Mechanical Behavior of Materials-1
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Lecture - 39
Grain Boundary Strengthening

(Refer Slide Time: 00:13)

Welcome back class to our course Mechanical Behavior of Materials. So we have already discussed about two strengthening mechanisms, precipitation strengthening and dispersion strengthening and the second one solid solution strengthening. Today we are going to discuss about grain boundary strengthening, okay. So here I have shown an optical micrograph of a polycrystalline alloy, okay.

So you can clearly see here grain boundaries. So these are your grain boundaries, okay. Now why do we see grain boundary between two grains. So here I have shown on the right side schematic where you have grain 1 on the left side and grain 2 on the right side. And what you can see here is a change in the orientation, right? So orientation changes from grain 2 to grain 2.

And you have a grain boundary here. So this is your grain boundary, okay. So what you see, you have an orientation here something like this, right? This is your orientation. But here it is in this direction, it changes. So there is angle here right? So the orientation between the two grains it is changing and that is why you are going to see a boundary between the two grains and that boundary is called grain boundary.

Now this grain boundary is also contributing towards the strengthening of a given alloy system. And we are going to learn about this particular concept why grain boundary is contributing towards the strengthening. What makes it you know, what makes it suitable to increase the strength of a given alloy system, okay.

(Refer Slide Time: 02:57)

So here I have shown a dislocation and then you can also see two grains. So we have grain 1 then grain 2, slip plane, grain boundary. So we have a dislocation because dislocation movements needs to be restricted so that we can increase the strength of the alloy and that is the crux of all strengthening mechanisms, we have discussed that before, okay. So what is happening now that suppose this dislocation is moving on the slip plane and reaches to grain boundary, right?

So what it says it has to change the path now, is it not because the orientation has completely changed from grain 1 to grain 2, right? That is point number one. And second, the grain boundary itself has an atomic disorder, right? So there is a discontinuity of the slip plane also from one grain to another. And this leads to the strengthening of the alloy because of the change in orientation between the two grains.

So strengthening occurs due to point number one, a dislocation passing into grain 2 from grain 1 will have to change its direction of motion, okay. Why because there is a crystallographic misorientation. So due to crystallographic misorientation, okay. That is point number one. And point number two is the atomic disorder within a grain boundary region will result in a discontinuity of slip plane from one grain to another, discontinuity of slip plane from one grain to another.

That means grain 1 to grain 2. So when a dislocation moves and it reaches towards the grain boundary right, it sees that the slip plane direction has changed, so it was like this now it is like this, okay. So the slip plane direction has changed, that is number one. And it has to also change the direction of motion is it not? So these two factors leads to the strengthening in the alloy and this is happening because there is a change in orientation of the alloy, okay.

Now what will happen when a dislocation is near the grain boundary and it is not able to change its position right, change the slip plane it is not able to traverse through the grain boundary, it is going to pile up at the grain boundary. So let us discuss about what is called pile-up.

(Refer Slide Time: 07:58)

So pile-up of dislocations, okay. So if dislocations are not able to traverse through obstacles such as a grain boundary in this case, what will happen it will lead to piling up, pileup of dislocations at gbs, so grain boundaries right? So what is happening, a dislocation is coming. So let me draw a grain boundary here. So suppose you have a grain boundary, so this is your grain boundary. So I will rally gb for grain boundary, becomes easier, okay.

So you have a gb here and then a slip plane. A dislocation is moving on a slip plane and it reaches towards to the grain boundary, it reaches at the grain boundary. Now it is not able to traverse through the grain boundary. So it cannot cross, so it will be staying there, right? Now suppose, so this is say dislocation number 1. Now this is dislocation number 2. A second dislocation is now moving right on the same slip plane.

And then it also reaches towards the grain boundary, right? But what will happen, the first dislocation is going to repel the second dislocation and mind it this is happening when there is a applied stress, right? So you have a applied stress and that is why dislocation is moving, right? So first dislocation is already at the grain boundary. Now second dislocation is moving along this direction.

As soon as it reaches towards the first dislocation, the first dislocation is going to repel the second dislocation, right? And second dislocation now will be at equilibrium due to the applied stress and the stress due to dislocation number 1. So they are going to maintain a certain distance here, okay. Now let us see what happens when we have third dislocation. So again the same grain boundary, slip plane.

(Refer Slide Time: 10:52)

So we already have dislocation number 1, then dislocation number 2. Now a third dislocation is coming. So we have dislocation number 3 and somewhere here you have source, source of dislocation which is ejecting dislocations continuously on this slip plane and because of the applied distress here dislocations are moving on the slip, okay. So the third dislocation is moving along this direction.

Now what will happen, as soon as it reaches to say this position nearby dislocation 1 and 2, the third dislocation sees a stress field because of the applied stress and the one and second dislocations they are going to repel the third dislocation. So again third dislocation will be at equilibrium position because of the applied stress and the repelling stress of 1 and 2, okay. So what will happen?

The third dislocation will come and then it will also occupy a position something like this. So what you are seeing that the distance between first and second dislocation is lower than the distance between second and third dislocation. Now the fourth dislocation will come and it will also be somewhere like this, okay. So now this distance if I mark this as a x 1, this is say x 2 or say x 23 and x 12 and this is x 34.

So you can clearly see that x 34 is greater than x 23 than greater than x 12, okay. So as you move away from the grain boundary the distance between the dislocation increases and this is happening why? Remember, whenever dislocation 2 comes it sees the repulsion only from dislocation number 1, okay. So there will be some distance. Now when the third dislocation comes, it will see a repulsion because of a combination of 1 and 2.

So it will have slightly higher repulsion stress compared to what was observed by dislocation number 2, okay? So it is going to maintain a slightly higher distance. Now when dislocation 4 comes, you are going to see distribution going to see the repulsion stress because of a combination of 1, 2 and 3. So there will be another larger distance between dislocation number 3 and dislocation number 4.

So what you see that these dislocations are actually piling up in front of the grain boundary, okay. And that is why we call this phenomenon as piling up of dislocation at a obstacle. In this case the obstacle is grain boundary, okay.

(Refer Slide Time: 14:05)

So we can write that dislocations piled up against the obstacle and in this case it is grain boundaries. They are going to produce a back stress. So when the dislocation 2 comes, dislocation 1 which is already there in the pile-up, it is going to produce a back stress on dislocation 2, okay. Similarly, when dislocation 3 comes both dislocation 1 and 2 they are going to produce combined back stress on dislocation number 3.

And this back stress is actually it is opposing the dislocation motion. So all the pile-up dislocation, they are going to produce back stress to oppose the motion of any additional dislocation, okay. That is point number 1. Point number 2 is dislocation will be tightly packed together near head or near grain boundary. And as you move away from the grain boundary, they will be widely spaced.

As one moves away from the grain boundary, the distance between dislocations increases, right? That means they are widely spaced, widely spaced right? And this is happening towards the source, okay. So you can clearly see here if I taught you about this 4 dislocations here, okay. So near the dislocation, near the grain boundary you can see dislocation 1 and 2 they are tightly spaced.

And as you move away from the grain boundary you can see that the dislocations are widely spaced and your source is here, okay. So there will be n number of dislocations, right? It is not only 1, 2, 3, 4. There will be lots of dislocations, which are going to be piled up at the obstacle. This obstacle in this case is grain boundary.

And mind it the dislocations are moving because you have already applied a stress. So there is application of stress also and because of that dislocation is moving on the slip plane, okay.

(Refer Slide Time: 17:49)

$$F_X = \frac{Gb^2x(x^2 - y^2)}{2\pi(1-\nu)(x^2+y^2)^2}$$

$$F_X = \frac{Gb^2x(x^3)}{2\pi(1-\nu)(x^2)^2}$$

$$F_X = \frac{Gb^2}{2\pi(1-\nu)x}$$

Now let me again draw this pile-up. So this is your gb. So I will give you one formula now. So this is a slip plane and then we have lots of dislocations and we have to now maintain the distance also, so something like this, okay. So it is increasing as you move away from the grain boundary and somewhere here you are going to have an edge dislocation. This is 0, 1, 2, 3, 4.

So n number of dislocations they are getting piled up at the grain boundary and the applied stress is tau, okay. And the distance I am measuring from the grain boundary. And I will tell you what is i here, okay. So you have all these edge dislocations. They are on the same slip plane, right?

So the general formula for edge dislocation, the force between two edge dislocation we can write as Gb square by 2 pi 1 minus mu x square minus y square divided by x square plus y square whole square, okay. So why it is 0 here? Because we are talking about the same slip plane and two dislocations those who are interacting they are on the same slip plane. So y becomes 0.

So force in this case we can write as Gb square by 2 pi 1 minus mu x cube by x 4. Or finally Gb square by 2 pi 1 minus mu into 1 by x. So in this particular pile-up case it depends, the force between two dislocation depends upon the x value right, distance between these two dislocation, okay.

(Refer Slide Time: 20:11)

$$\tau_b - \sum_{\substack{j=0 \\ i \neq j}}^{n} \frac{Gb^2}{2\pi(1-\nu)(x_i - x_j)} = 0$$

So now I have already mentioned that each dislocation in this pile-up as shown above is under equilibrium, okay. So if I consider dislocation number 3, it is under equilibrium because of what, because of the applied stress and then interaction between all the dislocations in the pile-up. So if I taught about say dislocation number 3 here, if I just choose this one, so you have, so this is dislocation number 3, so you have applied stress tau.

Then you have again stresses because of the dislocations which are present on the slip plane on the left side of the dislocation that will have the same direction as the applied stress. Then there will be repulsion from 0, 1 and 2 dislocation that will be on the opposite side, okay. So dislocation number 3 is under equilibrium because of effect of apply stress okay and then stresses due to the other dislocation in the pile-up, okay.

So now let us assume that this all dislocations are of edge character in the pile-up. So if I want to understand the force on a particular dislocation, so the resulting force acting on the ith dislocation we can obtain or we can say this is in equilibrium right? So we can write this as tau b minus sigma j equal to 0 to j equal to n, i is not equal to j and then Gb square divided by 2 pi 1 minus mu and then x.

So x i minus x j. And since this is in equilibrium it will be 0, okay. So a particular dislocation if you see I was talking about number 3 here, right? So this particular dislocation, number 3. This is under stresses of applied stress and then the stress is due to the other dislocation and this is under equilibrium, dislocation number 3, right? So the forces on dislocation number 3 should be 0, okay.

So now because of the applied stress the force on the dislocation will be given by tau b, okay. And the interaction forces between this dislocation number 3 and other dislocation will be given by this summation. Now the combination of these two should lead to 0 because dislocation number 3 is under equilibrium, okay. And you can calculate this, okay.

(Refer Slide Time: 24:24)

$$\tau_b - \sum_{\substack{j=0 \\ i \neq j}}^{n} \frac{Gb^2}{2\pi(1-\nu)(x_i - x_j)} = 0$$

So if you consider let us see I have a dislocation gb here and say we are just considering dislocation 1 and dislocation okay, only two, one dislocation in the pile-up. So here it will be now 0 and 1, okay. So it will start with 1. So 0 and 1 okay. So now if I want to calculate what is the force on first dislocation. So tau b, so I can write this equation. This equation will become tau b minus summation here okay.

So j equal to 0 and i is not equal to j. So i here is 1 because we are talking about the ith dislocation and here i is equal to 1, the first dislocation. So this will be Gb square divided by 2 pi 1 minus mu x i that means x 1 minus j equal to 0 to n, n is equal to 1 here so it can go to maximum 1 so x 1 minus x 0. And second term will be 0, x 1 minus x 1. That will be 0 anyway. That cannot be counted, i is not equal to j, okay.

So this is the equation for ith dislocation. Similarly, you can calculate for dislocation number 2 or say you have now added another dislocation, dislocation number 2 and now you want to calculate what is the force on dislocation number 1. You can use the same equation. So you are going to have tau b because of the applied stress. Then you are going to have a repulsion force from dislocation number 0.

So if I have number 1 here then number 2 and number 0 here, okay. So on number 1 if I want to calculate, so you are going to have tau b in this direction because of the applied stress. Now because of the dislocation 2 you are going to have another force from dislocation number 2 and then from 0 you are going to have force in this direction. And summation of all this should be 0 because 1 is in equilibrium.

And if you do that, if you use this particular equation on the top, you can solve this equation very nicely, okay. And remember n is the number of dislocation in the pile-up. So this is the generalized equation, this one, for resulting force on a ith dislocation if you have n number of dislocations in the pile-up, okay.

(Refer Slide Time: 27:35)

Now at the leading dislocation, the first dislocation in the grain boundary here, so I am talking about this particular dislocation okay. So what are the forces there now? You have forces from the dislocations in the pile-up, so you have lots of dislocation here, nth number of dislocation, right? And then you are also applying tau.

So on the leading dislocation in this region, you have a combination of stresses because of the applied stress as well as repulsive stresses from all the dislocations which are in the pile-up, okay. Now what will happen, this will lead to a very high stress concentration in this region at the leading dislocation is it not, okay? So the leading dislocation in the pile-up okay it will have stresses due to other dislocations in the pile-up and also applied stress right, stress due to applied stress.

And all of them are acting on the same direction, right? Okay? So what will happen this will lead to a very high stress concentration, okay? So this will lead to high stress concentration at the region of leading dislocations, okay. So now what can happen if you have a very high stress concentration? Two things can happen. One, the yielding can happen on the other side of the dislocation.

That means you can generate new dislocations on the other side of the dislocation and that other side of the grain and that grain can yield. That is point number 1. So if you have grains here and say you have piled up of dislocations and at the leading dislocation here you have a high stress concentration, okay.

So what can happen, this particular grain that has not yielded yet, because of the high stress concentration in the leading dislocation here, you know dislocation source such as Frank-Reed source can activate in the grain 2, if I name as grain 2 and grain 1 here. So yielding can occur in grain number 2. You can have nucleation of dislocations that is point number 1.

And second, instead of the yielding of the second grain you can initiate cracks at the grain boundary here, at this point, okay. So you can nucleate the crack. So these two things can happen if you have a high stress concentration at the leading dislocations because of the combination of applied stress as well as stresses due to other dislocations in the pile-up, okay.

(Refer Slide Time: 31:43)

So this will lead to two points. Point 1, yielding can occur on the other side of the grain boundary or say in this schematic here grain 2, okay. So you are going to see nucleation of dislocations; that is point number 1. And second you can see nucleation of crack at the grain boundary, okay.

And again this is happening because you have high stress concentration at the leading dislocation because of the pile-up of dislocations in the grain boundary, okay. Now what is the value of that high stress concentration we are talking about?

(Refer Slide Time: 33:18)

And that value was calculated approximately to be tau b and tau b sorry tau a where n is your number of dislocations in the pile-up okay and tau a is your applied resolved shear stress, okay. So the tau what I was mentioned before now it is tau a and that is what that is resolved shear stress on a particular slip plane, okay.

So if you have a grain boundary and then you have lots of pile-up of dislocations, so the stress concentration or the stress value with tau star here at the leading dislocations because of the applied stress and dislocations in the pile-up can be given by n multiplied by the applied resolved shear stress. So what is happening? It is actually multiplying with the number of dislocations in the pile-up with the applied resolved shear stress, okay.

So this derivation was introduced by okay and so tau star this is stress at the leading dislocation okay due to presence of other dislocations plus applied stress both. And that is how you are getting the stress concentration. So overall what you are seeing, the effect of n dislocations at the pile-up is to create a stress which is n times greater in the applied stress is it not?

(Refer Slide Time: 36:04)

So overall you can say the effect of n dislocations in the pile-up is to create a stress at the leading dislocation which is n times greater than the tau a applied stress, applied resolved shear stress because we are talking about the slip plane, on the slip plane; shear stress, okay. So if you have n number of dislocations, you are going to increase the stress at the leading dislocation which will be n times applied resolved shear stress, okay.

Now you have to also realize that all these dislocations which are piled up at the grain body they are also going to apply back stress at the source.

(Refer Slide Time: 37:41)

$$\boldsymbol{n = \frac{L\pi\tau_{\alpha}}{\alpha G b}}$$

So if you have again gb here and lots of dislocations and suppose you have a source here say Frank-Reed source which are generating dislocations continuously, okay. So all these dislocations in the pile-up here right, they are going to apply back stress on the Frank-Reed sources itself, right? So this does not mean that you know Frank-Reed source can generate dislocation continuously.

There will be certain number of dislocations this can generate because of the back stress from the dislocations in the pile-up, okay. So how many dislocations you can have in the pile-up there is a restriction and that number is given as n equal to L pi tau a by alpha Gb okay and where tau a is resolved shear stress, okay. Alpha is geometrical constant and it is given as 1 for screw dislocations or 1 minus mu for edge dislocation.

B is your Burgers vector okay and L is the length of the pile-up, okay. L is length of the pile-up. That means say if the pile-up is only up to this point, so we will call it as length. Typically it will be from the source, okay. So due to back stress you can generate only certain number of dislocations.

(Refer Slide Time: 40:13)

$$\boldsymbol{n = \frac{D\pi\tau_{\alpha}}{2\alpha G b}}$$

Let me write down that also. Due to back stress from the piled-up dislocations only certain number of dislocations generation okay and that is given by n and n is given by this equation here, okay. So now you know the number of dislocations which can be piled-up when you have a applied stress of tau a and length of the pile-up is given by L. So I can write what is the length of pile-up from this particular equation on the top.

That will be given as alpha n Gb by pi tau a. So this is the length of the pile-up, okay. Now suppose the dislocation source is at the center of the grain, so you have this say grain and here is your Frank-Reed source, okay. So length of the pile-up of dislocation, so this is your pile-up of dislocation. So this length L in this case can be given as D by 2 where D is your grain size, okay.

So we are assuming that the source which is generating the dislocations on that particular slip plane which is leading to finally the pile-up of dislocation, that source is sitting at the center of the grain, okay. So the pile-up length L will be given as D by 2 where D is your grain size. So I can write this as D by 2 equal to alpha n Gb by pi tau a. This is another equation, okay.

And assumption here is that the Frank-Reed source okay or the source which is generating dislocation is sitting at the center, okay.

(Refer Slide Time: 43:04)

$$\boldsymbol{n = \frac{D\pi\tau_\alpha}{2\alpha Gb}}$$

So now suppose you have reached to grain boundary here. So you have reached to grain boundary dislocations and now you want this dislocation to just traverse through the grain boundary. So that will also require some amount of stress is it not if you want to traverse this dislocations, not grain boundary.

If you want to traverse these dislocations through the grain boundary, you require some critical stress value to overcome that particular barrier. So if I assume that tau c is that critical stress which is required to overcome this particular barrier right, we can write an equation. So if tau c is the critical stress required to overcome the Gb okay obstacle. So in this case obstacle is Gb okay and this overcome Gb obstacle.

That means, dislocations in the pile-up are able to traverse through the grain boundary, okay. So dislocations in the pile-up are able to traverse through grain boundaries, okay. Then I can say that if I consider the leading dislocation, what was the stress at the leading dislocation, n tau a, okay. So n tau a has to be greater than equal to tau c.

So if the stress at the leading dislocation is higher than the critical stress required to overcome the grain boundary for these dislocations, these dislocations can actually burst from the grain boundary through the grain boundary. And that relationship will be given by this particular equation. It has to be minimum tau c. So it is greater than equal to tau c, okay. Now n we know.

We use this formula on the top. So n we can write as d pi tau a divided by 2 alpha Gb that is n from this particular equation we are using. And then tau a is already there greater than equal to tau c, okay.

(Refer Slide Time: 45:50)

$$\boldsymbol{\tau_\alpha \geq \sqrt{\frac{2\alpha G b \tau_c}{\pi}} D^{-1/2}}$$

And then we can write tau a square is greater than equal to root over 2 alpha Gb tau c divided by pi and D I am taking out, it will become D to the power minus half. This square will not be there, okay because I have already taking square root on the right hand side. So if I want to move dislocations through the grain boundary the applied stress should be this much, okay.

It is a function of diameter of the grain size and again remember we are assuming that the Frank-Reed source is at the center of the grain, okay. So we can overall write this as tau a greater than equal to tau naught plus K D to the power -1/2 where K is this particular constant. So this is your K, okay. And we have added tau naught because of the friction stress in the lattice.

That also needs to be overcome right? Even if you do not have grain boundaries the friction stress which is piled stress that has to be overcome if you want to move the dislocation, okay. So tau naught is added to take into account the friction stress needed to move a dislocation, okay. So overall you are getting this particular equation.

And you know you will see eventually that Hall-Petch relation is also given by this equation, almost this equation, where we are replacing tau with sigma, okay.

(Refer Slide Time: 48:19)

$$\tau_\alpha \geq \tau_O + KD^{-1/2}$$

So Hall-Petch Theory. So whenever we talk about grain boundaries means this particular theory will be always discussed about, okay. So what is Hall-Petch equation etc., etc. It gives a very nice description or nice relationship between the strain and the grain size, okay. And remember the grain size is related to grain boundary, okay. So in a given volume if you have larger grain size, the grain boundary density will be smaller, okay.

So Hall and Petch, both are scientists, they postulated that dislocations can burst through grain boundary. That is what we have discussed in the previous session also, okay. And to do that you require large number of dislocations in the pile-up so that you have large stress concentration which can overcome tau c, which is a critical stress required, right?. So valid for large number of dislocations in a pile-up, okay.

And we know tau a is greater than equal to tau naught plus K D to the power -1/2. So this is the genesis of Hall-Petch relation and this is what Hall-Petch actually postulated.

(Refer Slide Time: 50:21)

$$\tau_\alpha \geq \tau_O + 2\tau_c\, r^{1/2} D^{-1/2}$$

Later on what happened Cottrell came and he proposed another theory. So what was the proposition here? So you know he mentioned that it is virtually impossible for dislocations to burst through the boundaries, okay. Instead, what will happen, since you have a stress concentration at the leading dislocation in the pile-up you are going to activate dislocation sources in the adjacent grain such as Frank-Reed source, okay. So that is what they proposed and they also gave a formula for that.

So it is virtually impossible for dislocations to burst through grain boundaries. Then instead of bursting of dislocation you will have yielding of dislocations, yielding of grains, neighboring grains because of the nucleation of dislocations or say activation of Frank-Reed source in the neighboring grain.

So the stress concentration produced by pile-up in one grain activates dislocation source in the adjacent grain, okay. So if you have a grain and you have Frank-Reed source here, FR source. You then have pile-up of dislocations, something like this, okay. So in the neighboring grain okay, so somewhere here say, you are activating Frank-Reed source. So if it is grain number 1, this is grain number 2.

Because of the stress concentration in the leading dislocation you are activating a Frank-Reed source in the neighboring grain, say at a distance of r from the grain boundary. So this particular distance here is say r, okay. So the stress to activate the Frank-Reed source in the neighboring grain, this was given by Cottrell, okay.

Tau a should be greater than equal to tau naught plus 2 tau c r to the power 1/2, D to the power -1/2. So almost similar to what we just saw. Instead you have this term here, but it is also related to diameter of the grain, okay. So whether it is Cottrell theory or Hall-Petch theory, we just learned both are related to diameter of the grains and it says that increasing the grain size will require lower amount of stress for the plastic deformation, okay.

That means if you want to increase the strength of the alloy you have to reduce the grain size and that is what Hall-Petch relations discuss.

(Refer Slide Time: 55:01)

$$\sigma_y = \sigma_O + \frac{K}{\sqrt{D}}$$

Hall-Petch relation. So what is this relationship? Yield strength follows Hall-Petch relation for almost you know most of the materials and this relationship is given as sigma y equal to sigma naught plus K divided by root D okay, where sigma y is your yield strength of polycrystalline sample. Sigma naught is your frictional stress for dislocation movement.

And D, we have discussed this. This is grain size, okay. So what it predicts? If you reduce the grain size sigma y is going to increase, right? And that is the grain boundary strengthening concept. So if you want to increase the strength of a given alloy system where you say you cannot change the alloying elements, so solid solution strengthening you cannot modify.

And suppose that particular alloy system is also not able to give you precipitate, so precipitation strengthening is out of question. So one of the ways to increase the strength of that particular alloy system is to use grain boundary strengthening concept where if you reduce the grain size then you are going to increase the strength of the material and that is what Hall-Petch relation is all about, okay.

(Refer Slide Time: 57:38)

So decreasing the grain size will lead to increasing sigma y. So the more you decrease the grain size there will be increment in sigma y, okay. But this is not valid in nanometer range. That is called inverse Hall-Petch relation, but we will not go into that in this particular course. So if I plot say sigma y versus root D, suppose you take an alloy and you varied the grain size in that particular.

So you took different sample, you did some heat treatment, and you are varying the grain size. So in one sample you got D 1, in another sample you got D 2 and so on and then you also did tensile test and you figure out what is the yield point okay or sigma y here. So you can nicely plot something like this okay, where slope is going to be, so this is 1 by root D or say D to the power -1/2.

So slope is going to be K and this is going to be sigma naught, okay. So it is y equal to mx plus c. So x here is D to the power -1/2. So m or slope here is K which is a constant and then c here the intercept here is sigma naught. So this is what Hall-Petch relation is all about okay. And remember Hall-Petch relation is not a universal law. It is valid for certain range size range.

If you go to a nanometer size range, very small grains then this relationship is not going to be valid. But in most of the engineering alloys we are talking about 3 micrometer size of range. So we typically tend to use Hall-Petch relationship for most of the engineering materials, okay. So in this lecture we have discussed the details about grain size strengthening or grain boundary strengthening.

So we have till now completed three types of strengthening mechanisms, precipitation strengthening or dispersion strengthening, then solid solution strengthening and grain size strengthening. So in the next session we will discuss about strain hardening okay or work hardening. You have some little idea about it when Professor Shashank Shekhar discussed with you about the tensile curve, okay.

But we will discuss about this particular concept in more detail in the next lecture. Thank you.