Mechanical Behavior of Materials-1
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Lecture - 19
Stress and Strain Fields of Dislocations

Okay, so welcome back. We have now looked at the basics of dislocation. Now it is time to delve into the stress and strain field which is what will give us an understanding of how dislocations interact and how their interplay results in the overall behavior of a material, particularly metals and alloys. So from this point of view, let us start with understanding what is called as Volterra construction.

This is very useful from the point of view of understanding the stress and strain field, which will help us get the stress field around a screw dislocation. We will not be deriving the same thing about a edge dislocation, because it is a little bit more cumbersome. But nevertheless, I would strongly suggest you to look for similar kind of derivation for the edge dislocation. So in Volterra construction, what we have is that there are two types of defects described about a crystal.

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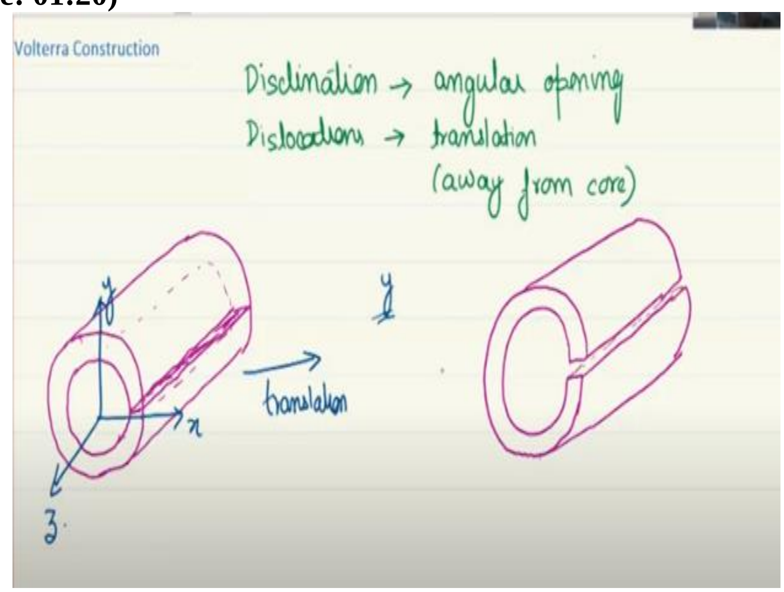

One is disclination. Disclination is when there is an angular opening and it is a very different type of defect and is also found in materials. But this is not what we are interested in right now. What we are interested in is dislocations. Here in the terms of Volterra construction, you would see that it results in translation. So translation and that too away from the core because at the very core you would not be able to define any translation.

Okay, so let us say for the purpose of this, we have a cylinder. So it is a material and we cut a cylinder out of it. So it is a single crystal, but it is a cylinder, hollow cylinder okay. So this is how it would look like and let us say that we cut this along one of the sides. So this is, now you can open it. It is now you can two different surfaces over here. Now the way you open it will and we are assuming that it is inside a crystal.

This is just for the purpose of construction, we have taken it separately, but it is happening inside a crystal. Now the way you translate or rotate, depending on that you can get different types of dislocations and disclinations. So first let us look at dislocation which is of our interest where the changes take place by translation. So let us say we have this as the x axis, this as the y axis and this as the z axis.

So if the translation takes place along the y direction, then this is how it will look like. So there is a gap or opening over here. This represents an edge dislocation.

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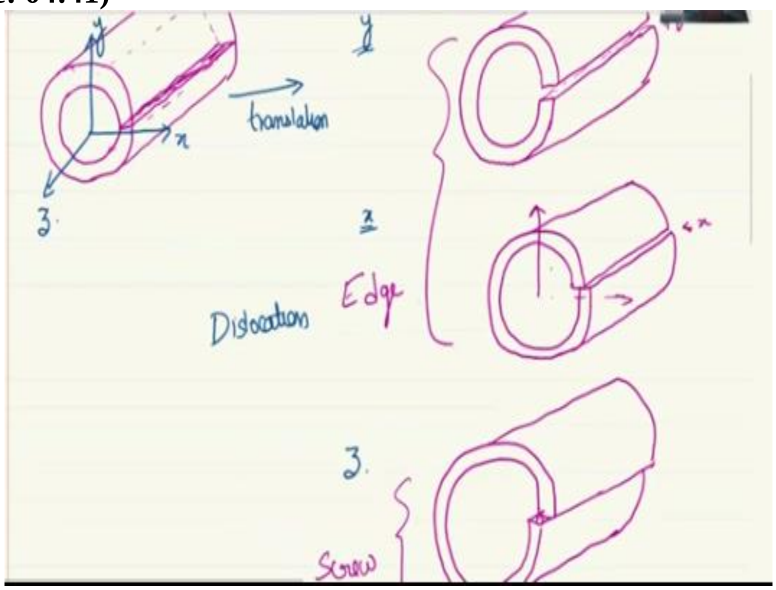

Now similarly, you can have the translation along x direction in which case this construction would look like, so this is moved along x direction. Here it is moved along y direction and this also represents an edge dislocation. However, if this translation were to occur, so you can clearly identify it with an edge dislocation. There is a gap over here. It is like missing plane over here.

Here again you can see that this is displaced. So this is like a missing plane over here. So both of these represent edge dislocation. But now when we see construction, when the translation takes place along z direction, it will clearly remind you of a screw dislocation. So you can clearly see that if you rotate around this, the layer goes back and this is how we had defined the screw dislocation.

So over here this represents a screw dislocation. So both of these are edge and this one is screw. And you would, this is the particular construction that we would use to derive the strain field around a screw dislocation, which can then help us also get this stress field around the screw dislocation. And this construction has been used by Ashby to explain the stress field around edge dislocation, which I earlier suggested that you can look at it on your own.

It is a little bit more complex than what we will look for the screw dislocation but not very unreachable. So this is for the translation. However, if the opening were made in an angular way. Now what do we mean by angular, we will see.

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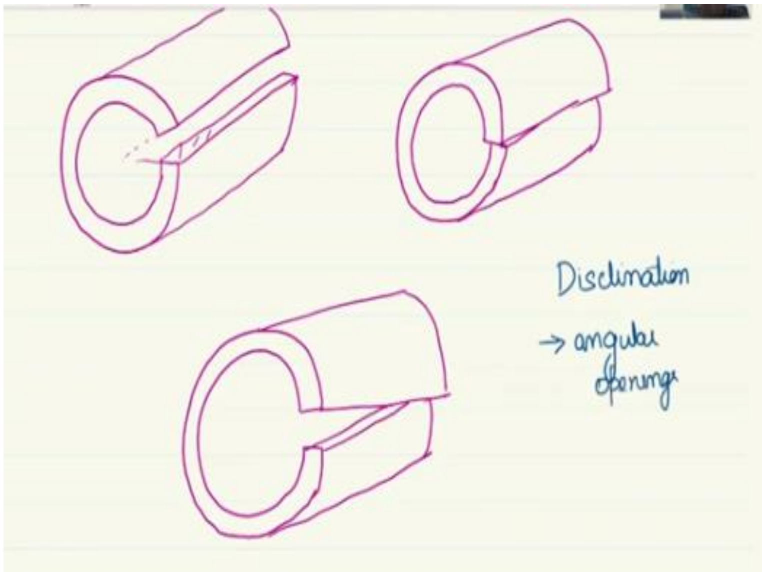

So you can have, so what do we see here is that there is an angular opening. So the two phases are not parallel unlike if you go over here when it was open the two phases were parallel. In this case the two phases are like this. And it has a very different implication or different effect when you look in terms of the stress and strain fields.

So in, something we will not discuss, but in disclination this results in a strain field which is ever increasing while the strain field around dislocation where, around a dislocation it saturates or it is bound. So this is one type of disclination where you have angular opening. The other type of disclination and be like this. So again you see the two faces are now twisted. So again there is a angular nature to this.

And in the third one we can, the third type of disclination can be like this. So here the two faces are like this, they are opening like a jaw. So that is these three represent the disclination with angular openings. On the other hand, we saw that these three we have translation which result in dislocations. So these two are in some sense similar, they result in edge dislocation and this one is a little bit different, which results in screw dislocation okay.

So now we understand Volterra construction which can be very useful to understand dislocation. From this we will use this particular structure and we will show the strain field around screw dislocation, okay. So but before that let us see why there should be a strain field or stress field around a dislocation.

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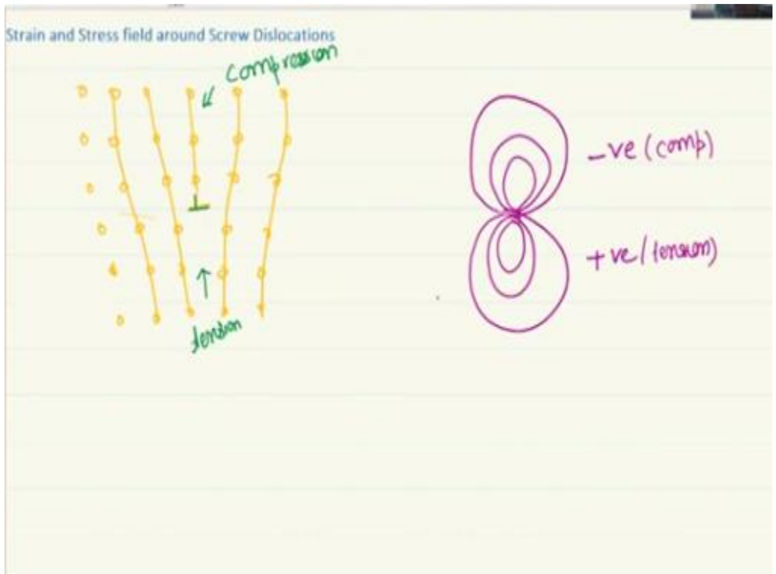

So let us say you have an edge dislocation. So here I am trying to draw an edge dislocation and here is your edge dislocation. Let me draw it with a different color, coming back to this color. So clearly you can see that there is a extra half plane on the top or you can say missing half plane on the bottom which clearly means that there is little bit extra space over here on the bottom side while on the top side you have a little tighter packing.

And because of this there should be compression over here. And there should be tension in this sorry, this is yeah this is right, because this will be stretched in some sense. And therefore, there will be tensile forces and if you were to later on when we look at the equations, when you draw it you would see that the stress fields would look like this. So here you will have positive and here negative assuming this is compression.

So negative is representing compression and positive is representing tension and these lines represent contours. So similar stresses. So these are much higher stresses, but in opposite sense. So this is how the strain fields and the stress fields look like for edge dislocation and similarly for screw dislocations, which we will see in the next slide, okay. So let us say we have used the construction, Volterra construction. Let me just pull these figures a little bit.

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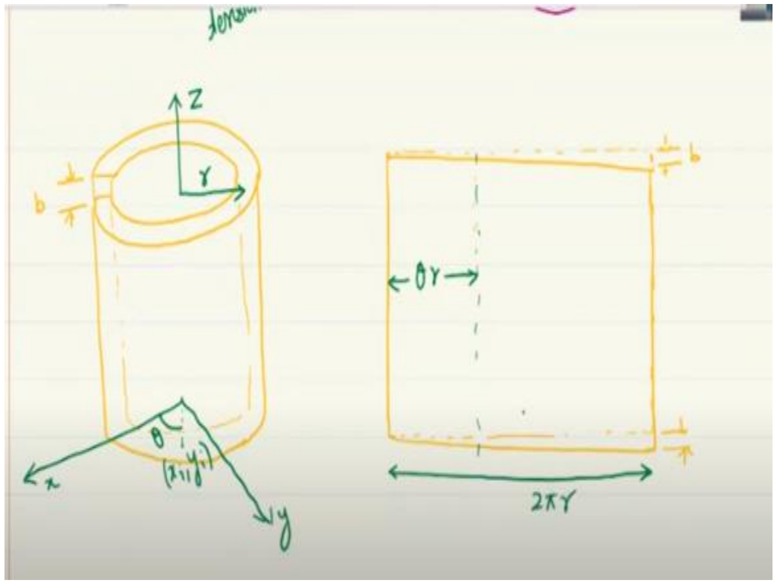

So you can see that this is translated along z direction. So this is your z direction. Let me draw it with different color. So this is part of the crystal which is translated by along z direction and this amount will always be fixed, which will be equal to Burgers vector because that is how one dislocation is defined where the shift if you remember from the Burgers circuit, it will be shifted by 1 unit, which is what we define as the Burgers vector.

So this is equal to 1 b. And let us again say along this cut, we have the x axis and perpendicular to this we have the y axis. And let us say we are looking at some point x_i, y_i which is at an angle theta from the x axis. Now if I were to unroll it, open this because it is cut and so if I opened it how would it look like? If it when completely no translation had taken place then this should have looked like this.

But now we know that translation has taken place, so the actual line would look like this. And we know how much it is moved down, this amount is equal to b and this point is somewhere over here. So if this radius is, it is a very thin cylinder. So will assume a constant radius. This radius is r and theta is in radians, therefore this is theta r length and this total length is $2\pi r$.

Okay so now I will have to jump a little bit, because I wrote this equation over here. Now if we, you can go back to our slides and look at how the strains are defined with respect to displacement. So first let us look at what are the values of the displacement.

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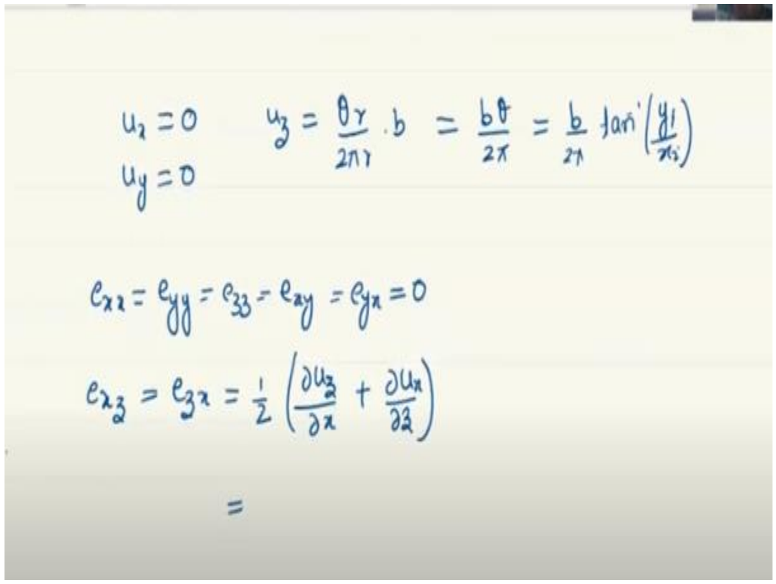

So we will see that along the x axis there is 0 displacement, along the y axis there is 0 displacement, but along the z axis, when we are talking about this particular point, along this particular line, there is a displacement and how do we calculate that? This is equal to $\frac{\theta r}{2\pi r}b$. So basically it has to be proportionate at a distance of $2\pi r$ the total displacement is b

So we know the displacement at this point, which is now equal to $b\theta/2\pi$ or we can say that since it is theta we can write it as $\frac{b}{2\pi}\tan^{-1} y_i/x_i$. So we can go back here and see this is theta which is tan theta is equal to y_i/x_i. Therefore, θ is equal to $\tan^{-1} y_i/x_i$. Now we know that e_{xx} if you go back and look at the equation with respect to displacement you would find that e_{xx}, e_{yy}, e_{zz}, e_{xy}, e_{yx} all of these will turn out to be 0.

However, what will not turn out to be 0 are e_{xz} equal to e_{zx} which is equal to $\frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)$. Over here again since u_x is 0, therefore this quantity goes away. What we are left with is only this quantity, which is equal to, so we have to differentiate this u_z with respect to x.

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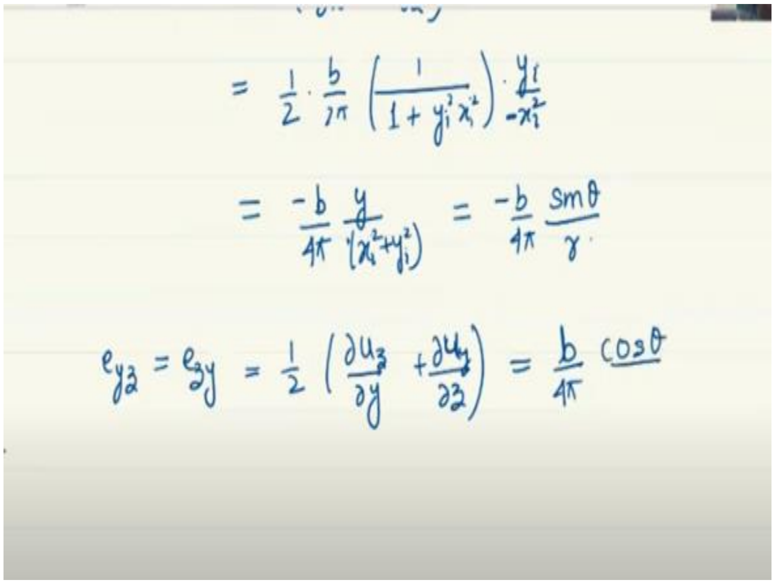

And what you will get is

$$\frac{1}{2}\left(\frac{b}{2\pi}\right)\left(\frac{1}{1+{y_i}^2{x_i}^2}\right)\left(\frac{y_i}{-{x_i}^2}\right)$$
$$=-\frac{b}{4\pi}\left[\frac{y}{{y_i}^2+{x_i}^2}\right]=-\frac{b}{4\pi}\left(\frac{\sin\theta}{r}\right)$$

But this is not the only component which is not 0.

We also have

$$e_{yz}=\frac{1}{2}\left(\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right).$$

So again ∂u_ycomponent is 0, but ∂u_z component is not 0. And therefore we differentiate the same equation with respect to y. And what you will get in this case by after all the simplification is $\frac{b}{4\pi}\left(\frac{\cos\theta}{r}\right)$

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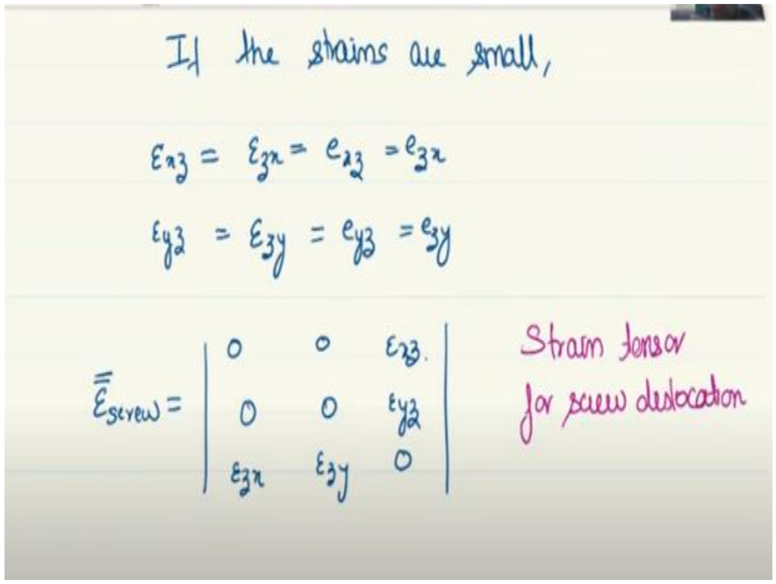

Now if the strains are very small we know that these engineering, these engineering strains can be considered equal to 2 strains. And therefore what we will have is epsilon $_{xz}$ equal to epsilon $_{zx}$ equal to e $_{xz}$ equal to e $_{zx}$. Similarly, epsilon $_{yz}$ equal to epsilon $_{zy}$ equal to e $_{yz}$ equal to e $_{zy}$. And overall the strain tensor would look like for a screw dislocation, so you will have this one 0, this one 0.

Here you will have epsilon $_{xz}$, here you will have epsilon $_{yz}$. Here you will have epsilon $_{zx}$, then $_{zy}$ 0, 0 and 0. So this translates to this strain tensor for screw dislocation.

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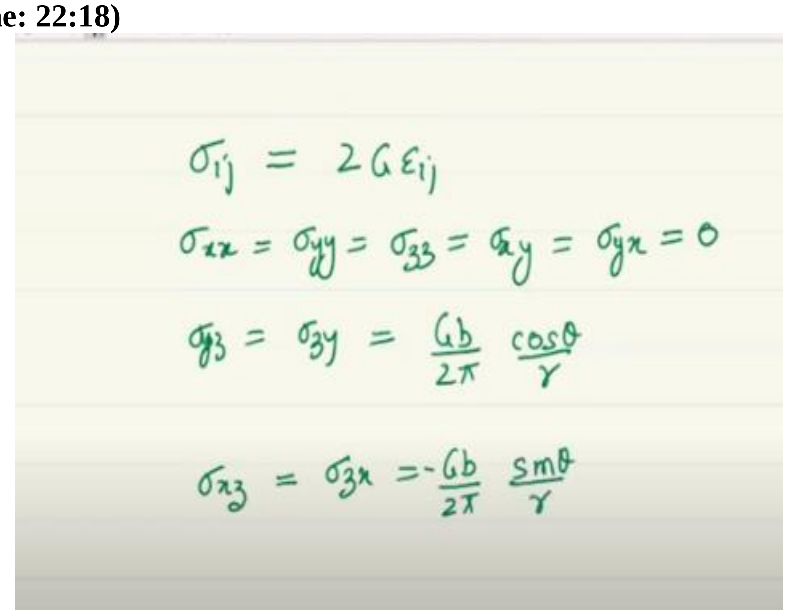

Now we also know that sigma ij is equal to 2G where G is the shear modulus times epsilon $_{ij}$. And therefore, what we get is that $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy}$ which were whose corresponding strains were 0 their corresponding stresses would also be 0.

And the corresponding stresses for the nonzero strains come out to yz equal to σ_{zy} equal to now you will have 2 into Gb multiplied with all this. So it comes out to $\frac{Gb}{4\pi}\left(\frac{\cos\theta}{r}\right)$. So very simple. Similarly, $\sigma_{xz} = \sigma_{zx} = -\frac{Gb}{2\pi}\frac{\sin\theta}{r}$

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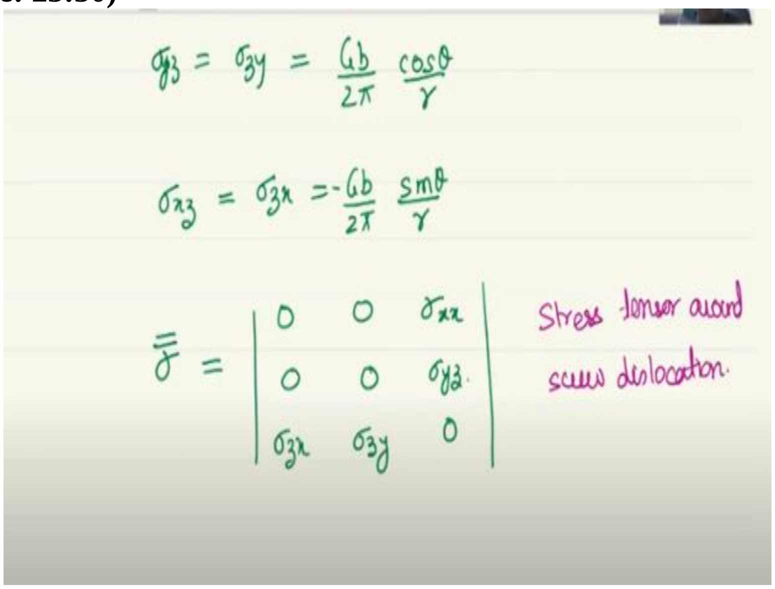

So overall what we get is a stress tensor which has components similar to the strain components five zero elements and four nonzero elements $\sigma_{xz}, \sigma_{yz}, \sigma_{zx}, \sigma_{zy}$ Defines the stress tensor around screw dislocation, okay.

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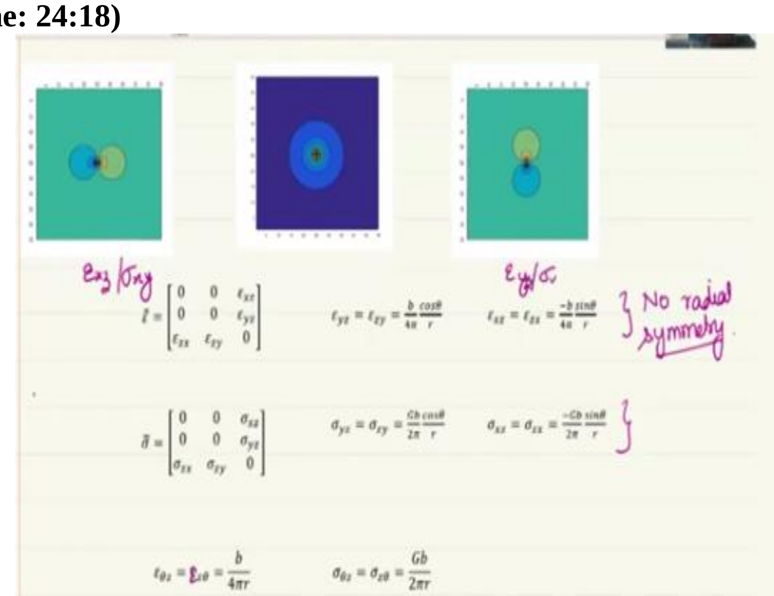

So this I have also given here in terms of, I have already typed it so that there is no mistake or any mistake does not creep into it. So you can clearly see this is the strain tensor. This is the stress tensor. These are the quantities and these are the quantities for the stress tensor, these are the nonzero stress tensor quantity. And the values are over here for the epsilon and sigma so this is also epsilon.

I did allow one mistake here. Okay so this is the strains and this is the stress, okay yeah. So yeah, one thing I am yet to come at is now that these are the nonzero components, what we see here is that epsilon yz or zy is equal $\frac{b}{4\pi}\left(\frac{\cos\theta}{r}\right)$. Now there is a cos theta term which means that the value changes with theta and that is it is not radially symmetric, okay.

But a dislocation line is a line, so should you not, particularly in the case of a screw dislocation it is a line. If you look at it for the purpose of Volterra construction, we may say that there is a cut at one point. But in real we do not say that this is where the cut is. This is location line and it is same all across and the construction is same all across. So there is no starting point.

And in that sense there should have been a radial symmetry. So what are we missing here? Similarly, we can look at the stresses. We look at it and we see that it is a function of cos theta and therefore, it is not really symmetry. In fact this is how the relation or when you plot this epsilon yz, so this is epsilon xz or sigma xz and this is epsilon yz or sigma yz. This is how it looks like.

You can clearly see this is not symmetric. And the clue or the key answer lies in the fact that this we this is a cylindrical quantity. But we have been using Cartesian coordinate x, y, z which is more suitable for a rectangular kind of object. Therefore, what we need is to translate everything into cylindrical coordinates and which is what is done over here.

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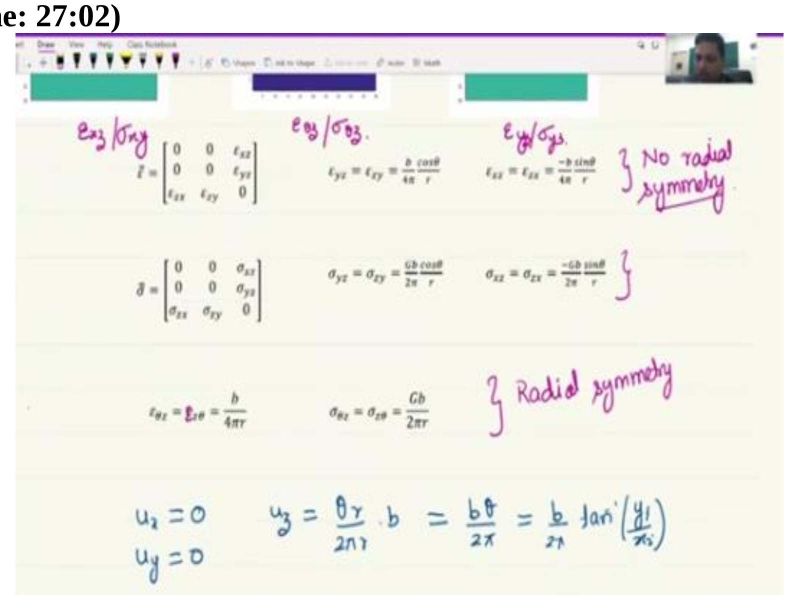

So when you translate it to cylindrical coordinate we have only these two quantities as nonzero,

$$e_{\theta z} = e_{z\theta} = \frac{b}{4\pi r}$$

And similarly $\sigma_{\theta z} = \sigma_{\theta z}$. And you can clearly see now it has radial symmetry. So our assertion or our understanding is right, there should be radial symmetry. But for that we need appropriate coordinates and that appropriate coordinate is the cylindrical coordinate.

So in here now you can see this is what we have here is $\varepsilon_{\theta z}$ or $\sigma_{\theta z}$. And we do really get a radial symmetry for this. So this is the overall stress field and strain field for screw dislocation.

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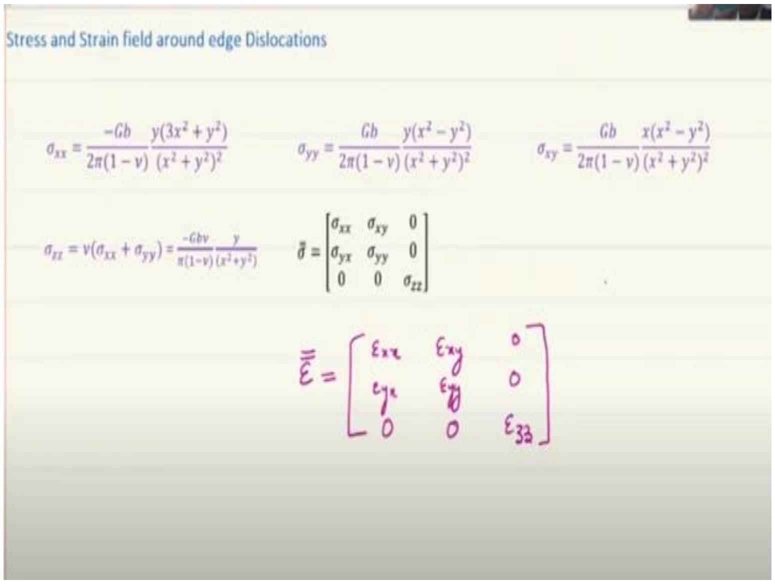

Now we will look at the stress and strain field around edge dislocation. So for edge dislocation you would see that the components which are nonzero are very different from the components which were nonzero in the screw dislocations. So here are the, then again as I mentioned earlier, we are not going to go through the derivation.

So here are the nonzero components. Sigma xx is given by this relation. Sigma yy is given by this relation. σ_{xy} is given by this relation. And σ_{zz} is basically given by the x and y, combination of x and y with a factor of ν. And therefore it turns out to be this. And when you look at the stress tensor, these are the five quantities which are nonzero.

And what you notice is that it is actually inverse, absolutely inverse of the stress tensor that you obtain for screw dislocation and same can be said about the strain tensor. So if we look at the strain tensor, if we had to write down the strain tensor for the edge dislocation it will be very similar to this. So these are the nonzero components for the strain.

Now if you want, if you wanted to plot how the σ_{xx} varies along x and y axis and here this does not have a cylindrical symmetry. So we need not worry about cylindrical coordinates. And in that case, if you want to, if you wanted to draw the variation of stress strain, so xx basically the normal stresses are acting along x.

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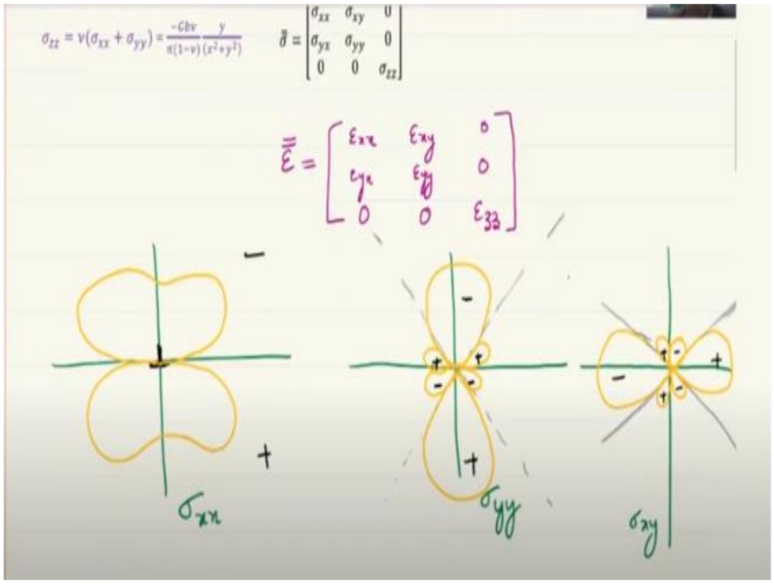

And if we draw it like this, there let us say the edge dislocation is there like this. We know that in this particular case, on the we have already looked at it in just one glance that there should be compressive stresses over here. So we will put it as minus sign and there should be tensile stresses, which we will put as positive sign. And if you plotted the contours, it would look something like this.

However, the fact is that all the plain above this middle line or this line dividing the extra are from where the extra half plane arises would have compressive strains and compressive stresses and everything below this will have tensile stresses. If you look at the tensor. if you look at the σ_{yy}, which is like this, it is a little bit more complicated than this and it will look like, so this is σ_{xx}.

This isσ_{yy}. This is roughly how the major component would look like but this is not all, there is more to it. So it would have if you draw 45 degree line, so along this 45 degree line, you would see that there are some smaller lobes. And when we are drawing the lines, you remember that it is contour line, meaning those of constant stresses.

Now here you would happen to have, if this is, so this would be compressive. So basically in this direction you will have compressive and this one will also be compressive and this one will also be compressive while this one will be tensile, while this one will be tensile and this one will be tensile. The third stress that we have is the σ_{xy}. So for the sake of completion, let us also look at this one.

It will it happens to be similar to the yy only that it is rotated by 90 degrees. And remember that this is shear stress not the normal stresses like σ_{xx} and σ_{yy}. So this is, these are called dumbbell type of structure or feature. So it has a dumbbell kind of distribution. And here again we will have smaller dumbbells. So whatever is the stress direction over here you would see shear stress direction in this particular case.

So this will be clockwise. So this one let us say this one represents the clockwise shear. So this one will be the anti-clockwise. And if this is clockwise this one is also clockwise and this one is

also clockwise. And this is anti-clockwise then this is also anti-clockwise and anti-clockwise. And this is the, these equations are very easy to plot in MATLAB and it is shown over here.
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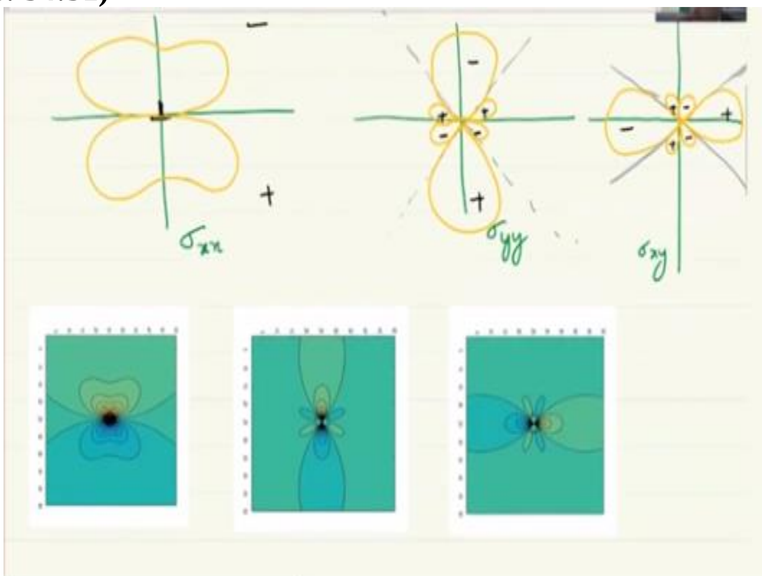

So this is the MATLAB plot which is shown. So this you can clearly see this is the compressive stresses which are reddish and bluish one are the tensile. So if you go closer and closer to the dislocation, there are lines that side looking dark otherwise it would have become redder and redder. And similarly over here. Over here you have the reddish one which is compressive and the blue ones are the tensile.

So these three lobes are tensile and here everything on the down bottom plane is tensile. And here the shear stresses, the negative ones are shown by blue. So these three lobes are blue and the positive ones are shown by green lobes, which are over here.
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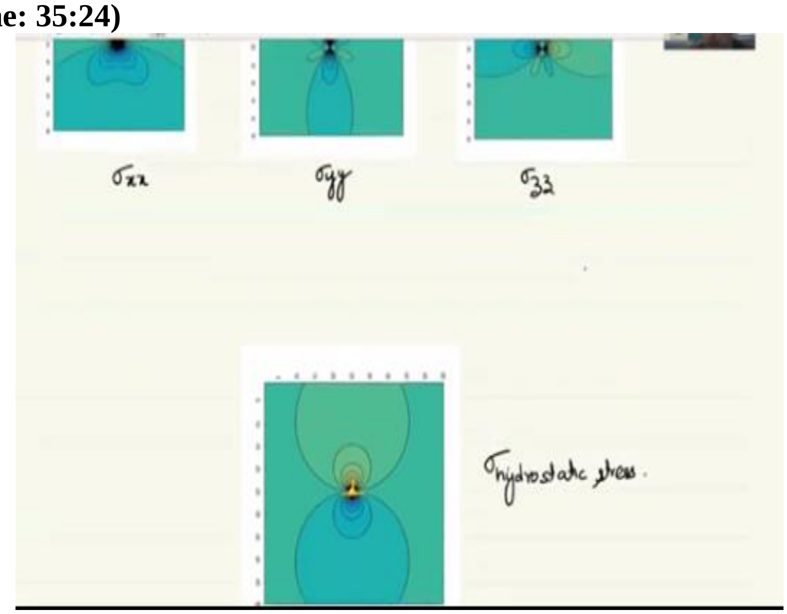

And since you have σ_{xx}, σ_{yy}, so this isσ_{xx}, this is $\sigma_{yy}$and this is σ_{zz}. Therefore, you can also plot the hydrostatic stress, which would look like this. So again this is where your dislocation is and overall you see that there is a compressive hydrostatic stress on the upper side and tensile hydrostatic stress on the bottom side. So that is the nature of thing. So now we have σ_{xx}, σ_{yy}.

And this was xy, sorry about this error. This is shear stress σ_{xy} and we see that overall we have three different zones now which can be divided or obtained by making 45 degree lines like we drew over here or basically 1, 2, 3, 4, 5, 6, 7, 8 different zones.

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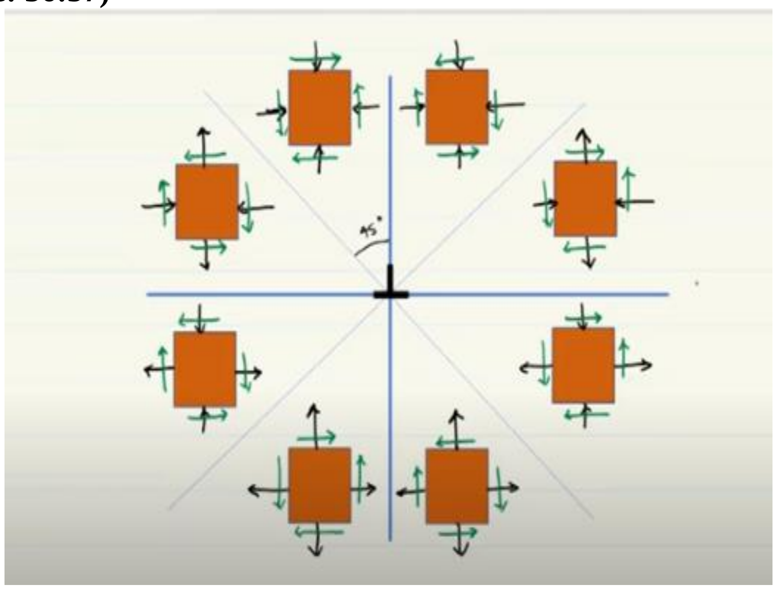

So these different eight different zones can be shown to have different stresses acting on them. So now we know that everything on the positive side in the x direction it will be compressive. So we can draw it like this and everything on below this line. So here is edge dislocation and we have drawn eight different zones where each of these zones are 45 degrees apart.

And you would see that in one particular zone you will have one constant stress state although the magnitude may vary, but the signs would not vary. So this is all compressive and over here we have all tensile. Now looking at the σ_{yy}, the normal stresses, we know that these two have the compressive and this one these two also happen to be compressive while these two are tensile and so are these two.

So we can accordingly draw like that. And then this is your tensile. So if these and these are tensile and so are these two. So these are the eight different zones and now we have identified the normal stresses on all of them. Now we can also identify the shear stresses on to them. So we said that, this one is, these two are the clockwise. So let us put it like this, clockwise on the x direction.

And so that would mean anti-clockwise in the y surface. So these two have to be opposite so as to balance. Otherwise, you can imagine that the object would start rotating which we know is not supposed to happen. Therefore, these two must be equal and opposite in sense. And whatever we have over here would be the same over here. So this end this would be same and it will be the very opposite in these two and in these two.

So this is how the stress state around edge dislocation looks like because we have the σ_{xx}, σ_{yy} and the σ_{xy}. So overall the stress state around the edge dislocation would look something like this. And that, so now we have, with that now we have the stress state around edge dislocation as well as the stress state around screw dislocation.

So we will conclude the stress state over here and we will come back and use this stress relations to calculate the energy of the dislocations. Thank you.