Mechanical Behaviour of Materials (Part –I)
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Module No # 01
Lecture No # 05
Mohr's Circle

Welcome back students so today's topic for this lecture is Mohr circle so you would remember that we derived a, equation which will allow us to obtain sigma x that is the normal stress and σ_{xy} which is the shear stress at various orientations. Given that you have the normal stress and the shear stress at one of the particular orientations. So here you will see that we can simplify that process so let us begin.

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Christian Otto Mohr (1882)

$$\sigma_x' = \overbrace{\left[\frac{\sigma_x + \sigma_y}{2}\right]}^{\sigma_{avg}} + \left[\frac{\sigma_x - \sigma_y}{2}\right] \cdot \cos 2\theta + \sigma_{xy} \sin 2\theta$$

$$\sigma_{xy}' = -\left[\frac{\sigma_x - \sigma_y}{2}\right] \cdot \sin 2\theta + \sigma_{xy} \cos 2\theta$$

$$(\sigma_x' - \sigma_{avg})^2 = \left[\frac{\sigma_x - \sigma_y}{2}\right]^2 \cos^2 2\theta + \sigma_{xy}^2 \sin^2 2\theta + 2\left[\frac{\sigma_x - \sigma_y}{2}\right] \cdot \cos 2\theta \cdot \sigma_{xy} \sin 2\theta$$

This process of simplification is obtained by using what is called as Mohr circle which is given here Mohr circle. So Mohr circle is basically a technique by which the transformation equations for plane stress can be represented in graphical form and this particular graphical form is circle. And hence it is called a Mohr circle the name Mohr comes from the person who gave this theory which is Christian Otto Mohr in 1882.

He developed a graphical method for analyzing stress which is now known as Mohr circle. This graphical representation is extremely useful because it enables you to visualize the relations between the normal and shear stresses acting on various planes at a point in a stress body. So first let me write down the name of the author which is Christian Otto Mohr and the year was 1882.

Using Mohr circle you can also calculate principle stresses maximum shear stresses and stresses on inclined plane. And if you have plotted it on graph circle then you would see that you can directly obtain an approximate value or even as accurate a value as your graph paper would be. So moving on let us look at the equation that we had and from there you would be able to obtain these equations.

So the equations that we are talking about are $\sigma_{x'}$ which is some particular orientation. So this is a basically an unknown or a variable and $\left[\frac{\sigma_x+\sigma_y}{2}\right]$. So this sigma x and sigma y are the normal stresses at a particular orientation. So these are known values. So these are not variables and the other term is $\left[\frac{\sigma_x-\sigma_y}{2}\right]\cos 2\theta + \sigma_{xy}$.

So this σ_{xy} is the shear stress also on the starting coordinate system and this hence this is also a known value $\sigma_{xy}\sin 2\theta$. And other equation is sigma xy prime so you want in the new coordinate you want to know the normal stresses as well as the shear stresses. And remember we are obtaining only one of the normal stresses the other is just rotate this θ would be rotates when anti which would mean that 2θ will become 180 degree.

So you put just 2 θ = 2 θ + 180 and you would be able to obtain equation for sigma y prime. So this equation was given in the form $\left[\frac{\sigma_x-\sigma_y}{2}\right]\cos 2\theta - \sigma_{xy}\sin 2\theta$. Now when you look carefully at this equation you would realize that this equation can be easily be transformed into an equation for circle. How let us see so let us call this value sigma average.

Now if I put $(\sigma_{x'} - \sigma_{avg})$ whole square then what I have so I brought this on to the left side this term and then I squared it. So what I have is a + b team so a^2+ b^2+ 2ab so which will become

$$\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 (\cos 2\theta)^2 + \sigma_{xy}{}^2(\sin 2\theta)^2 - 2\left[\frac{\sigma_x - \sigma_y}{2}\right](\cos 2\theta)\sigma_{xy}(\sin 2\theta)$$

So this is the first equation when I have brought this term to the left side and square it.

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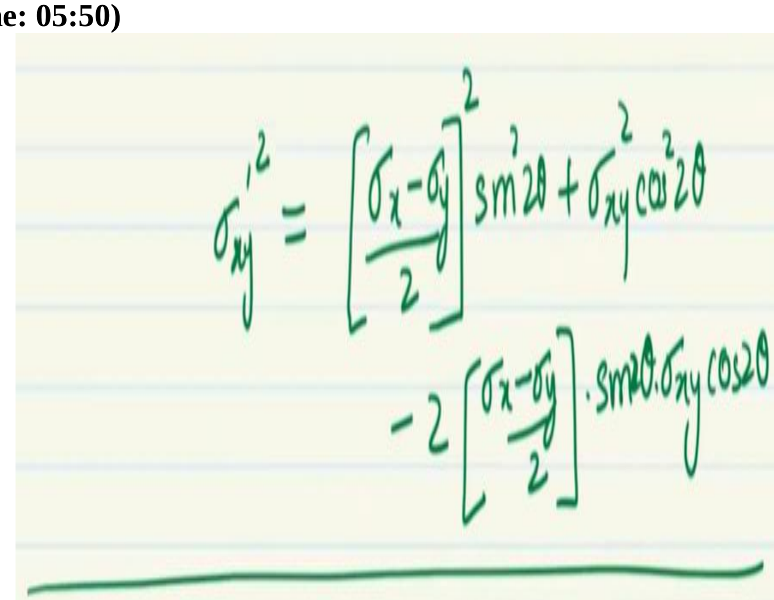

Now I will take second equation and I will directly square the term which is

$$\sigma_{x'y'}{}^2 = \left[\frac{\sigma_x - \sigma_y}{2}\right]^2 \sin 2\theta^2 + \sigma_{xy}{}^2 \cos 2\theta^2 - 2\left[\frac{\sigma_x - \sigma_y}{2}\right]\sin 2\theta\sigma_{xy}\ \cos 2\theta$$

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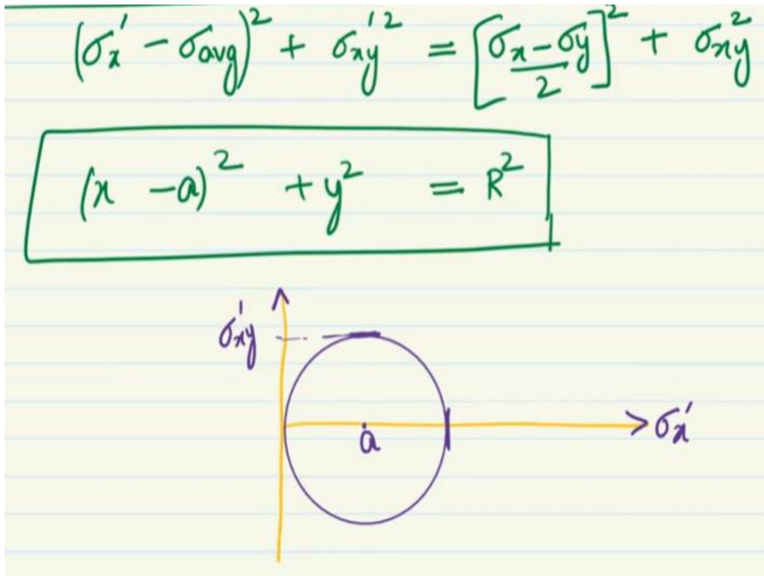

Now I will add these 2 relations and what I get is on the left hand side I have $\sigma_{x'}$ this is an unknown σ_{avg} this is a known quantity $(\sigma_{x'} - \sigma_{avg})$ square + $\sigma_{x'y'}{}^2$ this equal to. Now if you look at this term and this term these are the same terms but 1 is cos square 2 θ and another is sin square 2 θ. So when I add it becomes 1.Addition of both the equation leads to

$$\left(\sigma_{x'} - \sigma_{avg}\right)^2 + \sigma_{x'y'}{}^2 = \left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \sigma_{xy}{}^2$$

So the overall equation translates to this equation now overall here if you look this is unknown. This is something that you want to know the x-axis stresses and the normal stresses at particular orientation.

And this is shear stresses any particular orientation and there is known values. So this is some x normal stresses in the x and y directions which are given to you and similarly the shear stresses in our direction which is given you to you. Now what is very interesting as you can see that now that θ term is completely gone which means you would be able to find this sigma x prime and sigma xy prime for almost any particular orientation.

And that is the beauty of this another important thing to note here is that this equation has the form of a circle. So here this is like if you remember$(x - a)^2 + y^2 = r^2$ .so this equation is of that form. So your presenter of the circle would lie at some point on the x axis a distance away from the origin and on the y axis it will lie at the zeros of the y. And this term is the radius so here basically this becomes the radius of the r square term of the equation and this is the amount by which your center would be centered.

And if you draw it this how it would like so this is your circle and right now let us not worry about whether what is the end points and whether it intersects the axis or not. I am just trying to show you that the more circle is like this on the ever we know right now is that on the x axis you have theσ_x. Actually, it is $\sigma_{x'}$ and on the y axis you have the $\sigma_{xy'}$.

So this circle has x as the normal stresses and y is the shear stresses what is more that we are already said the circle center is shifted by a distance a. So this distance would be a, which was given by sigma average and there will be a maximum stress normal stress here. So because in the x axis we have the normal stress the maximum shear stress would be here. Because the shear stresses lie on the y axis so this would be your normal y axis shear stress.

So this gives you a glimpse of how easy it becomes to visualize the stresses or and the transformation of the stresses when we put it in the form of a Mohr circle. So moving ahead let us look at few more characteristic of the Mohr circle.

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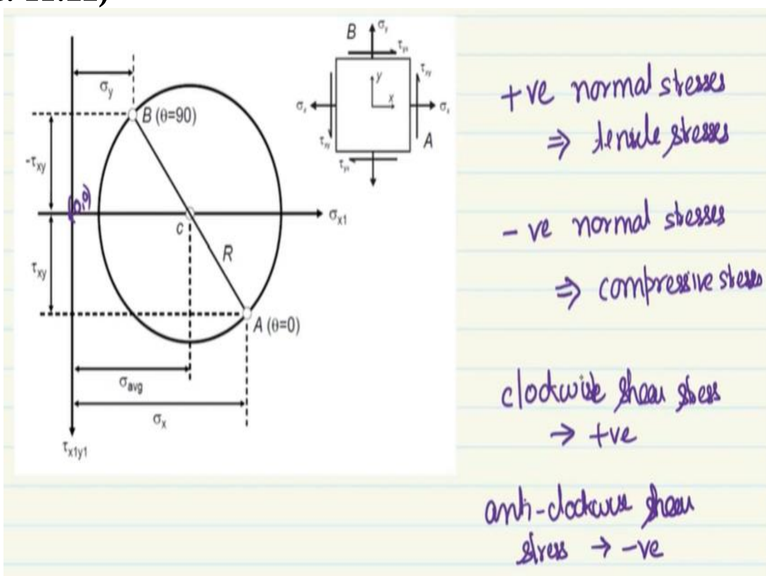

So let me copy image which will help us understand better so here is our Mohr circle. Like we mentioned on the x axis you have the normal stress and on the y axis you have the shear stress. Now for the Mohr circle normal stresses which are positive in values so positive normal stresses imply tensile stresses. And the negative normal stresses imply compressive stresses. So that is one important aspect when you are looking at the Mohr circle and in that respect.

If we assume that this point is 00 or the origin then it means that the normal stress throughout the circle is positive meaning this particular point in any orientation is experiencing only a tensile stress. And yes like I mentioned the circle is representing the stress state of one particular point. It may that all the points on the component are experiencing the same type of stress. So in that case this particular circle can represent the whole body but in general one Mohr circle represents one particular point.

And when you will look at it more carefully then there is one particular line for example this one and when you are looking at this line then it is showing you the stress so at one end it showing you the stress for one particular orientation. And at this particular value for example when let us look at a so here you have a, which is also shown in the orientation over here and this is a.

So this particular a point is representing this orientation and here you have 2 values one is normal stress one is the shear stress. So this is these 2 points that these 2 values will represent the normal stress which is this one and the shear stress like this one. So the x and y coordinate of a, are representing the normal stresses and shear stresses at this particular orientation a. Now if you look at the other point b which is over here so here again you will have 2 values x axis value and the y axis value.

So the x axis value will be representing the normal stress and the y axis value will be representing the shear stress. And important thing to note is that here when we rotate the orientation from 90 degrees so a to be as you can see the rotation orientation difference is by 90 degrees which would imply a 180 degree rotation in the Mohr circle. Because here it is rotation by θ and here it is rotation by 2 θ.

So when you rotate by 90 degrees here it gets rotated by 180 degrees here and that is where it helps when we write that equation if you remember we had changed the format of the equation so that our equation becomes into the format form of,= 2 θinstead of writing it as θ form. And if we had not done that then we would not get an equation like a circle. And if we do not get a circle like equation then we will not be able represent it in a very easy to visualize format like this.

So the overall we knew it beforehand without explicitly mentioning we actually included that 2 θ format into our equation. But now at this stage you would also be able to appreciate that why we move to the 2 θ format. And now you will now also be able to see that because of the fact that we move from θ to θ we also see that effect over here. So this when you rotate θ and the real space; in the Mohr circle it rotates by 2 θ.

So these are couple of important things to remember and another important thing is to remember is that if you have so for the normal stresses we know positive implies tensile and negative implies compressive. Similarly if you have clockwise shear stress so clock wise shear stress is represented by positive values. So in the picture you can see in the b we have clockwise rotation or of the shear stress which is positive.

So clockwise stress is given by positive values and for the, a you can see it is anti-clockwise and the value is negative. So that will always be the case so I got a little bit messed up but the main point is that one will remain positive and other will remain negative and it is a convention. So you can select a different convention but this is what is usually accepted so these are the 2 important aspects of the Mohr's circle another important aspect is that we see that the Mohr circle center lies on the x axis.

So if the questions may arise in your mind that is it possible that we can have a Mohr circle away from the x axis and the answer is no and it all boils down to this equation. So in this equation you can see that the equation is of the form (x – a) overall it comes out like this $(x - a)^2 + y^2 = r^2$ So the center would always lie shifted away from the x origin on the x axis by a distance a there is no term over here in the y which is as good as saying y – 0.

And therefore this circle remains confined on to the x axis it cannot move out to the x axis. So that is another important think to keep in mind now that we have little bit of understanding let us try to draw the Mohr circle for the first example of a vertical bar which was loaded. So let us see how that would look like.

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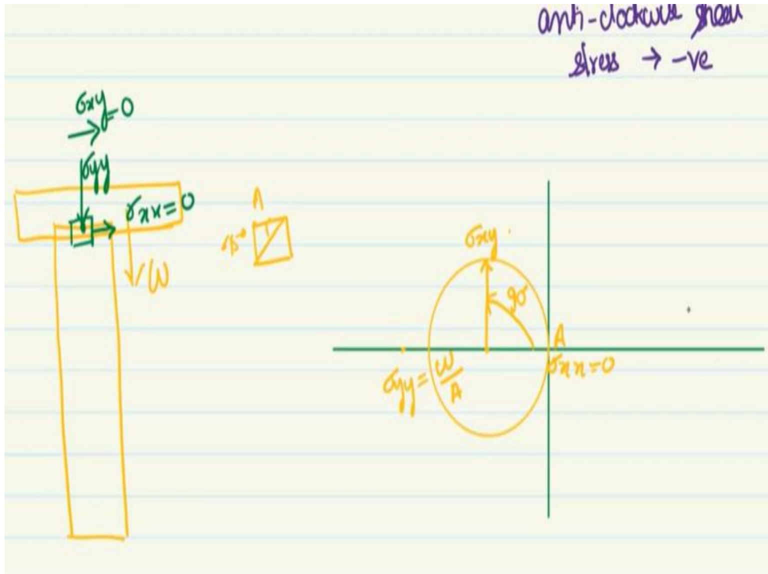

And it was loaded with force equal to w and the first thing that you have to realize is that where are we drawing the Mohr circle on. So let us say we are drawing the Mohr circle at this particular point not at this particular point this is how the force is acting in this direction. And if you look at the shear stress so the shear stress is equal to 0 for $\sigma_{xy} = 0$ let us say this is σ_{yy} and the sigma other element normal stress which is in this direction σ_{xx} now this is also equal to 0.

So we have compressive stress in the normal direction and 0 stress in the normal direction and the shear stress is equal to 0. So how does this look like so let us put a circle over here draw it like this because we have a circle to insert here. Now what will be the value of the circle and we know that one of them is negative normal stress on the other is 0. So these 2 points must differ by θ = 180 degrees and $\sigma_{xy} = 0$.

So we can say that this is also a point which point lies on the x axis so one of the point is this one and other is this one. And therefore now we can insert the circle over here to make it more precise let me move it and let it touch over here. So this is $\sigma_{xx} = 0$, $\sigma_{yy} = w/a$ whatever and you can see that shear stress is equal to 0 at this point. Now this also tells us lot of things for example this is a center of the circle and therefore this particular is the maximum value of shear stress that you can obtain so this is σ_{xy}.

So at what particular orientation it you can obtain this now this from here to here if you look at it so this is point if you are looking at. Now if you call this as point a, this particular orientation if you call this as a then we know that the σ_{xy} is 90 degrees away. In the remote circle which would mean that in the physical space it would be half of that because this is 2θ.

So in the physical space it is θ which is equal to 45 degrees so you can draw it like this so at this particular orientation which is 45 degree you would get shear stress as highest which is σ_{xy}. And you can even find the value why because you know the radius this is 0, 0 this is σ_{yy} so half of the radius which means σ_{xy} will be equal to $\frac{\sigma_{yy}}{2}$. So you that it becomes so easy to find out the maximum shear stress.

I did not have to do much calculation and that is the beauty of the Mohr circle so moving on to understanding other characteristics of the Mohr circle.

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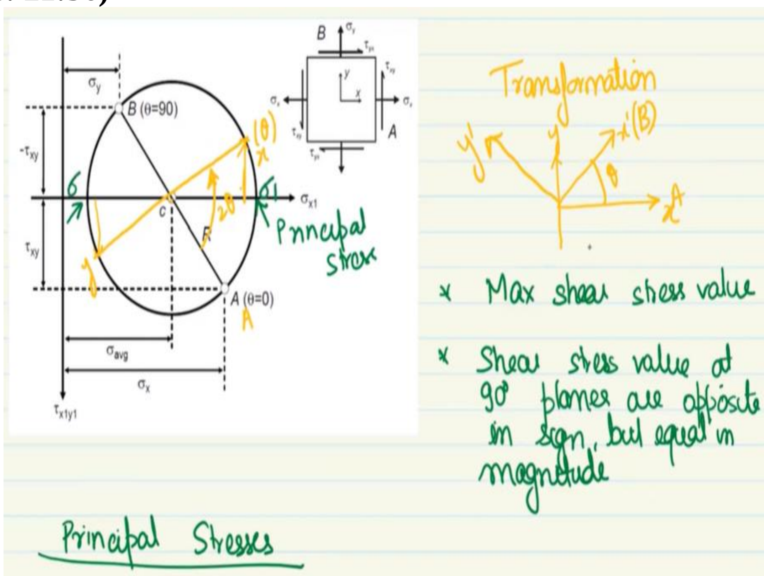

Let me again copy this image now we will look at the something that we have already touched upon and which is the transformation. So I just now mentioned that when we talked about the transformation that if you move from a to b in physical space it is moving by 90 degrees but in the rear in the Mohr circle it will move by 2 θ which is equal to 180 degrees.

So let us say this particular orientation and then so over here you have so let us say this is your orientation x and y and then you want to rotate to another orientation which is x prime y prime and let us say this is equal to θ this plane this rotation is θ. Therefore if your original one which is this one a x and y they were over here. So that let us say this s represented as a so somehow I am not able to select the color.

So I will move on and this is a, and this becomes my b so that this is rotated by θ so in here I will rotate by 2 θ. So this is θ is equal to 0 here some θ value which is actually equal to 2 θ away from this one and here you will have the in this direction it will be your x here you will get the y and another important thing you would notice is that. Since x and y are 180 degrees apart on the Mohr circle therefore the shear stress would always be negative for each other.

So for this one if this is the shear stress value then for one this will be the shear stress value and both of them are opposite of each other this is something. We already know basically on the basis of the fact that these 2 shear stresses have to counter balance. If they do not counter that balance what will happen, it is equivalent to saying that this element will start to rotate. And since it is in equilibrium so it cannot rotate and therefore whatever shear stress value you get or here it will be the inverse of the shear stress that you get over here or these 2 will be in equal in magnitude and opposite in sin.

So this is another important aspect about Mohr circle so you can get to know about the transformation very easily using this and if you want to calculate the value of the radius like I mentioned earlier whatever value you have over here. So you take (x + y)/2 that will give you the center and in the center you can add calculate the half of this distance that will give you the radius.

And if you have the radius then that is also equivalent to the maximum shear stress value. So you can get max. So we can get max shear stress value and we get to know that the shear stress value at 90 degree orientations 90 degree planes are opposite in sign but equal in magnitude. So this is another important aspect that we get to learn now another still I mean this Mohr circle is.

So versatile because you know we will get a lot more information so what is the other information that we can get we talked about principle stresses. Now you would see that we can also get principle stresses remember what was the principal stresses are basically the stress where no shear stresses are acting and it is the highest value of the stresses. Now if you look at the normal at this circle so each point the value of the x-axis represents the stress at that particular point.

And when you are on the x axis that is when the shear stresses are 0 and at one particular point the normal stress. The normal stress is highest and at the other point the normal stress is lowest and therefore it gives you directly gives you the value of the principle stresses. So this is one principle stress this is another one so this is σ_1 and this is σ_2 so we can get σ_1 , σ_2 values we are able to get maximum shear stress.

We can also see that the shear stresses are equal in magnitude opposite in sign we also see that you can get the values of normal stress and shear stresses at any other orientation. So this there are so much more that we can obtain when we are using always putting things in the Mohr circle format and that is why Mohr circle is. So versatile and so powerful now let us go through some simple examples to understand this concept in little bit more detail.

So let us say we want to calculate the shear the Mohr circle or we want to draw the Mohr circle for a uniaxial tensile condition.

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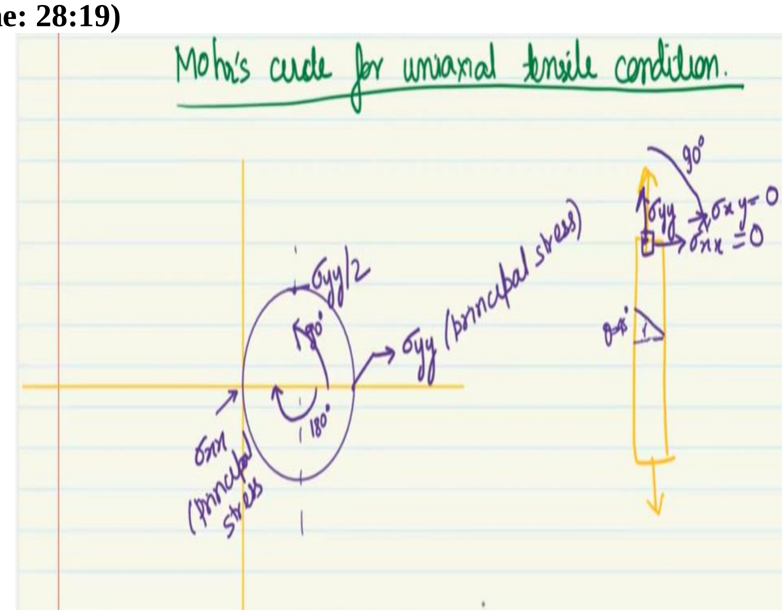

So how would it look like? So the Mohr circle for uniaxial condition not be very different from the one that we drew so we will just have a look at it what do we have in the uniaxial tensile condition what we have is something like this where you are pulling the material like this. And where are we trying to get the stress so let us say this is the a. So this is this particular plane this is the orientation and along this we have σ_{yy} sigma y y here this is σ_{xx} which is equal to 0 and $\sigma_{xy} = 0$.

σ_{yy} is the positive direction and so it is tensile stress and sigma axis is 0 so the simplest thing that, we it is one of the very simple conditions and therefore it will come out like this. So this is your σ_{yy} this is σ_{xx}which is also the principle stresses and this is also principle stress and clearly $\sigma_{xy} = 0$ at this particular point and where do we get the maximum shear stress.

So we get the maximum shear stress at this particular value and this is oriented by 90 degrees in the Mohr circle which would mean that it is oriented by θ equal to 45 degrees in this particular sample or in this condition. So at the 45 degree condition you get your the maximum shear stress in a tensile loading condition. So what else can we so what all did we obtain we obtained one the Mohr circle we obtained what are the principle stresses?

We obtained what is the maximum shear stress which is equal to $\sigma_{yy}/2$ and we obtain what is the particular orientation at which the shear stress is highest and how is it oriented with respect to the normal stress. And one more thing we know this is the maximum normal stress this is a minimum normal stress and this is rotated this is 180 degree apart in the Mohr circle which would mean that here it is 90 degrees apart which we already knew.

So this is 90 degrees apart so the Mohr circle is consistent that is again proved so it has been our examples show again and again that the Mohr circle is consistent and that it is very versatile we can obtain a lot of data from this small circle.

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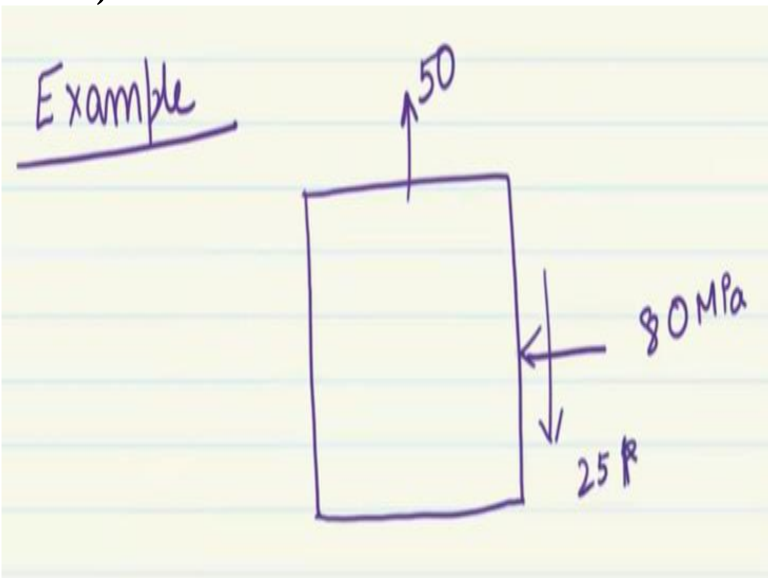

Now let us look at some a little bit more complex examples so let us say this is 50 MPa; this is compressive 80 MPa and the shear stress acting is 25 MPa.

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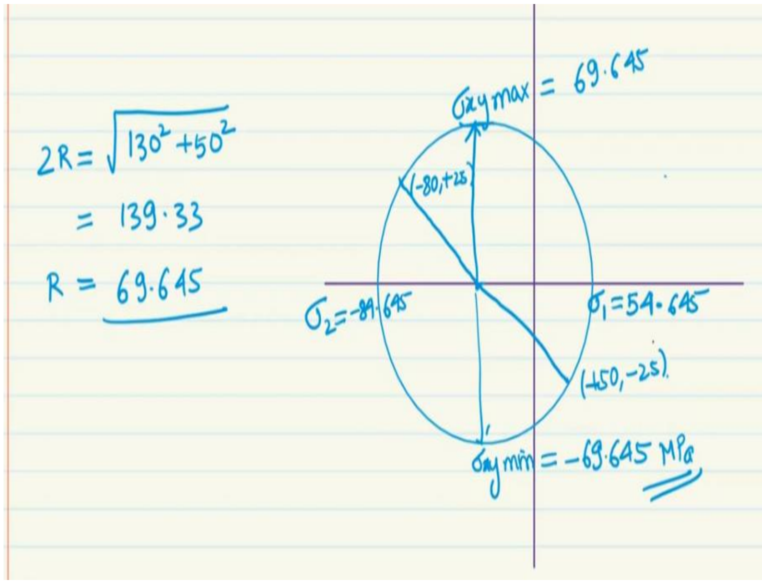

So if have to draw the Mohr circle for this how would it look like so one of them is negative minus a t and one of them is positive and there is also shear stress if it is here then it should also be here. And therefore both of them have shear stresses acting and therefore none of them are principle stresses. So the principle stress values would be higher than these. Now let us draw it and the condition what that we have here based on that we can say?

So here it is -80, 25 so here it is the clockwise direction so it is positive but this one is negative. So somewhere over here we are talking about so -80 on the x axis and +25 on the y axis. On the other hand this is +50 on the x axis and -25 so in terms of u axis it is symmetric +50, -25 and like we know that these 2 must be diametrically positive to each other. So although I have not drawn it very to the scale but I should be clear that it is diametrically opposite to each other.

And if it is diametrically opposite to each other then we know that we can obtain the value of radius by using these 2 coordinates. So basically $2R = \sqrt{130^2 + 50^2} = 69.645$ .Now if this is the radius how many things we can now obtain.

One first let us look at so there will be a center over here which will be between -80 to +50 and when we add the radius to this we will get sigma 1 and from there you can see that σ_1 will be equal to 54.645 and the σ_2 will be this point which is the center point minus this radius which will be equal to 6 – 84.645. And the radius is actually if you calculate it will come out to 69 so 69.645.

So this particular shear stress value σ_{xy} max = 69.645 and this point is also -69 other way actually it will be σ_{yx}. So we have obtained so many values we have obtained the principal stresses and we have obtained the maximum shear stresses we have obtained what is the sigma average radius and so many things just by drawing this circle. Because the relation that we obtain for σ_x and σ_{xy} had a particular relation which you could convert into a equation into a circle format.

And therefore it made our tasks so much easier and we are able to get so much data. So we will close this session and when we come back next we will talk about elastic stress thank you.