`Mechanical Behavior of Materials-1

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Lecture - 14
Plastic Instability

Good morning students. In this lecture we will talk about plastic instability. So on the reference of this you can read or go through chapter 5.

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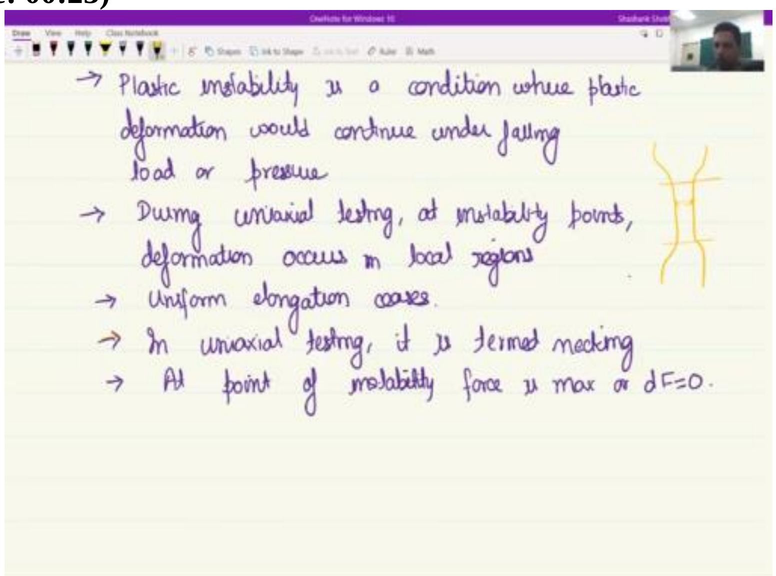

So what is plastic instability? This plastic instability is very obvious particularly when we are doing uniaxial tensile test. So when we see plastic instability and the uniaxial tensile condition, then we see that the load starts to decrease even though the strain is increasing. So we are talking as in terms of the overall component and not as the local stresses.

Local stresses are of course, increasing. So plastic instability relates to a condition of plastic deformation at which that deformation would continue under a falling load. So that is you can say working definition. So usually we expect that for increasing deformation we have to apply at least as much force as we were applying earlier and in most cases there would be increase in stress or pressure.

But after the point of plastic instability, the required force or pressure would decrease. During tensile uniaxial test the uniform elongation ceases and why does that happen? Because the instability occurs at some regions which are weak or where there are some variabilities. And therefore the stress gets localized in that region. But as a overall the body if you look at it, the load is falling and the deformation keeps on happening.

But what is happening is that the overall, the stress is not being applied or the elongation is not uniform, it is not happening throughout the gauge length. It is happening in the localized region. So during, the uniform elongations ceases. So if I had to draw it, this is how it would look like and something I have drawn earlier also. So this is our, the uniaxial test. This is the dog bone and probably this is the gauge length.

So until the uniform elongation or until before the point of instability, this whole region is getting deformed. But at the point of instability there is some, because of some variability in the material, some particular region there would be a necking that will have formed. And beyond this the overall stress and deformation would be limited to this region. And that is where the load requirement goes down even though the deformation keeps happening.

So in uniaxial condition this plastic instability is termed necking. Now since the load starts to decrease from this point, it means that there must have been a region where the maximum amount of load must have been applied. And therefore, if you look at the load curve then we can identify this point of instability by finding the region where load is maximum or where dF is equal to 0. So let me draw this to be able to appreciate this better.

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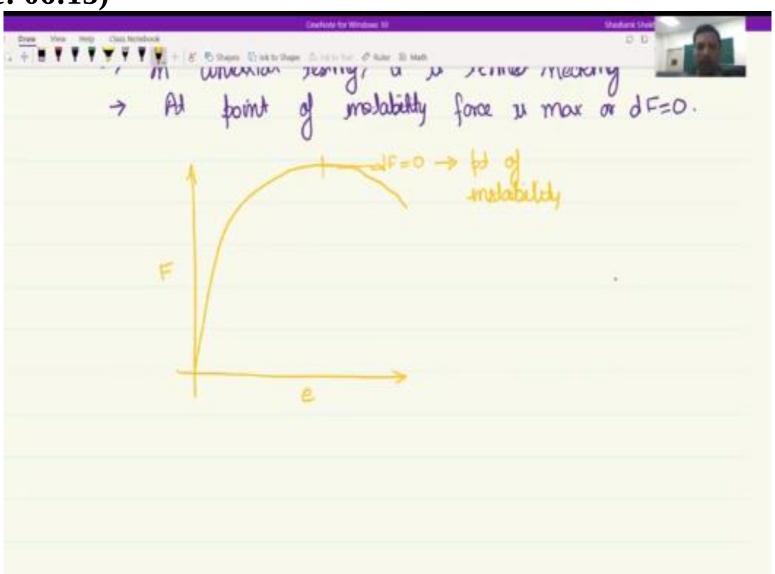

So let us say we are drawing the four, the load versus elongation plot. Then it is at this point where dF is equal to 0 is the point of instability because even though your overall force is decreasing, but elongation keeps on happening. So let us try to understand this with respect to one of the simplest condition which is the uniaxial tension condition.

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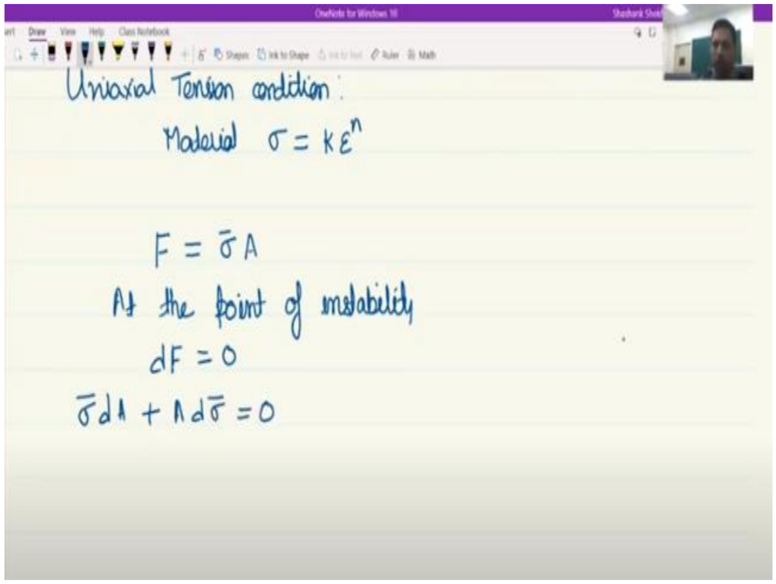

And let us assume that the material has a power law hardening behavior. So $\sigma = k\varepsilon^n$. Now we know that let us say we are applying a load F then in the gauge area F is equal to sigma where sigma has to be effective stress times cross sectional area A. Now at the point of instability we know that dF is equal to 0. But we know F is sigma times A. Therefore, we can write it as

$$\bar{\sigma} dA + A d\bar{\sigma} = 0$$

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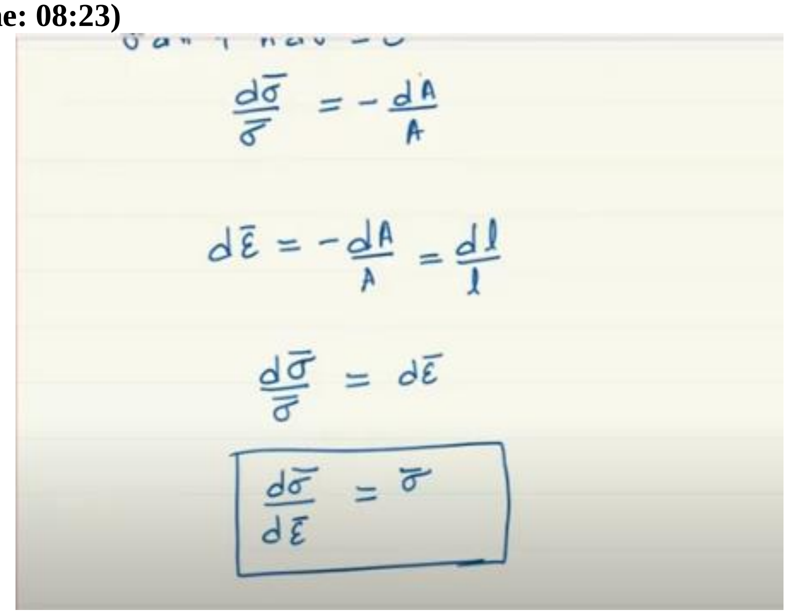

And now if we rearrange it, what will but we will find is that

$$\frac{d\bar{\sigma}}{\bar{\sigma}} = -\frac{dA}{A}$$

But then for uniaxial condition, we also know that because of the volume constraints

$$\frac{dl}{l} = -\frac{dA}{A}$$

which is equal to the differential strain. Hence, we can write this equation also as,

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \bar{\sigma}$$

So this defines our plastic instability condition for uniaxial condition.

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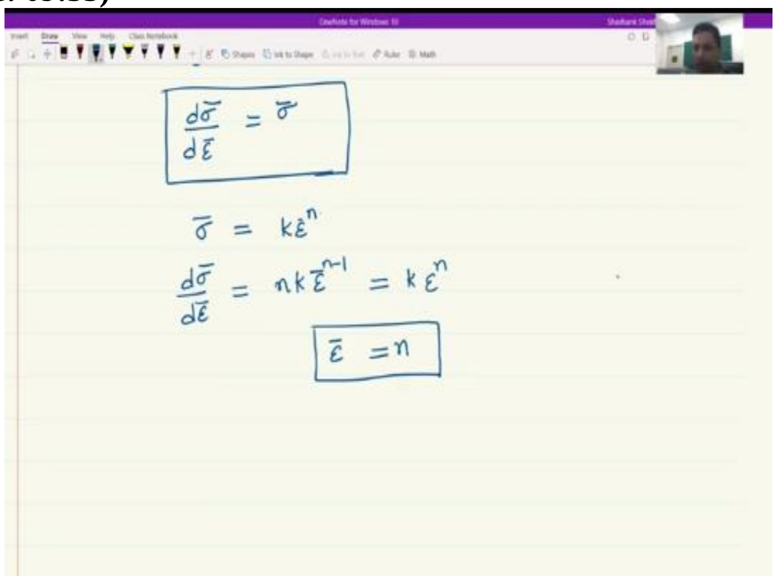

Now let us put the relation which we know is $\bar{\sigma} = k\bar{\varepsilon}^n$

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = nk\bar{\varepsilon}^{n-1} = k\varepsilon^n$$

Now when we put this together, what we will find is that $\bar{\varepsilon} = n$. So for a uniaxial condition, this is a very useful result.

What it says is that if we know the uniform, if we know the power law hardening relation and the strain hardening exponent, n also gives you the uniform elongation value that you can expect in this material. Or in other words, if I were to again, let me go back and use this one.

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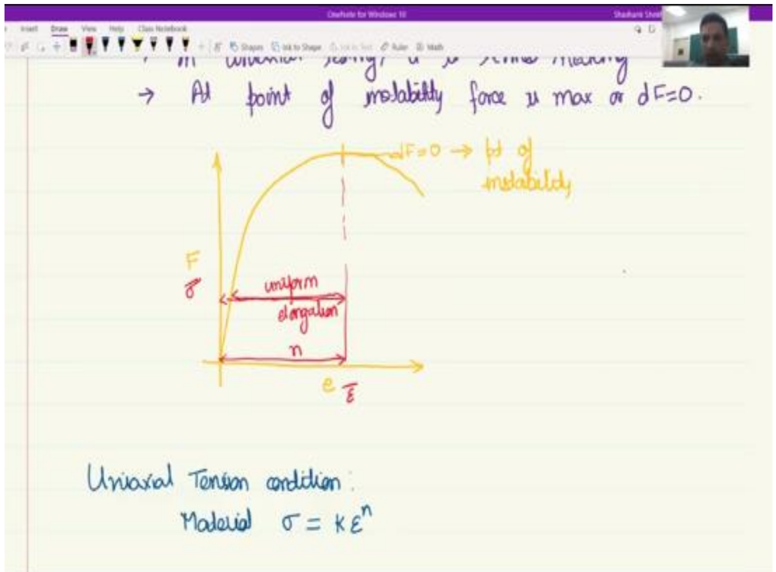

So this is the point. Only up to this point you get uniform elongation, that much we understand. And now this relation tells us that, if this were to be sigma bar versus epsilon bar, then this is also equal to n. So this is a new information that we are able to obtain if we apply this equation. And it also means that if we have different materials with different power law hardening behavior with approximately similar material, then we can say that as the n value increases, then the total elongation would also increase.

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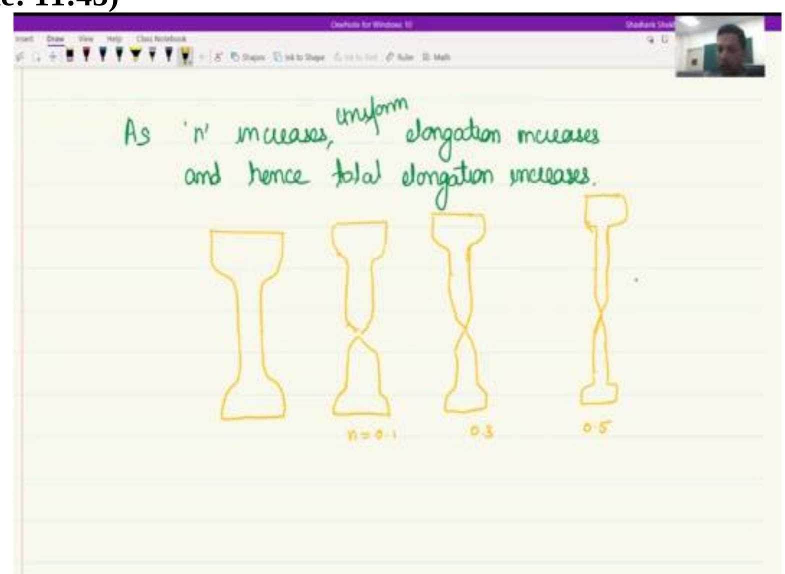

Or so what we are saying is that as n increases, total elongation, actually first we should say only about the uniform and hence the total. And hence total elongation increases. So let me just draw this. So let us say we have three different materials of starting same type of dog bone. So let us say this were the dog bone starting for three materials, where let us say n is equal to 0.1, 0.3, 0.5.

Then what you would see is that for 0.1 at the end of it, it will look like something like this. For 0.3, the total elongation is expected to be larger and for n equal to 0.5 it is expected to be even larger. So this is the schematic of what is the meaning of increasing n. So you can see that the sample thickness has become smaller and smaller, which means uniform elongation was larger.

And also the total elongation is larger, which is evident from the total height of the sample that I have shown over here. So this is the implication of n equal to epsilon bar for the plastic instability condition. There is something called as consider criterion.

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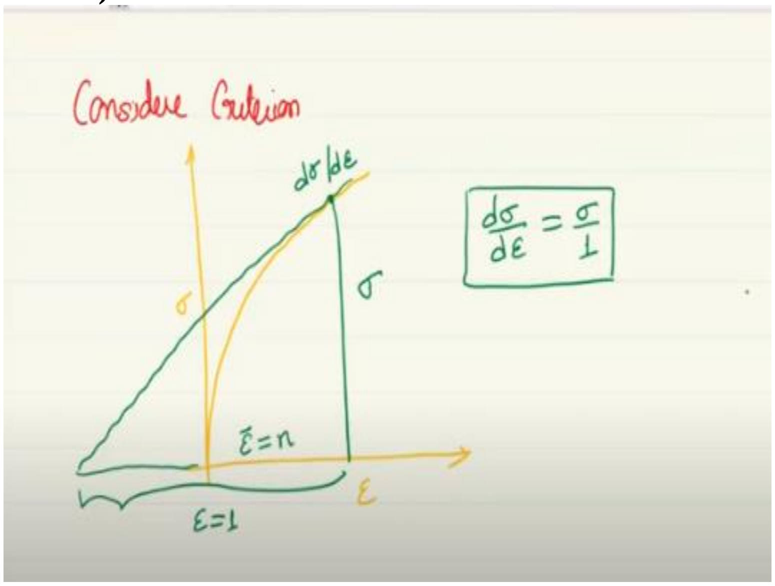

So from what we have already obtained, we can if you are to plot the true stress versus true strain plot so it will look like something like this. And at the point of plastic instability if you draw the slope, then what it is saying is that this is your sigma, this is strain equal to 1 and this is slope d sigma by d epsilon. So we know that $\frac{d\sigma}{d\varepsilon} = \frac{\sigma}{1}$

So this is geometric representation of what we have understood and up to this point if you look at epsilon, epsilon bar, it will be equal to n and we can also draw it a little differently if we

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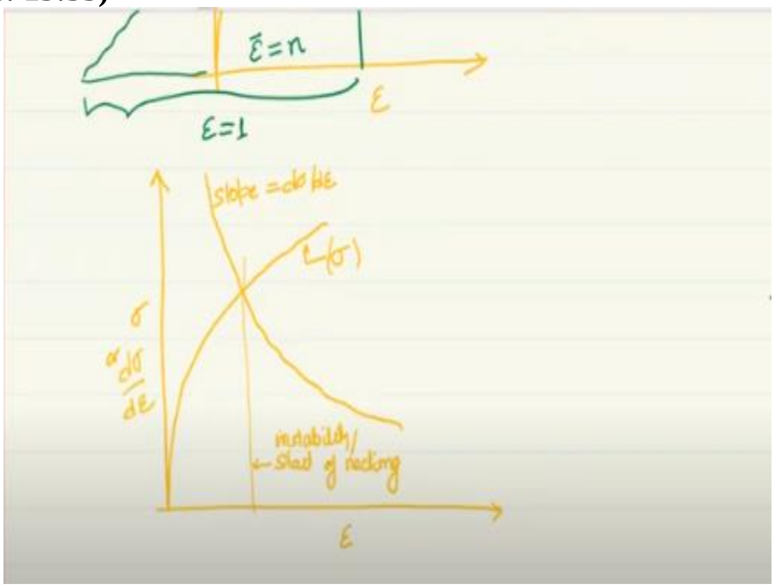

So let us say we are drawing the slope. So the slope is continuously decreasing as you can see, the slope is continuously decreasing, which is represented by this and this is the stress curve. So at one particular point they will intersect. So the slope is equal to d sigma by d epsilon. This is sigma and this is the point where instability or necking starts. So y axis represents stress and also the slope $\frac{d\sigma}{d\varepsilon}$.

So this has the unit of stress units and x axis as stream units. So the condition for necking in attention is met when the true stress sigma equals the slope d sigma by d epsilon of the true stress strain curve. So this is the true stress strain curve. So this is the condition for the necking. So this is the point where necking begins. Another important point about n.

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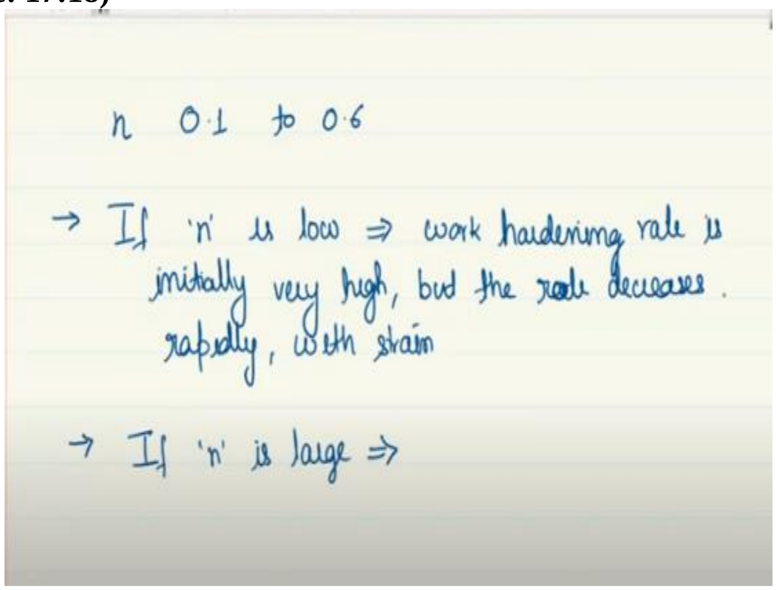

So we see that n can have a range of usually of the range of 0.1 to 0.6. If n is low, then what it means that the work hardening rate is initially high, but the rate decreases very rapidly, I will schematically draw to explain it, but the rate decreases rapidly with strain. On the other hand, if you have n as very large, relatively large, then it implies that work hardening rate initially is low, but it keeps on increasing for very high stresses until high stresses.

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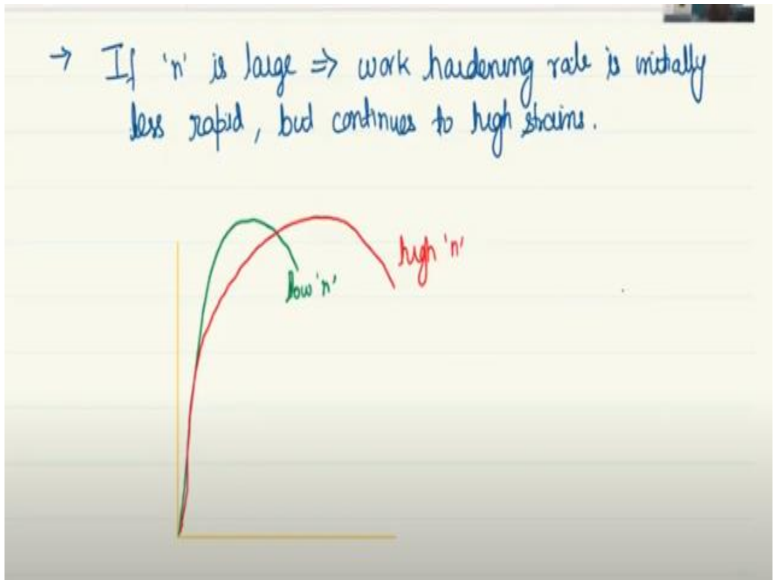

So initially it is less rapid but continues to rise. So if I were to schematically draw this, this is how it would look like. So this is for low n and for high n this is how it would look like assuming that it has almost same elastic constant. So this would be the difference between low n values and high n values. And it is clear why it should be so. Because for high n it means, uniform elongation is high and total elongation is high.

So initially it is less rapid, but it continues to high strains. On the other hand for low n, you will have low uniform elongation as well as total elongation would also be low. And therefore, it rises very rapidly, but then it saturates very quickly. So that is the implication of low n large n values.

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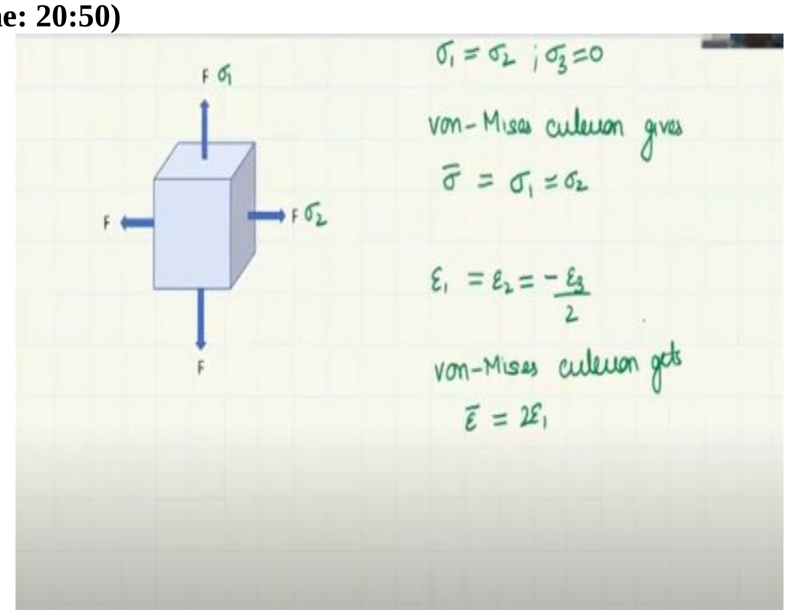

Now let us understand this with the help of one example, which is also similar type. Here it is given to you that the force condition is biaxial in nature and it is balanced, meaning both the forces are same. So if it were $\sigma_1$this is σ_2. Then what it is saying is that $\sigma_1 = \sigma_2$ and σ_3 is equal to 0 because it is biaxial, okay.

So using von Mises condition or von Mises criterion it gives sigma bar equal to sigma, $\sigma_1 or\ \sigma_2$ whatever because both of them are equal. Now the since it is symmetric, therefore $\varepsilon_1$1 is also equal to ε_2 and it is a plane strain, plane stress condition. Therefore, the third one will have to be such that the total volume strain is equal to 0 and therefore, the $\varepsilon_1 = -\frac{\varepsilon_3}{2}$

So ε_3 is basically twice as ε_1 and 2. Now if you sum it up you would see that it will come out to 0. And from there we get the relation between ε_1 and 3. And now if you put this also in von Mises criterion, so it is similar to the uniaxial condition. And there we know that the effective strain is equal to ε_1. So it will be similar here.$\bar{\varepsilon}$ would be equal to ε_1.

I am sorry, it is not exactly the same as uniaxial condition, because here the third one is in compression in some sense and the other two are in tensile condition. And therefore, if we look at the third one, which is if it has to be equivalent to the third one as in uniaxial condition, then it will come out to $2\varepsilon_1$. And it will be advisable if you actually go through the steps, use the von Mises relation to get this value.

And it will give you a hands on experience on calculating these values. So you would get that $\bar{\varepsilon}$ =$2\varepsilon_1$.

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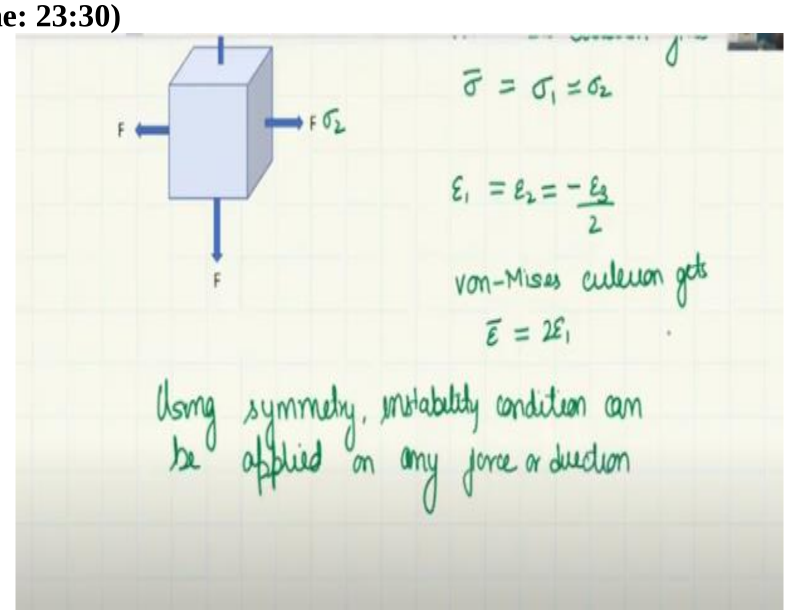

Now using symmetry we can apply the instability condition on any of the forces. Apply it on any force or direction.

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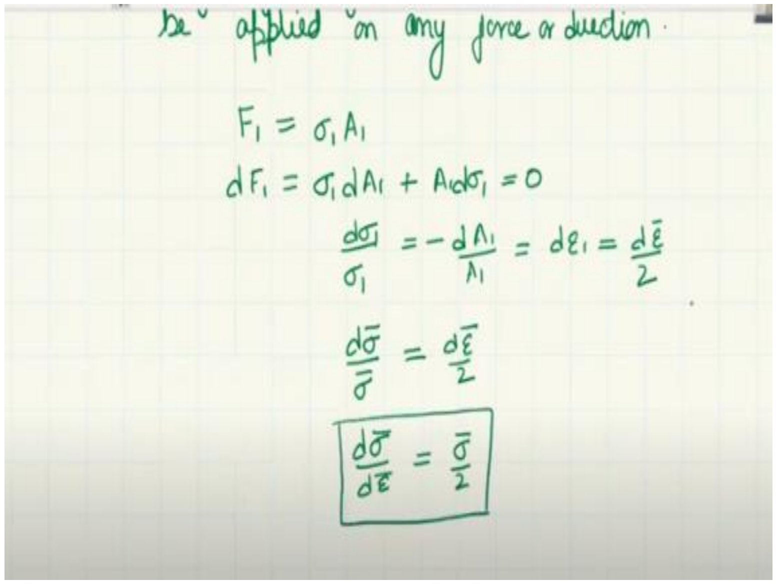

So we have $F_1 = \sigma_1 A_1$ and again we know that for instability dF_1 it should be equal to which is equal to

$$\sigma_1 dA_1 + A_1 d\sigma_1 = 0$$

And again rearranging what we get is

$$\frac{d\sigma_1}{\sigma_1} = \frac{dA_1}{A_1} = d\varepsilon_1 = \frac{d\bar{\varepsilon}}{2}$$

So for the biaxial loading condition, this is the relation that we get.

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{\bar{\sigma}}{2}$$

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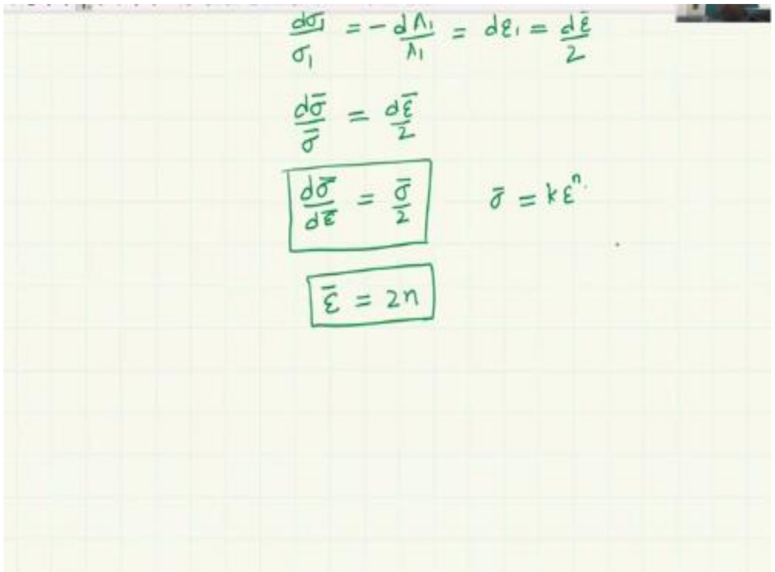

And now if you put $\bar{\sigma} = k\varepsilon^n$ what you would get is epsilon bar is equal to 2n. So in this particular case you would get a uniform elongation to a much larger strain value. So this is what this relation is telling us. In fact, it will be twice of what you would get in uniaxial stress condition. So with this example, we will come to end of this particular topic that is of plastic instability. Thank you.