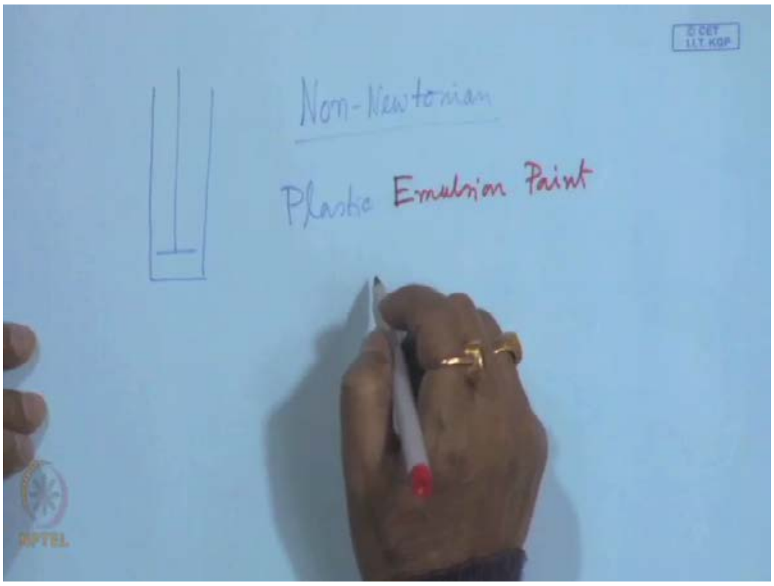


A Newtonian fluid, being completely devoid of elasticity, is considered as purely viscous material, and by mechanical analogy, its deformation and flow under stress is represented by a weightless piston moving in a cylinder or a dash pot filled with a medium that offers some resistance. The amount of flow, γ_s (i.e., strain) is a linear function of time, t, i.e.,

$$\gamma_s = \frac{\tau_s}{\eta}.\, t = \phi.\, \tau_s.t$$

So, a Newtonian fluid being completely devoid of elasticity, which is which shows a continues deformation as we have seen in this case this diagram this diagram, viscosity behaviour of a ideal liquid, this contains linear increase of strain with time. So, that Newtonian fluid being completely devoid of elasticity without having any component of solid, that is considered as purely viscous material. And by mechanical analogy its deformation and flow under stress is represented by weightless piston moving in a cylinder or a dash pot filled with a medium that offers some resistance.

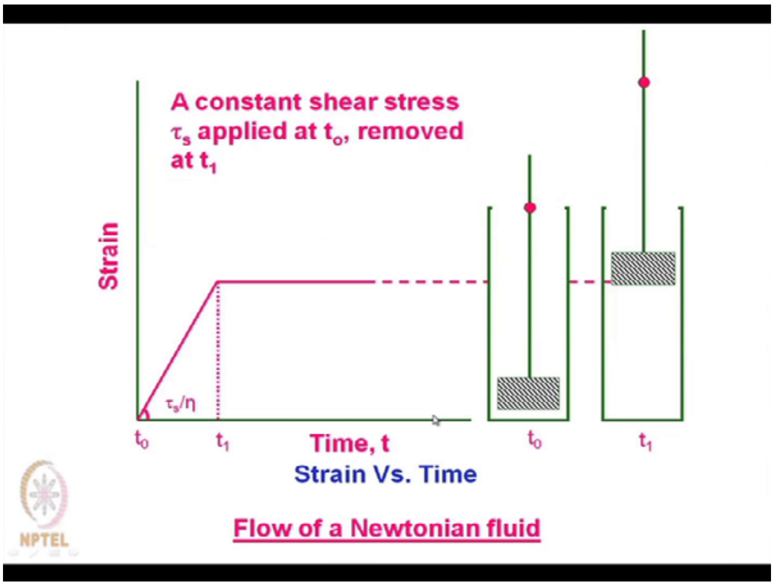
(Refer Slide Time: 39:07)



The amount of flow says, it can be viewed like this is a cylinder and dash pot, this is piston like this. So, is this piston is workless this piston workless piston moving in a cylinder or dash pot, this is the dashpot, this is the piston with a medium that offers some resistance.

So, this movement of this piston with this cylinder or dash pot can be compared with the flow behaviour of a fluid. The amount of flow gamma s, that is the strain is a linear function of time, say gamma s is radial linear of function of time. So, gamma is equal to tau s by eta into t and tau s or tau s in to 5 into t.

(Refer Slide Time: 40:14)



A constant shear stress τ_s applied at t_o, removed at t_1

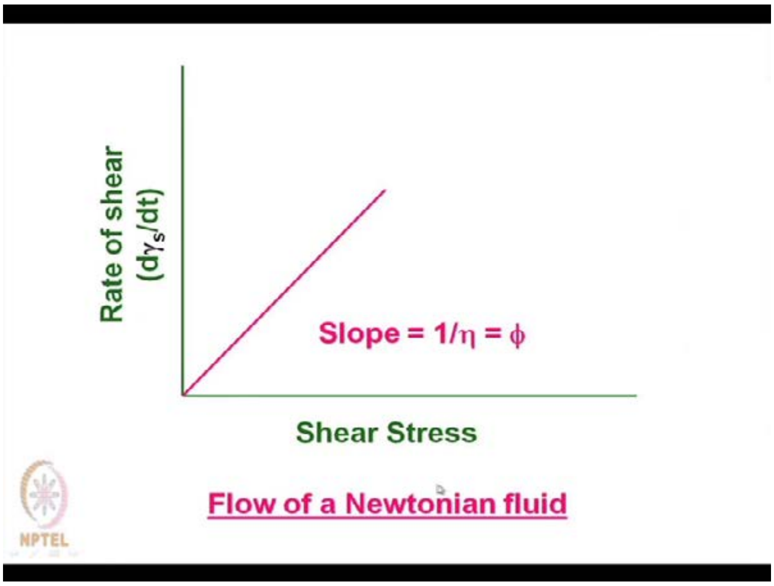
Strain Vs. Time

Flow of a Newtonian fluid

This diagram shows the flow card, flow of a Newtonian fluid with the help of dash pot piston despot model, sketch show a where you see constant shear stress tau s, tau s is applied at t 0. On application of stress deformation increases linearly and at time t 1, that tau s stress tau s is removed, how it can, we visualize with the help of piston and this cylinder. Here this is the application of where the piston remains at the bottom, it is moved top at time t 1, it was at t 0 t, it was at the bottom at time t 1 it is moved up.

Now here by doing, so this piston will get some resistance from this wall of the cylinder, for which that stress tau s is equal to moved top from the bottom from the bottom to take it up that is stress tau s is required. And the slope that means, the ratio of tau s eta viscocity is the slow. So, this is a flow card for Newtonian fluid or is strain process type profile.

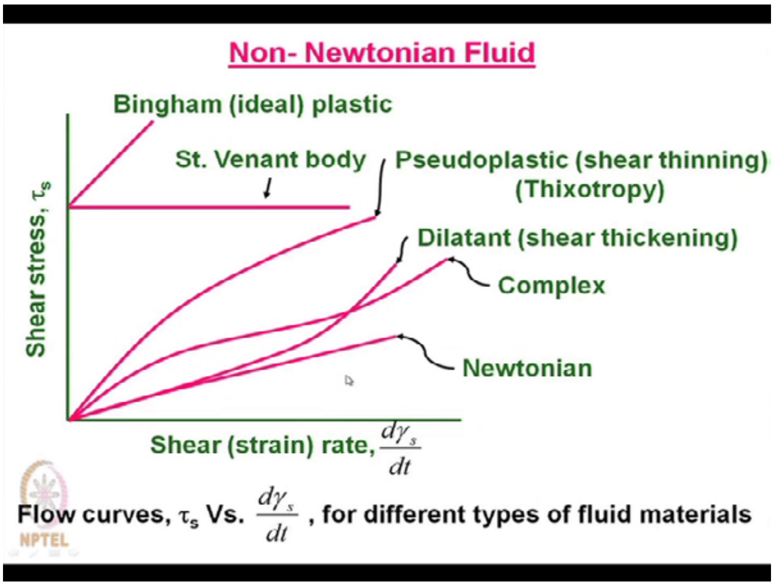
(Refer Slide Time: 41:54)



Flow of a Newtonian fluid

And, if we make a plot of rate of shear stress rate of shear rate of shear d gamma s dt rate of shear is rate of shear d gamma s dt deformation rate, rate of deformation we can say and this is shear stress. So, shear strain rate, we can also say shear strain rate versus shear stress card, these also linear in case of a non Newtonian fluid, a Newtonian fluid. So, we see the, we see that, this is the a Newtonian fluid or an ideal fluid.

(Refer Slide Time: 42:44)


Non- Newtonian Fluid

Flow curves, τ_s Vs. $\frac{d\gamma_s}{dt}$, for different types of fluid materials

Now, our polymer materials are Nonnewtonian materials or non Newtonian fluid. So, the peculiar behaviour where, now here if we make plot of shear stress versus shear strain rate. Now, ideal plastic bodies ideal plastic bodies known as Bingham plastic bodies, if we explain the behaviour such kind of plastic material, what happens on application of stress, there is some instantness your deformation, you can say that means, this much of stress, it can bear without any application of these this strain.

So, shear strain at the 0 shear strain at the 0 shear strain rate shear strain rate, this is the stage value it can and bound, which it requires increase in stress value that means, the variation of shear stress with the shear rate, increases linearly. That variation occurs at much higher shear stress, that variation of shear stress shear strain rate occurs for Bingham plastic occurs at much higher stress after, which it becomes linear.

There can be other materials, which are termed as state venant body state venant body, which does not show any increase of that, does not require show any increase of shear strain rate shear strain rate remains constant at any particular shear stress. These are the 2 cases and these are some real situations, which we object with polymer materials.

Now, if you look at this curve, which is concave downward, this sido plastic behaviour, sido plastic means, it behaves like a plastic, but on application of shear strain shear stress its viscosity decreases. We can bring an analogy to the sand bead on seashore on seashore, we find there is sand bead, which apparently looks like dry sand bead. But, on application of throbbing force with the help of your foot will find, that slowly water is coming out from the sand bead, although it appears that there is no water.

Where from that water is come now, there is properties of sand and water and that water molecules form an envelope or coating over sand particle, that that becomes set that water envelop just remains tightly bound over each and every sand particle, when it is under static situation, when there is no lot. But, on application of pulsating load what happens those envelop what envelop layers breaks and water molecules kept released from sand particle water comes out and the sand particle appears to be little freely, that some water molecules, water comes out it appears to be fleet.

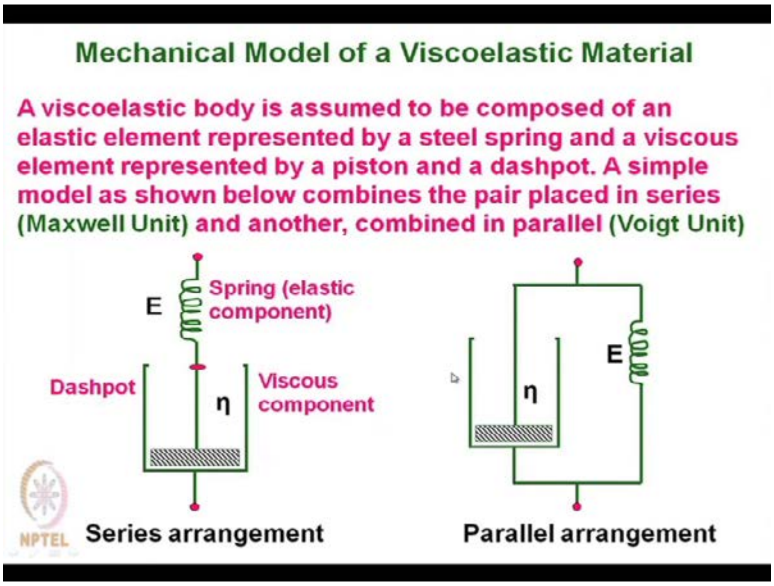
So, that kind of behaviour is known as thixotropy, it is a kind of shear thinning phenomena, shear thinning phenomena means on application of stress on application of stress what happens this this viscosity is reduced, viscosity decreases. The decrease of viscosity on application of stress that means, on or increase of shear strain rate increase in shear sorry, shear strain on application of shear stress gradually, the requirement of stress, shear stress decreases gradually, with the increase of shear strain rate.

So, that kind of behaviour is known as thixotropy, this is also this phenomenon is also visualize in case of plastic emulsion paint, in which some pigment particles are disbursed in a vehicle, that vehicle is made of some polymer water soluble polymer water and some other organic solvents. There pigments are disbursed and when it is kept static condition in a container, that remains almost in a solid like mask, if you open the lid and just make the ups and down, the bend does not go down, that is due to the settling of those pigment particles by this vehicle molecules are due to water molecules.

Now, on application of stress, what happens it becomes thin, it becomes the viscosity is reduced, viscosity decreases and then with the help of glass reduce the paint over wall. The opposite behaviour is found in case of dilatant material, which is known as shear thickening that means, on application, if you increase the studding, rate of studding, thick as it is called as gradually increase of so, increase of viscosity with shearing is known as dilatant properties decrease of viscosity shear is known as thixotrophy properties.

And one there can be material showing both the behaviours of thixotrophy as well as dilitant and also known as complex and if it is purely linear, that is a Newtonian fluid.

(Refer Slide Time: 49:55)



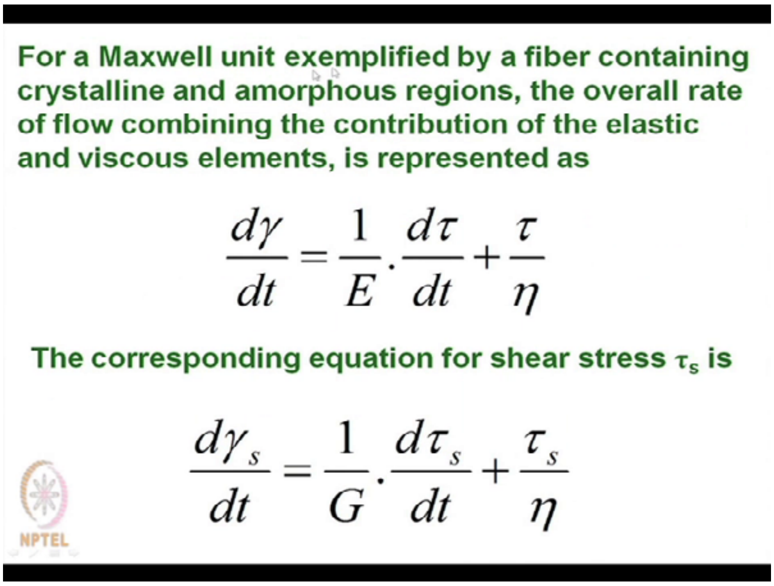
Mechanical Model of a Viscoelastic Material

A viscoelastic body is assumed to be composed of an elastic element represented by a steel spring and a viscous element represented by a piston and a dashpot. A simple model as shown below combines the pair placed in series (Maxwell Unit) and another, combined in parallel (Voigt Unit)

And that viscoelastic phenomena can be explained with the help of mechanical products one is Maxwell model and another wire model. This is the Maxwell model where, that elastic component is represented by string viscous components is represented by dashpot. So, on application of stress that means, is on stress is applied, what will happen deformation will occur, in this a part, this viscous part and that can be accommodated by that that can be accommodated by this spring the deformation of that deformation can be accommodated by this spring.

And this is in this serial connection this is parallel connection, so a viscous bodies is assumed to be composed of an elastic element represented by a steel spring and viscous element represented by a piston and dashpot. A simple model is shown below, which combines the pair, placed in series Maxwell unit, this case another combined parallel arrangement unit.

(Refer Slide Time: 51:10)



For a Maxwell unit exemplified by a fiber containing crystalline and amorphous regions, the overall rate of flow combining the contribution of the elastic and viscous elements, is represented as

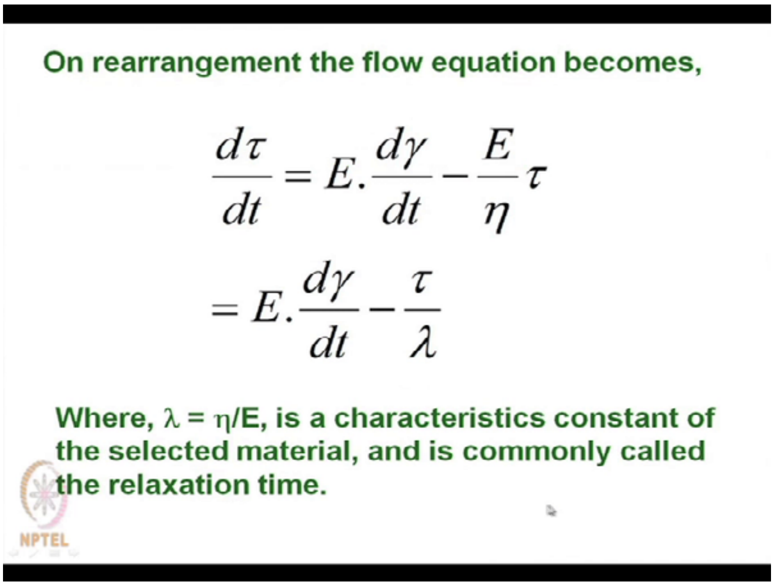
$$\frac{d\gamma}{dt} = \frac{1}{E} \cdot \frac{d\tau}{dt} + \frac{\tau}{\eta}$$

The corresponding equation for shear stress τ_s is

$$\frac{d\gamma_s}{dt} = \frac{1}{G} \cdot \frac{d\tau_s}{dt} + \frac{\tau_s}{\eta}$$

For a Maxwell unit exemplified by a fiber containing crystalline and amorphous regions, the overall rate of flow the overall rate of flow combining the contribution of the elastic viscous elements is represented by this equation. This is the this is the a elastic element, this is the viscous element and the corresponding equation for shear stress tau s is given by this d gamma s dt is equal to 1 by G in to d tau s dt plus tau s by eta.

(Refer Slide Time: 51:41)



On rearrangement the flow equation becomes,

$$\frac{d\tau}{dt} = E.\frac{d\gamma}{dt} - \frac{E}{\eta}\tau$$

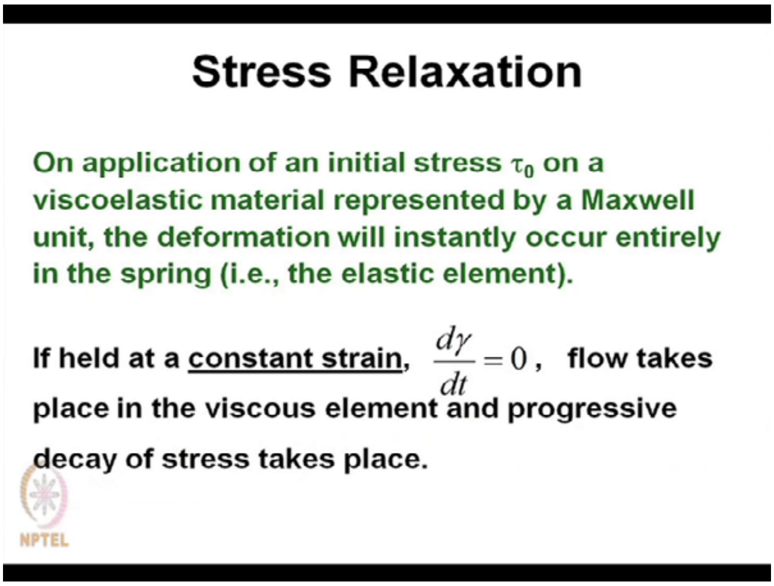
$$= E.\frac{d\gamma}{dt} - \frac{\tau}{\lambda}$$

Where, $\lambda = \eta/E$, is a characteristics constant of the selected material, and is commonly called the relaxation time.

On the rearrangement of that flow equation becomes d tau dt is equal to E in to d gamma dt minus E by in to eta tau s by and that equal to E into d gamma dt minus tau by eta tau by lambda. Now, here a new parameter has come this lambda, which equal to eta by E is characteristics of constant of these selected material and that is known as relaxation time relaxation time, that is some load is applied, then it is released before coming to it is original configuration, it should get some minimum time.

So, minimum time should be allowed to the slowly, that segment is relaxed relaxed and it compact with tries to come back to original confirmation.

(Refer Slide Time: 52:31)



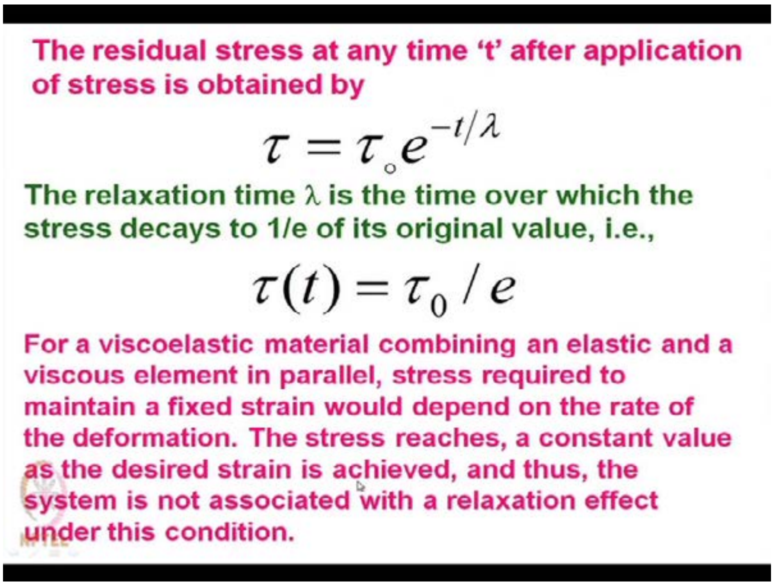
Stress Relaxation

On application of an initial stress τ_0 on a viscoelastic material represented by a Maxwell unit, the deformation will instantly occur entirely in the spring (i.e., the elastic element).

If held at a constant strain, $\frac{d\gamma}{dt} = 0$, flow takes place in the viscous element and progressive decay of stress takes place.

That is known as stress relaxation, now this stress relaxation on application of an initial stress tau 0, on viscoelastic material represented by a Maxwell unit, as I shown the series and the combination of the dash pot deformation will instantly occur entirely in the spring. Deformation occur in in case of entirely on a spring, that is elastic deformation, if held at a constant strain, the this constant strain d gamma dt would be equal to 0 and flow take place in viscous element only and progressive decay of stress take place, that progressive decay is called relaxation.

(Refer Slide Time: 53:17)



The residual stress at any time 't' after application of stress is obtained by

$$\tau = \tau_{\circ} e^{-t/\lambda}$$

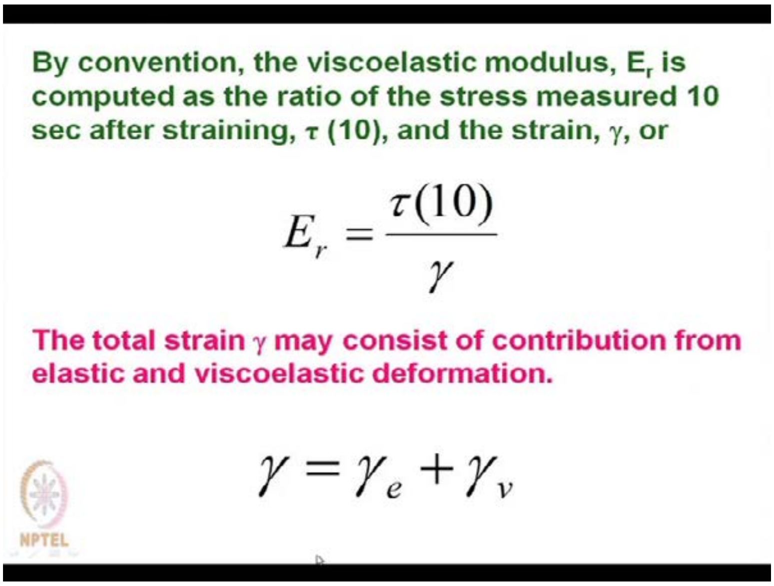
The relaxation time λ is the time over which the stress decays to 1/e of its original value, i.e.,

$$\tau(t) = \tau_0 / e$$

For a viscoelastic material combining an elastic and a viscous element in parallel, stress required to maintain a fixed strain would depend on the rate of the deformation. The stress reaches, a constant value as the desired strain is achieved, and thus, the system is not associated with a relaxation effect under this condition.

The residual stress at any time t, after application of stress is obtained by opted from this equation tau is equal to tau 0 in to t minus ty lambda, this relaxation time lambda is the time, over which the stress decays to 1 by E of its original value, that is tau t is equal to tau 0 by E. For a viscoelastic material combining an elastic and viscous element in parallel, now the stress required to maintain a fixed strain would depend on the rate of the deformation, the stress reaches a constant value as the desired strain is achieved and thus, the system is not associated with a relaxation effect under this condition.

(Refer Slide Time: 54:14)



By convention, the viscoelastic modulus, E_r is computed as the ratio of the stress measured 10 sec after straining, τ (10), and the strain, γ, or

$$E_r = \frac{\tau(10)}{\gamma}$$

The total strain γ may consist of contribution from elastic and viscoelastic deformation.

$$\gamma = \gamma_e + \gamma_v$$

By convention the viscoelastic modulus Er can be computed as the ratio of the stress measured at 10 seconds after straining, designated as tau 10 and the strain gamma. So, Er becomes is equal to is equal to tau 10 by gamma, the total strain gamma consist of contribution from elastic and viscoelastic deformation. So, this elastic elastic gamma elastic component and gamma v from the viscous component, so this so, both elastic and viscous components contributes to the total strain of the body curve.