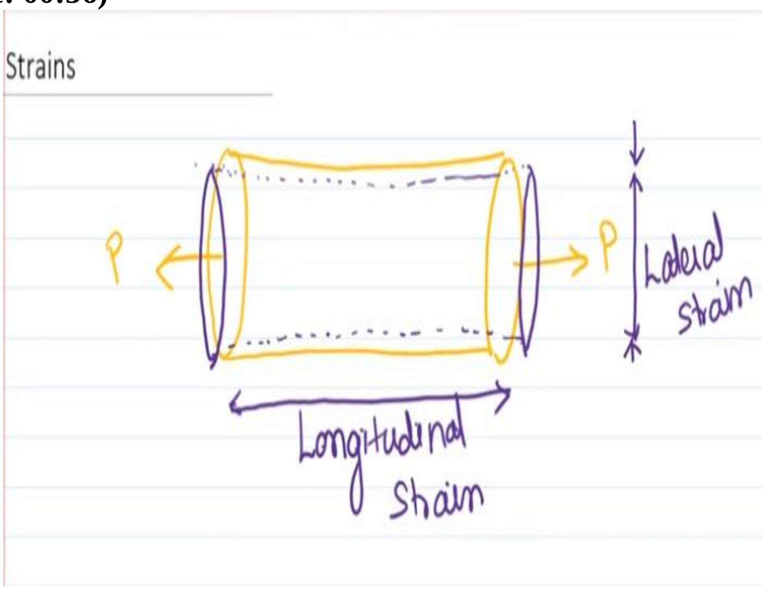
Mechanical Behaviour of Materials (Part –I)
Prof. Shashank Shekhar
Department of Materials Science and Engineering
Indian Institute of Technology - Kanpur

Module No # 02
Lecture No # 06
Stress-Strain Relation

Good morning students so today we will be discussing another parameter which is strains so strains as you would see the definition that we give you here that it would be valid for the elastic as well as the plastic condition. So strains like the stresses are also a tensor quantity as you would see you may not have used it as a tensile quantity so far. But from now on we will see that it is also tensor quantity so let us begin with our understanding of this content.

(Refer Slide Time: 00:56)



So if you have a cylinder let us say and you apply a force on it and pull it out then the cylinder gets elongated and in the process there is some strain. So how does the strain look like so let me draw it here it is not difficult to draw. So let us say this is a cylinder and although it looks like it is not parallel but the sides are parallel and then you are applying some force which will convert to stress.

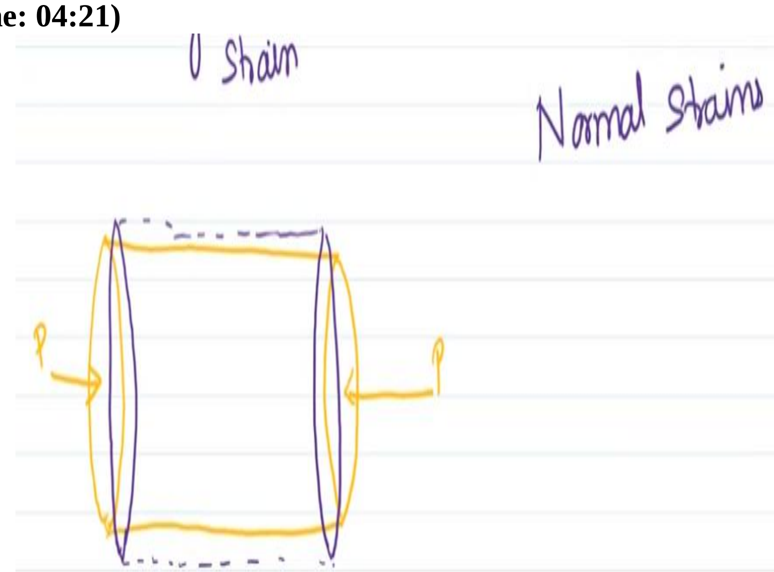
So let us say this is the force acting on the 2 sides so it is a uniaxial stress condition uniaxial loading condition. And we are intentionally making keeping things simpler now after you have exerted some force on it. Then now I will again exaggerate a little bit there will be elongation in this cylinder and again keep in mind there is a cylinder although it may not look like one.

But assume that it is a cylinder so there is some amount of paper that has come out in my drawing but that is not the intention it is inadvertently it has gotten added. So the length has gotten increased so this is after you have applied stress and at this particular position the internal resistance force that is acting is same as the force that you are acting up from outside. And therefore the material does not extend any further and it stops at this particular position.

And therefore we see that the length has increased now this longitudinal increase in length is measured per unit length. And it is called longitudinal strain we will come back to the formal definition in a moment for now let me just draw it here. So this is the longitudinal strain but at the same time you would see that there is also a strain in the other direction and that is called the lateral strain.

So here you can see there is a decrease in length so this has you see that this new cylinder is lower in diameter compared to the outer diameter and therefore this is called lateral strain. So you would also be able to realize from this drawing that since longitudinal strain is positive or in other words it is increasing length. So the other one would have to be decrease in length for constancy in volume this is when you are applying a tensile load.

(Refer Slide Time: 04:21)

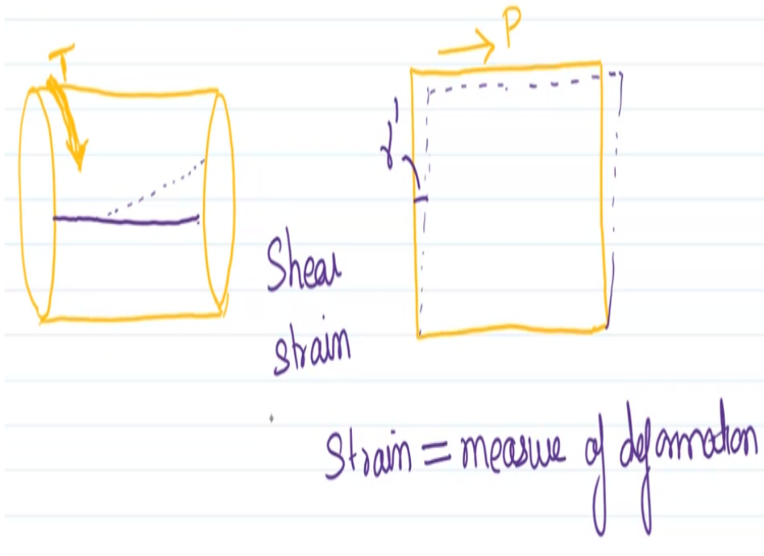


But you may also have a compressive loading and in that case it the overall drawing will look like and this time. Let me try to make it more cylindrical than the previous time so thankfully it has come out to a little better cylinder. Now this time let us say I am applying stress in the opposite direction our load has been applied it is still uniaxial to keep things simple and without loss of any generality.

But the stresses are in the opposite direction and this time again exaggerating things but not the meaning of things. So the cylinder would have decreased in length and the diameter would have increased a little bit. So this is again loading and there is a strain so this is a, the strain in the top one is opposite in sense to the strain in the bottom one. Similarly if you talk about the lateral then again the lateral strain is also opposite from here to over here.

So these are called normal strains however you may also get strains which are shear in nature meaning the plane. So the direction of the strain is in the plane of the force where it is being applied and therefore these are called shear strains. So for a cylinder type of condition it may look like twist.

(Refer Slide Time: 05:59)

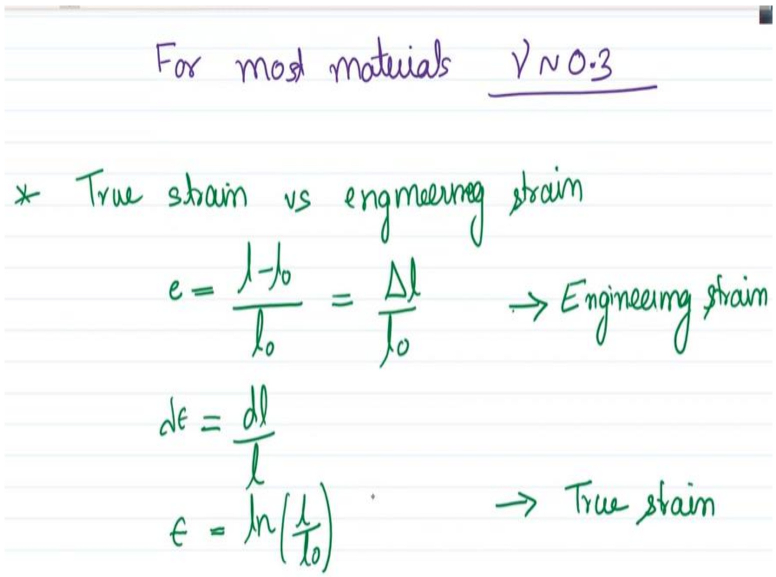


So let us draw again the cylinder so the torque is being applied so this is a torque and the strain is also this is the plane of the torque and the strain is also in this plane and therefore it may look like this so what it is showing is that originally the line may have been like this. But now that you have twisted it like this so this line may appear bent like this and if you open it up if it were a hollow cylinder then how would it look like.

Let us look at it so here the stress or the load is acting in this direction and the strain is acting like this. So this is called shear strain and strain so for what we have understood what is strain? It is a measure of deformation so we have seen that there can be 2 types of strains the normal strain and the shear strain and shear strain is acting in the plane where the stress is being applied similar to what we had as the shear stress.

And usually it is given by the angle and the normal stress is basically just a, elongation or compression. Now that you have seen that there is relation you must have observed that when you have a positive longitudinal stress strain the lateral strain is negative and vice versa. There this ratio is it is not by accident it has to happen it is because of the constancy of volume meaning the volume as to remain conserved.

(Refer Slide time: 09:04)



And this ratio is called ν this is given by negative of lateral strain by longitudinal strain. So there is already a negative sign added over here and therefore when you look at the value of ν it will mostly be positive and for ideal plastic deformation where volume remain constant you can easily show that this must come out equal to 0.5. So you can look at the ν ratio for lot of materials and what you would see is that for most material you can find these on Wikipedia or elsewhere.

And what you will find is that for most materials ν is of the order of not equal to but of the order of 0.3. So what it is telling is that volume is not conserved particularly when we are taking about the elastic regime many ways we know that volume is not conserved need not be conserved. And the value comes out to approximately 0.3 for most of the material like copper will come out 0.33 aluminium 0.32 stainless steel 0.30 and so on.

So most of the materials will give you ν of 0.3 and now when we are talking about the strain there are few more aspects that we need to be aware of in terms of strain. So what are those let us look at those one is true stress versus engineering strain how is the engineering strain define which is given usually by the symbol e which his simply the change in length over the original length therefore this is $\Delta l/l_{\circ}$.

But if you had put it in the differential form then it would come out to sorry this is not the e represented here sorry it is $\varepsilon = \partial l/l$ .whatever is the instantaneous length change in that instantaneous length with respect to that instantaneous length. And if we integrate it with this definition it at every infinite decimal point then you would see that the strain equation will come out to ln $l/l_{\circ}$.

So this is called an engineering strain or basically a working value of strain and this is true strain. So in this particular strain the strains have been added with respect to the length at that particular time and whatever increment has taken place and in that sense it is the true representation of the strain value.

(Refer Slide Time: 12:44)

* $\varepsilon = \ln(1+e)$

$$= e - \frac{e^2}{2} + \frac{e^3}{3} + \cdots$$

as $e \to 0$, then $\varepsilon \to e$.

for very small values of strain $\varepsilon \to e$

Now that we are talking about true strain and engineering strain let us look at few more things. For example if you are given that the strain values are very small then you would find that true strain and engineering strain values are very close to each other are almost equal to each other. The relation between true strain epsilon and the engineering strain is ln if you look at it $l/l_\circ$.

So it is basically (1 + e) and therefore this can be expanded to the form $e - \frac{e^2}{2} + \frac{e^3}{3} -$, factorial and so on. Now from this it is it is clear that as e tends to 0 then epsilon tends to e and therefore we know that for very small values a strain epsilon approaches e or engineering strain approaches true strain. So that is one another fact; or important point when we are talking about true strain and engineering strain.

(Refer Slide Time: 14:01)

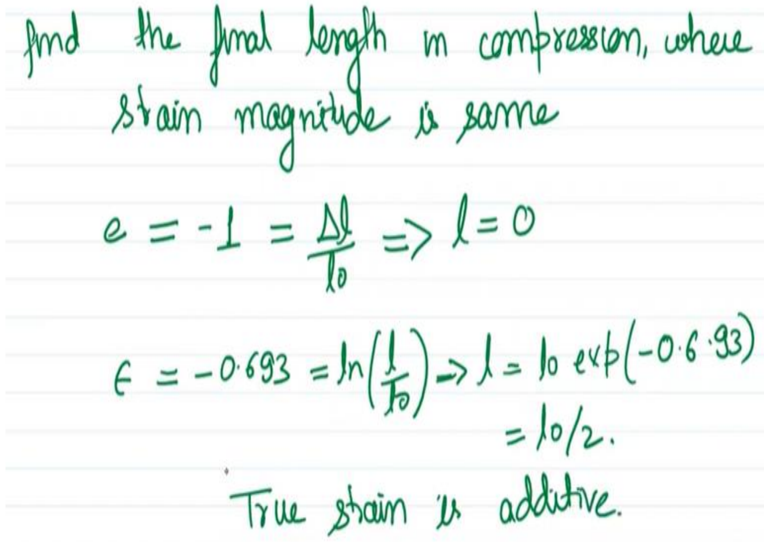
* Rod of length l to 2l.

$$e = \frac{\Delta l}{l_0} = 1$$

$$\varepsilon = \ln\left(\frac{l}{l_0}\right) = 0.693$$

Another important aspect is that let us say we have we extend a rod of length l to 2 l so what will be the true strain and engineering strain. So let us calculate e which will be equal to $\frac{\Delta l}{l_\circ}$.so it is delta l is also equal to $l_{\circ}$ and this is $l_{\circ}$. Therefore this is l on other hand epsilon is equal to ln $l/l_{\circ}$. which I = 2 $l_{\circ}$ and therefore this becomes ln 2 which is 0.693.

(Refer Slide Time: 14:47)



Now if you are given to find the final length for which final length in compression where strain magnitude is same. Remember we are saying strain magnitude not value because sine would obviously change. So what you are given is that magnitude is same so it is 1 but the sine would be -1 and this is equal to $\frac{\Delta l}{l_{\circ}}$. So you have to find what is the $l \Delta l - l_{\circ}$ so you will see that this is possible only when l = 0.

So you will have to reduce the length to 0 to get a strain value of in compression if you are doing the compression to get the strain value of -1. So now do let us do the other thing I have used a wrong expression from here it should have been epsilon sorry about this. I am changing it because we keep using this and it will become difficult to follow if I keep changing it.

So remember we use epsilon for true strain now we know epsilon = -0.693 which is equal to ln $l/l_{\circ}$ and if you keep this l = $l_{\circ}$exp (– 0.693) which is equal to $l_{\circ}/2$. So the final length should be $l_{\circ}$ by 2 meaning that cylinder should be compressed to half the length to get the value of 0.693 in compression but we have also added the sin which is -0.693.

Now look at it what is the implication if you take a rod you double it the true strain is 0.693 you compress it bring it back to original length which is now the new length half. Then you are giving a strain of -0.693 so the final change in length is 0 and if you could calculate the total strain is +0.693 -0.693 which is equal to 0. Therefore true strain is also you can do algebraic sum and subtraction on it and that way true strain looks to be more useful when we are doing mathematical calculation.

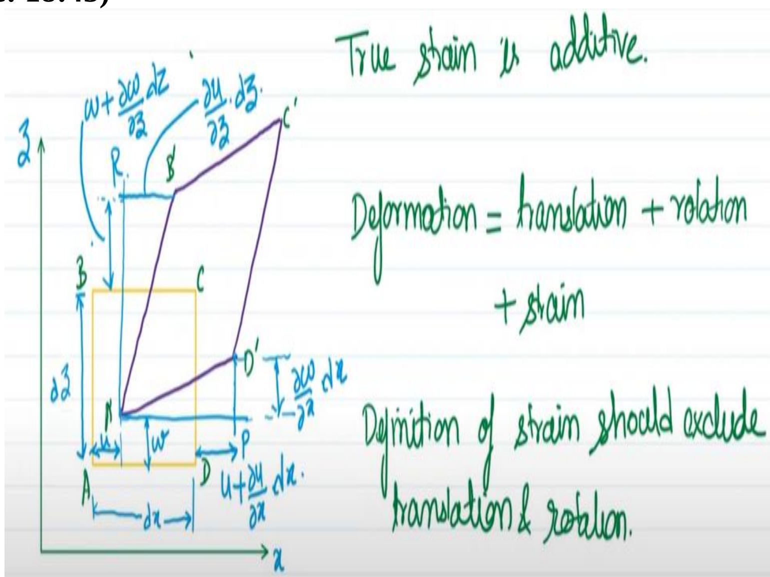
So now so we have looked at the fact that what should be the strain value what should be final length if you want to get the same magnitude of engineering strain and true strain. Now few thing basically what it is showing is that true strain is additive. And later on we will see that true strain is more meaningful in terms of strengthening than engineering strain and hence more meaningful for plastic deformation.

Volume change is related to the sum of 3 normal strains and with volume constancy

$$\epsilon_x + \epsilon_y + \epsilon_z = 0$$

for plastic strains. And again when we are calculating in terms of the true strains so now that we have some fundamental understanding of the strain. Let us look for a formal definition of the string.

(Refer Slide Time: 18:45)



So let us so now I have obtained the axis and let us say we have a simple shape like this. So let us name these corners so let us say we are looking a 3 dimensional object or maybe a 2 dimensional object but whose third dimension is constant and it does not get deformed but whatever relations that we derive over here would be equally valid. So let us say we start with the nomenclature of the corners so it is a, b, c and d.

Now after deformation it may look like so the share has not come out right let me so these are the new corners a prime, b prime, c prime and d prime. Now first thing to realize is that whenever you are doing deformation the deformation will always have some amount of rotation some amount of lateral shift and some amount of strain. So deformation will always lead to some translation.

Translation is not really changing the shape of the object so the strain equation should not take into account that. Similarly rotation is not any share change so it should the formal definition of strain should be able to keep that out and the strain. So this for definition of strain should be such that it is able to isolate translation and rotation or basically exclude translation and rotation.

Now here this is a very infinite small element of some big object and therefore this one can be represented as dx. Yes, I have forget to mention that the axis is x and z so we are assuming that the third axis is y and whatever we derive for x and y will equally be valid for y. Now this is x axis this is d x and similarly this is dz and displacement let us called the displacement over here.

This is the displacement of point a not overall displacement so this displacement is u and the displacement in this direction is w. Now this point as gone from here to here d has gone from D to D prime. Now if we look at only the extension along the x axis which is the so we will have let us name this point P and similarly let us name this point R. Now what is the value of so this is dx this is u so what is this?

D has gone to D prime and what is the overall displacement over here this is equal to u + so this point is going away by u must be shifted by D. But at the same time there would also be elongation in this so that must also be taken into account over here and that would be del u by del x times d x. Similarly this point P has gone to D prime and therefore this must be shifted by B + whatever the extension has been over in B prime.

So this equal to $w + \frac{\partial w}{\partial z} dz$ so far we have looked at only the normal displacement and extensions. Now let us look at the other ones like B prime and P so the D prime P is distance is equal to so this one is so the displacement or extension of w along the x direction. So w is originally this direction. But in this direction this the gap keeps increasing and that would be given by $\frac{\partial w}{\partial x} dx$.

Similarly this one would be the extension of u along z direction so this one will be given by $\frac{\partial u}{\partial z} dz$. Now with this if you are now comfortable with whatever the values you have written over here calculating the strains becomes straight forward.

(Refer Slide Time: 24:51)

Normal Strains $e_{xx} = \frac{A'P}{AD} - 1 = \frac{dx - \cancel{u} + \cancel{u} + \frac{\partial u}{\partial x} \cdot dx}{dx} - 1$

$= 1 + \frac{\partial u}{\partial x} - 1 = \frac{\partial u}{\partial x}.$

$$\boxed{e_{xx} = \frac{\partial u}{\partial x}; \quad e_{yy} = \frac{\partial v}{\partial y}; \quad e_{zz} = \frac{\partial w}{\partial z}}$$

Shear Strains

$$\angle PAD' \approx \arctan\left(\frac{\frac{\partial w}{\partial x} \cdot dx}{dx + \frac{\partial u}{\partial x} \cdot dx}\right)$$

For example let us look at $e_{xx} = \frac{\acute{A}P}{AD} - 1$. So basically this is your new length along x direction this is the old length so if we take the change in length $\acute{A}P - AD$ my original length is AD. Now we will just insert the values $\acute{A}P$ which is nothing but this is dx – u we come to this point and this much length now we have to add this much which is $u + \frac{\partial u}{\partial x} dx$.

So this is dx – $u + \frac{\partial u}{\partial x} dx$ this is A prime P and A D is nothing but dx – 1. So you can see this and this get cancelled out and when you have $dx + \frac{\partial u}{\partial x} dx$. So this becomes del u by so you can cancel out the dx team in all of these and it becomes 1 + $\frac{\partial u}{\partial x}$ - 1 which is equal to $\frac{\partial u}{\partial x}$. So the $e_{xx} = \frac{\partial u}{\partial x}$ and with this understanding we can now extend to the others which is $e_{yy} = \frac{\partial v}{\partial y}$ and $e_{zz} = \frac{\partial w}{\partial z}$

(Refer Slide Time: 28:35)

$$\approx \arctan\left(\frac{\partial w}{\partial x}\right)$$

$$\angle PAD' \approx \frac{\partial w}{\partial x}$$

$$\angle RA'B \approx \frac{\partial u}{\partial z}$$

$$e_{xz} = \frac{1}{2}\gamma_{xz} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \frac{1}{2}\gamma_{zx} = e_{zx}.$$

So that is the normal strains now if you are looking at the shear strains so the shear strain is nothing but this angle which is P A Dˈ actually. So P A Dˈ which is approximately equal to arc tan now if you are talking the tan of this we assume this is the theta. So what we are trying to find is theta so this is Tan inverse of this is the perpendicular this is the base so this value divided by this value.

So the numerator is $\frac{\partial w}{\partial x} dx$ and the denominator is nothing but AP which we had already written here which was d x + $\frac{\partial u}{\partial x} dx$. Now again you would see that we will be able to cancel out d x and this will be become 1 + $\frac{\partial u}{\partial x}$. But then $\frac{\partial u}{\partial x}$ is a very small quantity and therefore as compared to 1 and therefore this can be simplified to just 1 and therefore it becomes arc tan($\frac{\partial w}{\partial x}$).

So this is a approximately arc and we also know that theta is approximately equal to sin theta for very small values and sin theta is approximately equal to Cos tan theta. And therefore we can say that this is approximately equal to $\frac{\partial w}{\partial x}$ which is nothing but your ex set. Now similarly we can calculate the value of the other theta which is R A Bˈand R Aˈ Bˈ.

Actually here also we have this prime so similarly we can calculate RAˈBˈ and you would be able to show that this is nothing but $\partial U/\partial Z$. Now coming to the shear strain we know the relation that

$e_{xz} = 1/2\gamma$ we do not know yet in the sense of this particular course. We will come back to this a little later so e_{xz} which is that tensor quantity is equal to this shear strength $\gamma_{xz} = 1/2\,(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z})$.

And this is also equal to (½) $\gamma_{zx} = e_{zx}$ so these we have seen is nothing but the theta that exist between the 2 axis of the formation. So this is the definition for the normal strain as well as the shear strains. There was a little bit of break and we looked at this equation for the normal strains as well as the shear strains. So this is the normal strain giving over here and the shear strain. so basically what we are doing is you have 2 angles P A D' and R A B'. So R A' B so this these 2 angles R A'B' let me write it clearly and this is also P A'B'. So the shear strain is basically average these 2 so it may be that in some cases because of the way that you select the coordinates this may be 0 one of these may be 0 the other may show the total angle or the strain.

And that is why the way it is defined is that e_{xz} is the average of these 2 so you sum it up and take the half of it this is what we are doing over here. And this way you have the strains for the x y plane and you can get the strains similarly in the other planes but right now we are limited to only these plane.

(Refer Slide Time: 32:29)

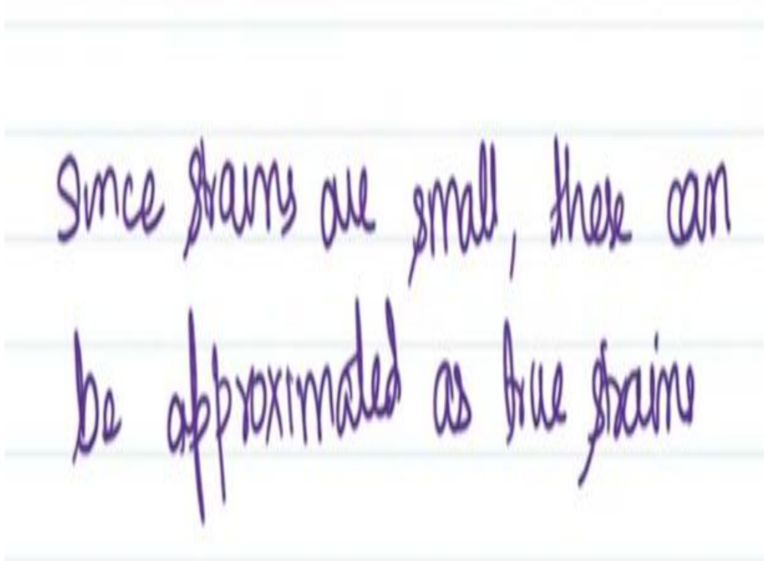
$$e_{xx} = \frac{\partial u}{\partial x};\; e_{yy} = \frac{\partial v}{\partial y};\; e_{zz} = \frac{\partial w}{\partial z}$$

$$e_{xz} = \frac{1}{2}\gamma_{xz} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \frac{1}{2}\gamma_{zx} = e_{zx}$$

So we will talk only about this plane now in summary you can look at the overall how the equation would look like. So these are the e_{xx}, e_{yy} again I made a mistake here it should be so let me just overwrite here so it should be e_{zz} similarly here it is e_{xz} which is equal to (½)γ_{xz}. So γ_{xz} just represents that particular angle so for example γ_{xz} will represent either d it will represent either this angle P A' B' or it will represent R A' B'.

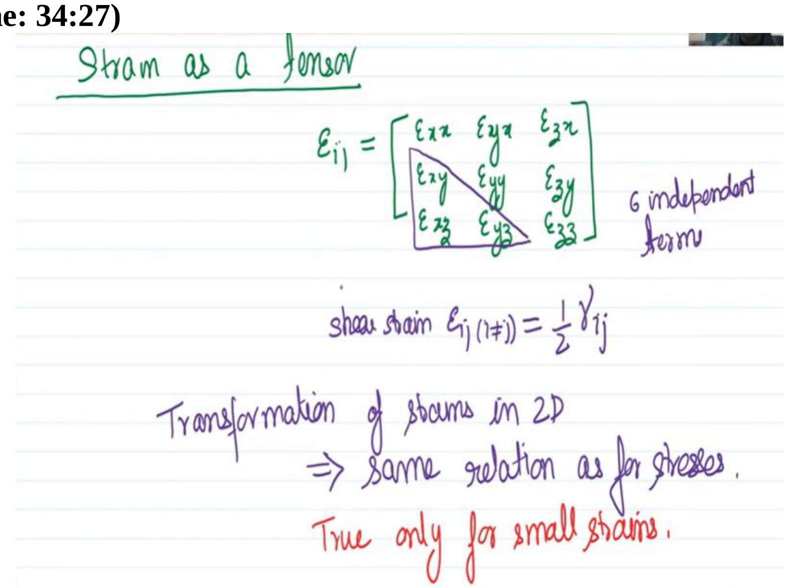
And on the other hand epsilon x z represents average of these 2 so that you must keep in mind and with that we have this equation over here. So moving on we have the equations and we must always keep in mind that we would be using these as true strains which would means that the strains are.

(Refer Slide Time: 33:36)



So since strains we are assuming that or the other way down we are assuming that the strains are small. So we can take them to be equal to true strains so moving on now we will have seen the formal definition of strings now let us look at strain as a tensor quantity.

(Refer Slide Time: 34:27)



So you have in the previous definition we have seen that already seen that there are normal strains and there are shear strains and for each of the strain there will be 2 planes so basically there will be 6 shear springs but at the same time the material is not rotating therefore 2 of these strains the pair of the shear strain should be equal. And therefore independent number of shear strains will only be 3.

So we will have 3 shear strains + 3 normal strains and therefore you will have something if you want to represent as a tensor quantity this is how you will represent it epsilon xx and like I said this 2 must be equal and therefore overall if you look at it. These 3 are basically just the same values as the other 3 meaning they are not interdependent term and therefore you have 6 independent in this matrix in general cases.

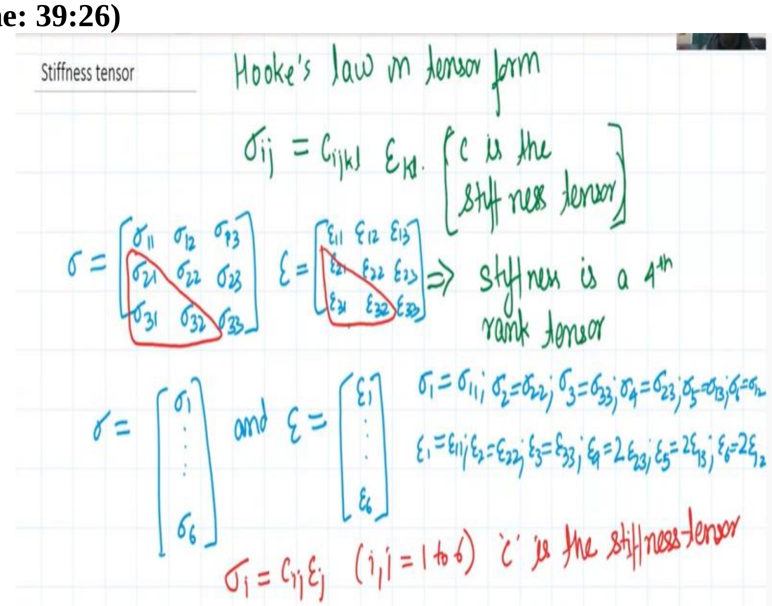
And another thing to keep in mind is that when we have already seen that if you are talking about epsilon shear strain where is I is not equal to j. So for shear strain epsilon i j ,i not equal to j then this is equal to 1/2 of gamma i j so this is the way we have defined it. So that it fits into this matrix form or in this tensor form. So now that we have the strain as a tensor form we can also apply the transformation.

So the transformation if you remember we had relation for transformation of stresses in 2d. Similarly we have transformation of strains in 2d and the relationship is same as same relation as you had for stresses. So that is the beauty of defining it this way we are able to use the same relations and it have. It is as if you do not have to derive it separately and everything can be used as it is whatever you use for stresses you can use it for strings.

And just that you have to keep in mind that epsilon i j = 1 by 2 gamma i j and another thing is that this is true only for small strains. So this is also something that you need to keep in mind because as I said again and again that we are making assumption that these when the strings are small not assumption. But we know that when the strains are small then engineering strains are equal to 2 strain and these relations that we have carried out are true for only the true strains.

So this will be valid only when the strains are small in value now another thing is that when we are using the stresses as tensor strains as tensors then the Hooke's law can also be written in that form. And it would mean that the compliance or the stiffness itself would be on much tensor quantity. So there are 9 quantities in general in the stresses and 9 quantities in general in the strain and therefore the proportionality constant which the stiffness or compliance the inverse of it will have lot more vmber of elements in it.

(Refer Slide Time: 39:26)



Let us see let us have a look at it so first let us write Hooke's law in tensor form so if you write the Hooke's law in tensor form then we know that's it so you have σ_{ij} which is basically σ_{12}, 2 sigma 1 x or you can see 1, 2, 3 for i j can be 1, 2, 3 or i j can be x y z and therefore it can be σ_{xx}, σ_{yy}

and σ_{zz}, σ_{xy}, σ_{yz} and so on. And here you have the strain so it is ε_{kl} and this is again k l is another pair of x y z another set of x y z or 1, 2, 3.

And therefore the proportionality constant will have to relate i j to k l and therefore the proportionality constant will have c_{ijkl} where c is the stiffness tensor. Now this from this what we see is that stiffness tensor would be fourth rank tensor because the stress is 2 second rank tensor strain is second rank tensor. So the proportionality constant between these will have to be a fourth rank tensor.

But then you would also remember that we have the simplification that sigma if you remember go to sigma then for or if we have for the strain then we have something like this. Now here as you can see there is second rank tensor there is a second rank tensor. So obviously if something has to relate them then it will be a Fortran tensor. But we know that there are 3 quantities which are not which are not independent similarly which are not independent.

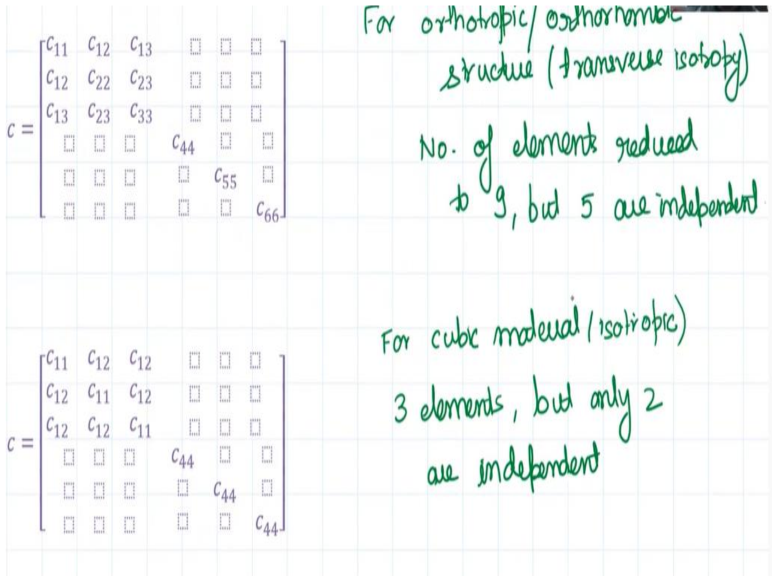
Here similarly there are 3 quantities which are not independent here which means we can write sigma as something like this. So there are 6 quantities over there similarly we can write strain as this where $\sigma_1 = \sigma_{11}$, $\sigma_2 = \sigma_{22}$, $\sigma_3 = \sigma_{33}$, $\sigma_4 = \sigma_{23}$, $\sigma_5 = \sigma_{13}$, $\sigma_6 = \sigma_{12}$. Similarly $\varepsilon_1 = \varepsilon_{11}$, $\varepsilon_2 = \varepsilon_{22}$, $\varepsilon_3 = \varepsilon_{33}$ we have to clearly define it.

So that there is no ambiguity about which particular strain does epsilon 3 or epsilon 4 represents or likewise which particular entity does or which particular component are sigma 2 or sigma 3 represent. Because we are changing the notation from second rank tensor to a first rank tensor and coming to this epsilon 4 = 2 epsilon 2 3, epsilon 5 = 2 epsilon 1 3 and epsilon 6 = 2 epsilon 1 2.

So now the second rank tensor has been converted to a first rank tensor and which means that even our stiffness can now be translated to a much lower rank tensor. How so now we can write sigma i = c_{ij} epsilon j and where i and j go from 1 to 6 and obviously c is the stiffness tensor. So, now c_{ij} or the stiffness tensor is relating between first rank tensor to first rank tensor.

Therefore c $i\ j$ itself is now only a second rank tensor where there is 6 elements in this 1, 6 elements in this one it will have 36 elements in general case. And this is what is represented over here so this is your very general case stiffness tensor elements. So there are 36 elements over there now just like a in stresses and strain we know that certain elements are same as the other meaning it is not independent and this happened to be over here.

(Refer Slide Time: 45:51)



So your independent quantity is so this can be reduced to so basically let me be more specific based on symmetry. Sorry this is not symmetry based on equivalence of value this matrix can be reduced to 21 elements earlier we had 36 elements. However we are not satisfied with this one because we know that there are something some more conditions that we can apply and hence we can reduce it.

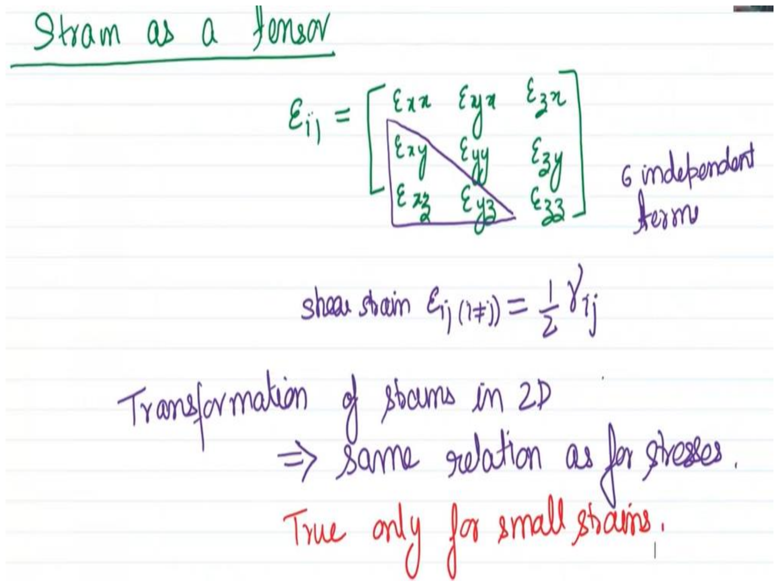
So for example for now we will apply some amount of symmetry we know that it is not a completely random structure it is a cuboidal type of structure. So for that type of cuboidal structure for orthotropic or orthohombic structure what we say what we call it is that has a transverse isotropy. So it has now the number of elements are reduced to 5 elements you can see even some of these elements are equal and rest of them are 0 elements.

Actually number of elements reduced is to 9 but 5 are independent so you can see there are 9 elements over here but only 5 are independent elements. So you can see there are 1 1, 1 2, 1 3 and again 2 2, 2 3 and then 3 3 so there are 6 here 1 2 9. So there are total 9 elements here but there are interdependence amongst them and only 5 will happen to be independent. Now that is for a very low symmetry material we would be dealing with in metals mostly cubic materials.

And for cubic materials and if you assume that it is isotropic then these number of elements can be reduced even further for cubic material isotropic. We will have now it has 3 elements 1 1, 1 2 and 4 4. But there is again interdependence amongst them and only 2 are independent. We are not going into the details of deriving how we get to these because it is a very long mathematics and you will probably but you can take courses on mechanics of material to understand this.

However since it is useful for us so we taking these equation over here for our purpose because, it is good have a comprehensive understanding about one that stress strains are actually a tensor quantity and also the stiffness and what are the various elements that are present in them. And what do these different elements represent now that we have stress strain as a tensor quantity and we have also simplified it for the cubic materials with isotropic symmetry and we have seen that it has only 3 elements and 2 are independent.

(Refer Slide Time: 50:44)



Now it is time we write down Hooke's law in the matrix form and this is what we will have but we will get. So you can see that this is why we were saying you said you saw we have 3 different elements but only 2 are independent. Now this becomes clear over here so you can see there is 1 – ν that is 1 element ν another element and 1 – 2 ν /2. So there are 3 elements but all of these can be represented in only in terms of ν.

So basically there are ν and so in this particular case there is only 1 independent element. So one there is 1 element inside the matrix and the other one exists outside it which is e. So in terms of e and ν you are able to describe all the different elements and in fact the whole stiffness matrix and here it is given in terms of stiffness and also written this equation in terms of compliance which is how this is represented here.

So this quantity over here represents the compliance so epsilon is equal to compliance sigma and both of this equation represents the same Hooke's law. So one is written in the form where you get the stiffness tensor for other where you get the compliance tensor. So you can see now that basically this is a very simple and easy way to represent stress and strains and even though there are in the tensor form but they are not very scary as it may seem at first.

So now moving on and now what we want to do now that we know that stresses and strain are tensors we have looked at the relation Hooke's law in the tensor. Now let us look at the individual components in a scalar way and then we will look at plan strain condition and the plane stress condition.

(Refer Slide Time: 52:49)

Stress	Strain-x	Stain-y	Strain-z.
σ_x	$\varepsilon_x = \frac{\sigma_x}{E}$	$\varepsilon_y = \frac{-\nu\sigma_x}{E}$	$\varepsilon_z = \frac{-\nu\sigma_x}{E}$
σ_y	$\varepsilon_x = \frac{-\nu\sigma_y}{E}$	$\varepsilon_y = \frac{\sigma_y}{E}$	$\varepsilon_z = \frac{-\nu\sigma_y}{E}$
σ_z	$\varepsilon_x = \frac{-\nu\sigma_z}{E}$	$\varepsilon_y = \frac{-\nu\sigma_z}{E}$	$\varepsilon_z = \frac{\sigma_z}{E}$

So the next thing we will look at is stress strain relation in isotropic materials which was the last case that we saw over there but now in a scalar form. So we know that there are σ_x, σ_y , σ_z these which are acting on a material and because of that there will be normal stresses on that and we also know that there will be shear stresses acting on it and there will be shear strains and that will be taking in the material in response to that.

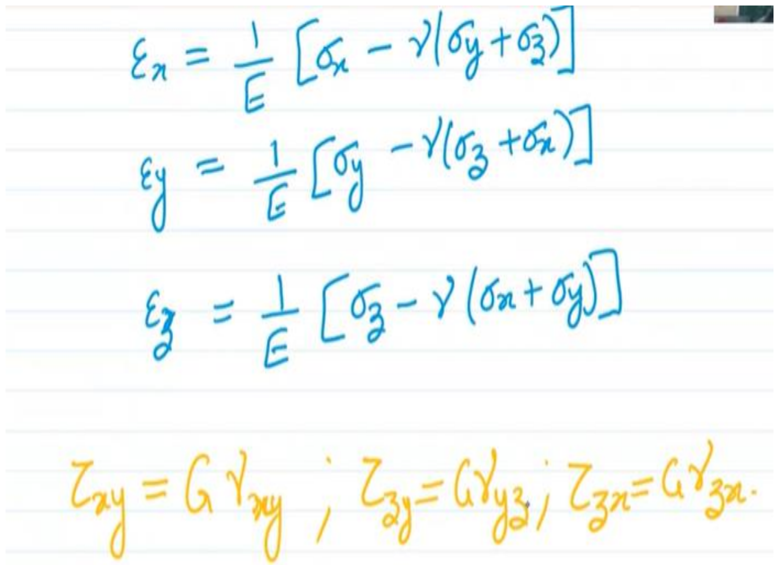
So we will look at the scalar relation so first let us look at the normal stresses so let us say these are the stresses that are acting over here, and the strain in the x direction strain in the y direction because of the normal stresses and strain in the z direction because of the normal stresses. So stress if it is acting in the x direction or it can be acting in the y direction or it can be acting in the z direction.

And what will be the strain in the x direction so strain in the x direction because of the stress in the because of the stress in the x direction would be given by σ_x/E strain in the x direction because of the strain in the y direction would be given by epsilon y and it will be in the perpendicular direction. If you remember we have strain which is longitudinal and then lateral.

So this one is the lateral strain and therefore this will be related by the Poisson's ratio ν we had described earlier there was the. So this one we have not written the Poisson's ratio ν so the strain would be in terms of that ν. So it will be $–\nu\sigma_y/E$ and for the z similarly we will have epsilon z = -ν σ_z/E. And that sets up the template we can write the equation similarly for the other ones.

So here strain in the y direction now it is the lateral strain so there will be the ν over here $–\nu\sigma_x/E$. This is strain in the y because of σ_y so it is normal so it is similarly here epsilon z this is lateral –$\nu\sigma_x/E$,epsilon z = lateral so $–\nu\sigma_y/E$. Epsilon z this is normal so this is σ_z/E. So overall if you look at the strain it will be because of if in a general case then it will be because of stress in the direction x, y and z.

(Refer Slide Time: 57:10)



And therefore we can write that

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_y + \sigma_x\right)\right]$$

this is for the normal stress for the shear we can also write tau x y. Now here it is conventional to represent this modulus in terms of the shear modulus this was the elastic model this was the shear modulus.

So it is represented like this there will be tau so we have the relation for epsilon and the normal strains and the shear strains and now we will also look at some more relations which come in handy when we are talking about the stresses about the strains and material. So one of the other so, we have looked at elastic modulus and shear modulus another constant that is dependent or is interrelated with E and G is the bulk modulus k.

(Refer Slide Time: 59:09)

Bulk modulus (K) is the ratio of hydrostatic pressure to the dilation it produces. (volume strain)

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}\underbrace{(\sigma_x + \sigma_y + \sigma_z)}_{3\sigma_m}$$

$$= \frac{1-2\nu}{E} \cdot 3\sigma_m$$

It is the ratio of hydrostatic pressure we are not yet gone into the details of hydrostatic what is the hydrostatic and what is deviatoric? For now just we will define and then we will come back and revisit what is hydrostatic? So hydrostatic pressure to the dilation it produces dilation meaning volume strain. So volume strain is what basically $\varepsilon_x + \varepsilon_y + \varepsilon_z$. Now if you look at the above relation then you can add these and what you will get is this

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{1-2\nu}{E}(3\sigma_m)$$

Now bulk modulus the way we are defined it above will come back and now we will write it.

(Refer Slide Time: 01:01:14)

$$K = \frac{\sigma_m}{\Delta} \qquad \Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

$$= \frac{1-2\nu}{E} \cdot 3\sigma_m$$

$$\boxed{K = \frac{E}{3(1-2\nu)}}$$

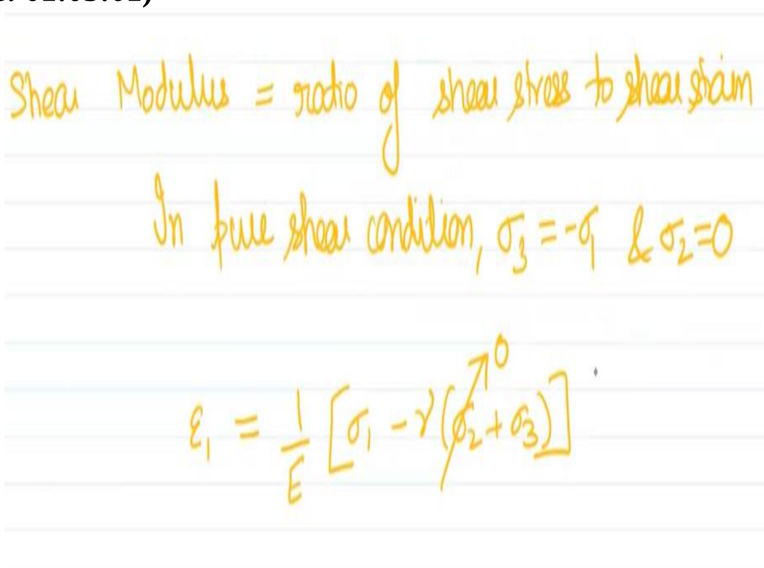
So k = σ_m/Δ? Basically delta is nothing but $\varepsilon_x + \varepsilon_y + \varepsilon_z$which we have already seen over here. And since again like the invariance of stresses there is also invariance of strains so it can also be written as $\varepsilon_x + \varepsilon_y + \varepsilon_z$. In fact all the relation that we have written over here instead you can replace this x y and z by 1 2 and 3.

And in fact we will use it and you will be able to and standard better. So now that we have like this so we can write is equal to so we know that epsilon 1 2 3 this 1 – 2 ν. So this becomes $\frac{1-2\nu}{E}(3\sigma_m)$ and now coming to this equation we have the value of sigma. And we know have this sigma m so sigma m and by delta and therefore from this relation what it comes is that

$$k = \frac{\sigma_m}{\Delta} = \frac{E}{3(1-2\nu)}$$

So this is another handy relation, which, we use for relating the stresses and strains. So k is called the bulk modulus the other one is the e elastic modulus and g the shear modulus. And still another relation so now we have 3 quantities k,E so we can also relate G with this and this one we can do with the help of our pure shear condition.

(Refer Slide Time: 01:03:01)



So shear modulus so this is the ratio of shear stress to shear strain to get to the equation for this what we will do is? We will go to the pure shear condition in pure condition sigma 3 = -sigma 1 and sigma 2 = 0. Now if you write sigma 3 = -(sigma 1) then we can go to this relation and put the value over here. So again like I said that here what you are doing is you can instead of epsilon x; you can say it is

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

So we will take this relation and over there we will put these conditions now sigma 2 is 0 because we are talking about the shear condition and sigma 3 is basically – (sigma 1) so this becomes sigma 1 + mu sigma 1.

(Refer Slide Time: 01:04:48)

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\cancelto{0}{\sigma_2} + \sigma_3)\right]$$

$$= \frac{1}{E}(1+\nu)\cdot\sigma_1$$

$$G = \frac{\tau_{max}}{\gamma_{max}} = \frac{\sigma_1}{2\varepsilon_1}$$

$$\boxed{G = \frac{E}{2(1+\nu)}}$$

And therefore this is $\frac{1}{E}(1+\nu)\sigma_1$. Now for getting the relation for the shear modulus we know that shear modulus is equal to τ_{max}/γ_{max}. But what is γ_{max}? γ_{max} is nothing but 2 times epsilon 1 and τ_{max} is nothing but σ_1. So whatever relation we have derived is for the condition of pure shear but then if it is the relation between G, E and ν then it must be valid for all conditions.

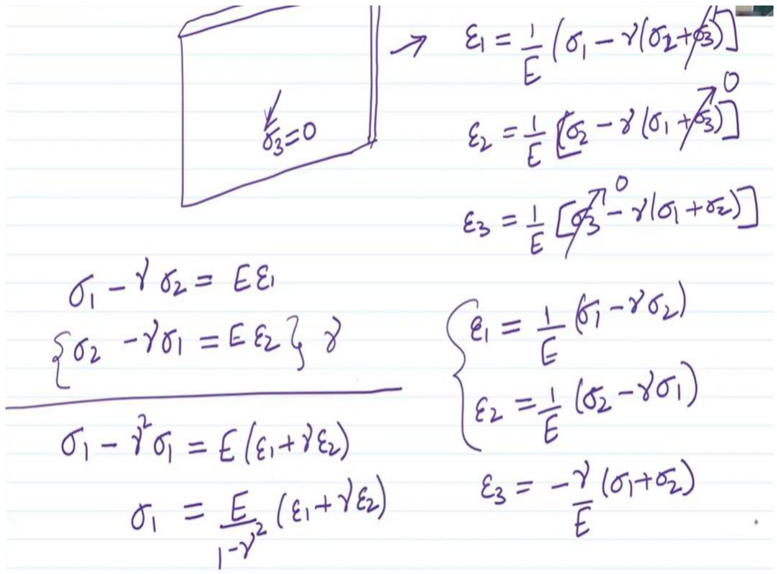
So

$$G = \frac{E}{2(1+\nu)}$$

this is another relation this may which may come in handy when we are talking about elastic deformation behavior of materials. So you sometimes you may be given E sometimes you may have obtained G which is the shear modulus and you can translate or convert from 1 form to the other.

As long as we are in the elastic deformation limit now that we have this now let us since we are talking about the elastic properties. So let us also look at 2 special condition one is called the plane strain condition and the other is called a plane stress condition.

(Refer Slide Time: 01:06:35)



So first we will look at the plane stress condition so what is the plane stress condition? This condition defines that your stresses are they are only in the plane there is no out of plane stress. So for example you may have a thin sheet like this and both these cases are in very useful in engineering because they simply things without much loss of accuracy of the results. So let us say there is a thin sheet and if you are pulling it or as long as the stresses are limited in the plane so the stresses would stay remitted in this plane.

And if you are not bending it or folding it therefore the stresses in the other direction or normal to this would remain 0. So basically sigma 3 = 0 that is what is plane stress condition and all the stresses are inside this plane. Now since sigma 3 = 0 meaning one of the principle stresses here is 0 so the sigma 1 and sigma 23 must lie in this particular plane. Now with the conditions sigma 3 = 0 and now we can use again the relation that we had earlier.

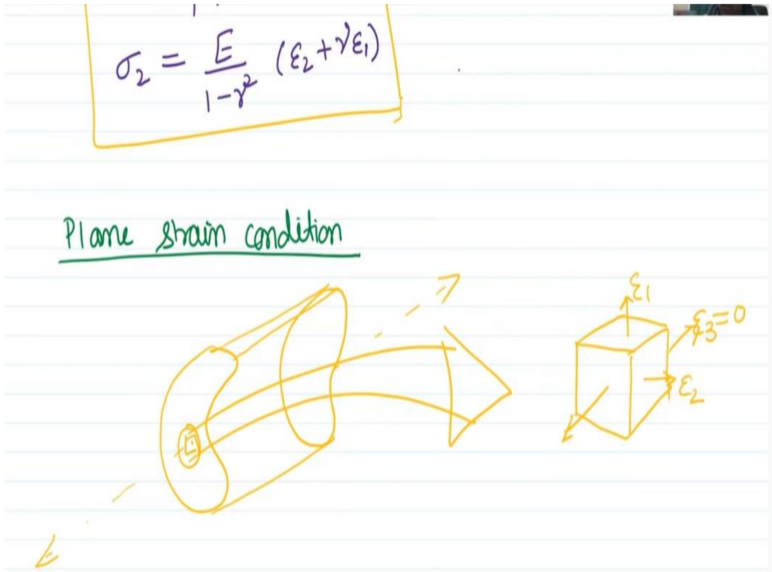
If we put epsilon 1 so again we will go back and write those equations as

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2)]$$
$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1)]$$
$$\varepsilon_3 = \frac{1}{E}[-\nu(\sigma_2 + \sigma_1)]$$

And you can put these values in the equation for stresses so we can take these 2 equations and try to get equation for sigma 1 and sigma 2. So if you write it down in a explicit form then what you will get is

$$\sigma_1 = \frac{E}{1 - \nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

(Refer Slide Time: 01:11:02)



And similarly you can get a relation for

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

So we have a simplified relation for epsilon 1 and 2 and we have specified relation for sigma 1 and 2. This is for the plane stress condition now we move on to the plane the other technology you use full simplification which is plane strain condition. So when does the plane strain condition take place.

So for the example let us assume that you have a very long wire or rod now along so it is so long that the stresses are being applied only in the perpendicular to the length of the wire and strains are also meaningful only in that direction. But the strain along the wire becomes negligible or meaningless because it is. So long that not much change is taking place along the length.

So that condition what will happen is that you have a plane strain condition so for example let me draw here simple schematic. So let say this is some section of that long wire so this extends along this direction and along this direction. Now if you look at this the epsilon along this direction so if you let us take an element from here. So the strain in this direction is equal to 0. So there are strains over here there are strains over here some.

Let us call it that we have picked up our orientation such that we have epsilon1 and 2. So there is strain in the 1 direction and the 2 directions but epsilon 3 = 0. So this is the condition which is called plane strain condition and this is also something as I said mentioned that this is also very meaningful and you find it very frequently in engineering application. So now when you have a condition like this when you put in the values you would see that the equations again simplify.

(Refer Slide Time: 01:14:03)

$$\varepsilon_3 = 0$$

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu(\sigma_1 + \sigma_2)\right] = 0$$

$$\sigma_3 = \nu(\sigma_1 + \sigma_2)$$

$$\varepsilon_1 = \frac{1}{E}\left[(1-\nu^2)\sigma_1 - \nu(1+\nu)\sigma_2\right]$$

$$\varepsilon_2 = \frac{1}{E}\left[(1-\nu^2)\sigma_2 - \nu(1+\nu)\sigma_1\right]$$

$$\varepsilon_3 = 0$$

So now that we have epsilon 3 = 0 so we can write the equation like this but

$$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \nu(\sigma_1 + \sigma_2)] = 0$$

So basically what you get is that

$$\sigma_3 = \nu(\sigma_1 + \sigma_2)$$

Now that you have the relation our new relation for sigma 3 you can insert it into the equations for epsilon 1 and 2. And therefore what you get for epsilon 1 and 2 are like this

$$\varepsilon_1 = \frac{1}{E}[(1 - \nu^2)\sigma_1 - \nu(1 + \nu)\sigma_2]$$

And epsilon 3 we already know is equal to 0 so this is again another simplification that we can obtain for engineering applications. So this is for plane strain condition and these are very useful and helpful to simplify the problems and solve our equations and we will not to solve our condition right now for this one.

(Refer Slide Time: 01:15:34)

Examples

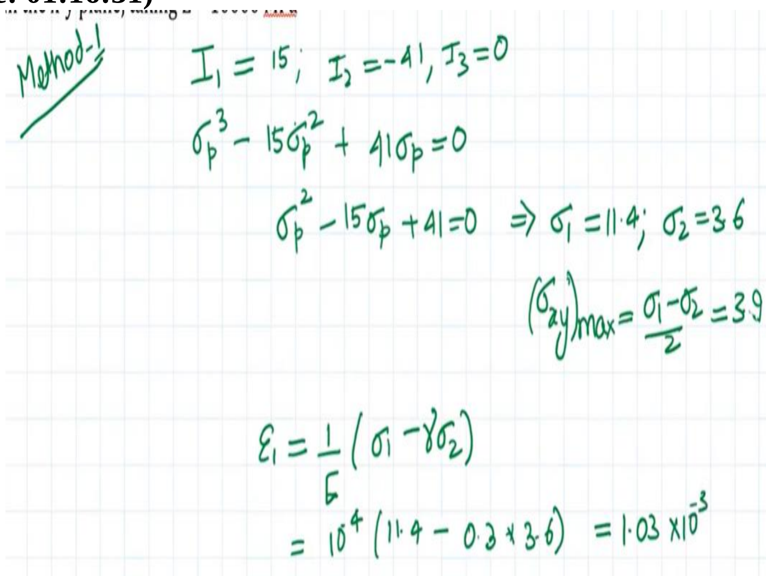
Consider a stress state where σ_x= 10 Mpa, σ_y= 5MPa and σ_{xy}= 3MPa. (nu=0.3) $\nu = 0.3$

1. Assume plane stress condition and find the principal stresses in the x-y plane
2. Determine the principal strains and the largest shear strain in the x-y plane, taking E = 10000 MPa

But let us solve a problem solve an example so let us say that you are given that consider a stress state where sigma this is a example based on what we have learnt overall for this chapter. So it is given that consider stress state where sigma x is equal to 10 MPa, sigma y = 5MPa and sigma x y = 3MPa and ν or basically it is ν is equal to 0.3 assume plane stress condition and find the principal stresses in the x y plane determine the principle strength.

And the largest shear strain in the x y plane taking equal to 10,000 MPa so basically you will have 10 GPa elastic modulus e has been given. Now that you have this these values actually you can solve it 2 different ways

(Refer Slide Time: 01:16:31)



So let us look at it method 1 here we have given sigma x sigma y and of course assuming that sigma z = 0 we can first find the principle stresses from the principle stresses, we can find the principals strains and from the principle strains we can find the largest shear strain. So let us try it like that so for that we will need like invariants which are I_1 and you can clearly see that I_1 will be equal to 15 and everything we are measuring here is in terms of MPa I_2 = -41 and I_3 = 0.

And then applying it into that equation which gives us the values of the principles stresses

$$\sigma_p{}^3 - 15\sigma_p{}^2 + 41\sigma_p = 0$$

it boils down to because there is the third stress is 0. So it automatically will translate to a quadratic equation .And when you solve it and I am not going through that solving process but you can show that sigma 1 = 11.4 and sigma 2 = 3.6.

And obviously sigma 3 = 0 and if sigma 1 is 11.4 and sigma 2 = 3.6 then we also know that sigma xy max meaning that circle if you remember from the Mohr circle. So the sigma x y max is basically sigma 1 – sigma 2 by 2 which is again 3.9. So, 3.6 MPa is the maximum shear stress and principle stresses are 11.4 and 3.6 so this is the value for the first part now we want to find the strains.

So for the principal strains we had said that we can apply that equation for epsilon x as well as in the epsilon 1. So we have sigma 3 = 0 so this translates to minus gamma sigma 2 and 1/E in terms of MPa is 10^{-4} (11.4 - 0.3×3.6) =1.03×10^{-3}

(Refer Slide Time: 01:19:16)

$$\varepsilon_2 = \frac{1}{E}\left(\sigma_2 - \nu\sigma_1\right)$$
$$= 10^{-4}\left(3.6 - 0.3 \times 11.4\right) = 0.018 \times 10^{-3}.$$

that is the strain value 1 principle strain and similarly for the principle strain 2 1 over E. This time it will be equal to sigma 2 minus gamma m – ν sigma 1. So $10^{-4}(3.6 – 0.3\times 11.4)= 0.018\times 10^{-3}$; so this is the strain in the second principle strain. Now we also need for if you want to calculate the shear strain we also need a value of shear modulus.

Shear modulus G= E/2(1 + ν) is equal to you can put in the value and you would find it is 3846 MPa and therefore gamma 1 2 = sigma x y max by G which is equal to 1.01×10^{-3} and therefore epsilon 1 2 is half of this which is 0.52×10^{-3}. So these are strain values and ratio so there are no units for the string. So that is the first method where we have calculated the principle stresses and from where there we calculated.

(Refer Slide Time: 01:20:50)

Method-2

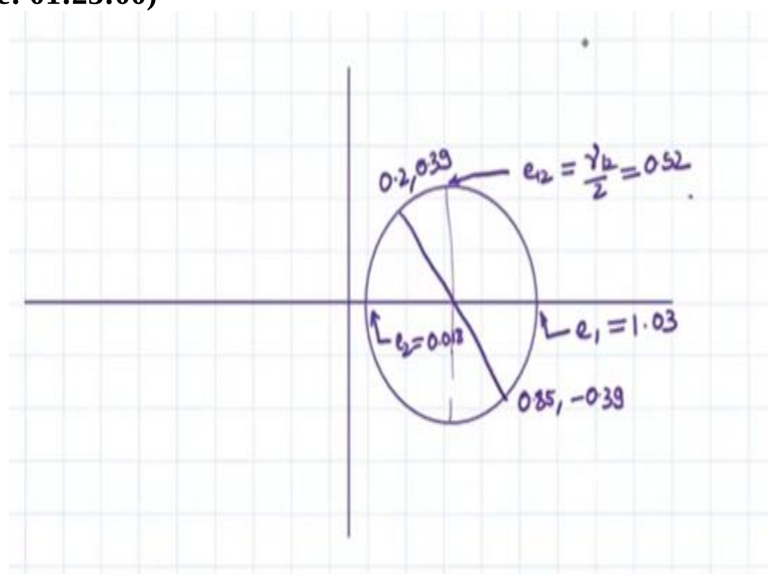
$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$$
$$= 0.85 \times 10^{-3}$$
$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right)$$
$$= 0.2 \times 10^{-3}$$
$$G = 3846$$
$$\gamma_{xy} = \frac{\sigma_{xy}}{G} = \frac{3}{3846} = 0.78 \times 10^{-3}$$
$$e_{xy} = 0.39 \times 10^{-3}$$

But the other method is where you do not need to directly find the principle stresses if it were only to so in the first part of this we are actually asking for the find principle stresses in the x-y plane.

But if the question was only about the second part then you would see that we can skip the first part and directly go to the second part without calculating the principle stresses and that is what this method is about.

So this is method 2 and $\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y\right)\right]$ and when you put in the values you would find it is 0.85×10^{-3}. And similarly $\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x)\right]$ and that gives you the strains the epsilon x and epsilon y. And now we need to go to find the shear strain so the shear strains for that we will still need the g. So we will still use the same equation therefore G is still equal to 3846 using the same equation as above.

So everything is given E and ν are given so we can directly calculate G and to find gamma x y we can directly find from x and y sigma x y/G and sigma x y is also given over here which is 3 mega pascals. So 3 by 3846 which is equal to 0.078×10^{-3} and therefore $\varepsilon_{xy} = 0.39\times 10^{-3}$. And once you have these values just like for the stresses we can draw the Mohr circle.
(Refer Slide Time: 01:23:00)



So we here is our circle just shift it a little bit here now let me zoom it a little bit. And one particular condition is known to us so the values are like this so we have calculated f strain x 0.85 strain y 0.2 and epsilon x y 0.39. So one of the x plane x y will be 0.39 and other will be -0.39 so the minus we have the way we have drawn here is in the higher side of x normal stress normal strain.

So this one will be zero point and once you have these values you would be able to also denote where the principle strains 1.03 here and the principle strain here which will be 0.018. And just from the circle you would be able to derive these values for calculating the stresses and strains from the given condition. So what we have seen here are that there are 2 different methods.

So that completes our understanding about the elastic behavior of the material next in the next chapter we will start discussing about the plasticity of the material. So with that we close this chapter thank you