Mechanical Behaviour of Materials - I
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Module No # 03
Lecture No # 13
Effective Stress Effective Strain

Good morning students so welcome back and today we will be discussing about a relatively simpler or smaller topic. We will be talking about what is effective stress and what is effective strain? So far what we have seen is that uniaxial stress condition is easy to define in terms of when the yielding occurs and when it comes to multi-axial stress condition then we have a little bit more complexity in defining when the ending occurs**.**

So, we saw that it has to be defined in terms of shear stress and that there were 2 different models or criterions that we called that we used one were the Von – Mises criterion and the other was the Tresca criterion. Now how do we use this equation to define a unique value or a unique parameter? Why we want a unique parameter? So that we can say then when this unique parameter reaches the critical value then the yielding occurs.

And then we can call this unique value as the effective stress. Similarly, when the effective strain when the unique whatever equation we derive if we are able to get a unique value for that overall strain state and define it as a effective strain. And when this effective strain reaches a critical value we can say this particular amount of strain causes yielding or this particular strain is what is implemented or has been imposed on the Material.

So, we want to have a concept or a parameter which can be used to define the overall state of stress and that is what we are going to call as effective stress and effective strain.
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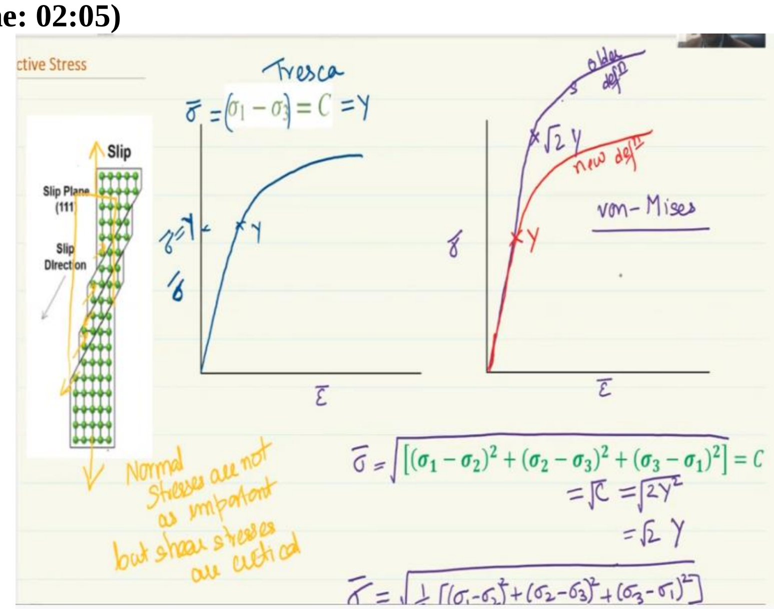

So let us begin so first of all one question that may have been coming to you again and again**.** And Although I have briefly touched upon that topic but that may have still been coming to you. And the question is why does shear stress only cause the plastic deformation and not the normal stress? And the answer is simple because deformation occurs by movement of dislocations and not by complete tearing of atomic bonds.

So, for example here look at an example and let us say you initially you had along bar like this. So let me draw it like this initially probably it this bar was like this and when you applied a, stress like this. Then there was slip occurring on different planes and this is how the deformation occurred. So do not assume or this idea that the bonds are getting broken between layers complete the detachment of bonds is taking place is not correct.

What is actually happening is that some kind of defect which we will explain later on in couple of lectures is called as Dislocation. They get generated on the planes and their movement causes the movement of one plane with respect to the other plane. So here you can see this is 1 plane and there is another plane. So, this one moves in this direction with respect to this one. And therefore this has caused a little bit of deformation similarly this has moved with respect to this in this direction and so on.

And all of these will have dislocations moving and it is not that the overall plane the bonds between the atoms in the 2 planes got sheared at one moment. If it were to be there then it would have required a much larger stress and therefore the stress that we actually require is much lower than theoretical stress which also we will look at when we introduce the concept of dislocations.

And it is also the reason that last class we saw that in the yield locus some stresses higher than yield strength or the yield stress of the material can also be present and still in the multi-state axial stress state and still yielding will not occur. That is still possible because what we are talking about uniaxial not uniaxial but normal tensile condition.

But when the shear stress at a particular plane exceeds a critical value there has to be shearing and there will be slip and there will be deformation which again, we will understand later when we get to the concept of dislocations. So, for now it is sufficient to understand that deformation occurs by movement of dislocations and not by complete tearing of atomic bonds and hence normal stresses are not as important but shear stresses are critical**.**

So, that is the first part, now moving on to explaining the effective stress. Now you remember when we were talking about this one this particular criterion which is the Tresca criterion. So, we said that when σ_1- σ_3 which is also a shear stress reaches a critical value C that is when yielding occurs. And we showed that this is what is equal to y so now if we define this term as effective stress**.**

So, when you keep increasing the effective stress until the value it reaches a value Y. So, it will have a very similar stress strain plot like this and at this point you can define that this is Y, so at this point your $\bar{\sigma} = Y$ which means σ_1- σ_3 has reached a critical value Y. Now the other criterion we had was the Von – Mises Criterion so this was given by this and we said that when this reaches a critical value this is where the yielding will occur.

So, now let us say if so, this is in square format now let us say if I had defined this as or the square root of this as the and before I go to there, so in this condition we know that this C= 2 Y^2 **.** Now if I define this or square root of this as effective stress meaning sigma bar is equal to this so this is also equal to this or this will be equal to $\sqrt{2}Y$. Meaning if I had to draw the stress strain curve and we have not yet defined the effective strain so on.

But on the x axis we assume that it is an effective strain we have not yet defined it but let us say on the x axis you have effective strain. So, if we were to plot it(Refer time: 07:48)you know plot the Von - Mises then we would see that for the same material it would come out like this. I will write Von - Mises elsewhere so that it does not guess get hot watch. So, in this particular case this is where the yielding is occurring and, so in this particular case this is the point this is not Y this would be$\sqrt{2}Y$.

Because we have said that when the sigma bar reaches $\sqrt{2}Y$ this is this particular value that is when yielding will occur and this is your sigma bar which is given by this equation. So, what we see is that the plot if we were to compare it with the uniaxial tensile stress strain for the Tresca von it looks very much like the uniaxial stress strain. Because here also yielding occurs at Y and in the uniaxial condition also it happens at Y.

But when we go to Von - Mises then the yielding here occurs as$\sqrt{2}Y$. While in the uniaxial stress strain condition the yielding occurs as to Y. And therefore, there is a little bit of offset so why not define our effective stress a little bit with a factor**.**

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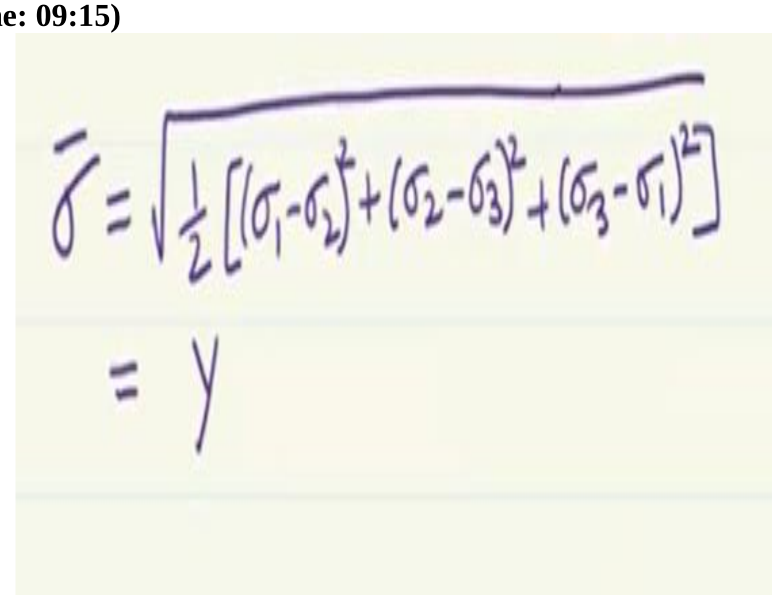

So instead of this being the effective stress why not define effective stress as and now if we put the values over here which is this under root item has this is the whole thing is C so this is 1 by 2 C and C = 2 Y^2. So overall this if you put in the values in terms of Y it comes out to Y. And now this equation or this plot also translates to same as on the left and here also if you keep increasing the effective stress now with this new definition not the older definition.

So, this one was with the older definition and this is new definition. So, with the new definition we see that the stress strain curve or the effective stress strain curve for Tresca and Von – Mises matches not only that if you look at only for the stress part right now. If you look at the stress part then they also match with the uniaxial stress strain condition. And therefore, this meaning this definition of effective stress looks much more meaningful.

And that is how effective stress for Von – Mises is described and the reason of putting this factor 1/2 is only this that you can have a good correlation with the uniaxial stress strain condition. So, this stress strain curve condition the stress strain effective stress strain curve would be similar to the true stress strain curve**.** So, but we have only talked about stress part so far.

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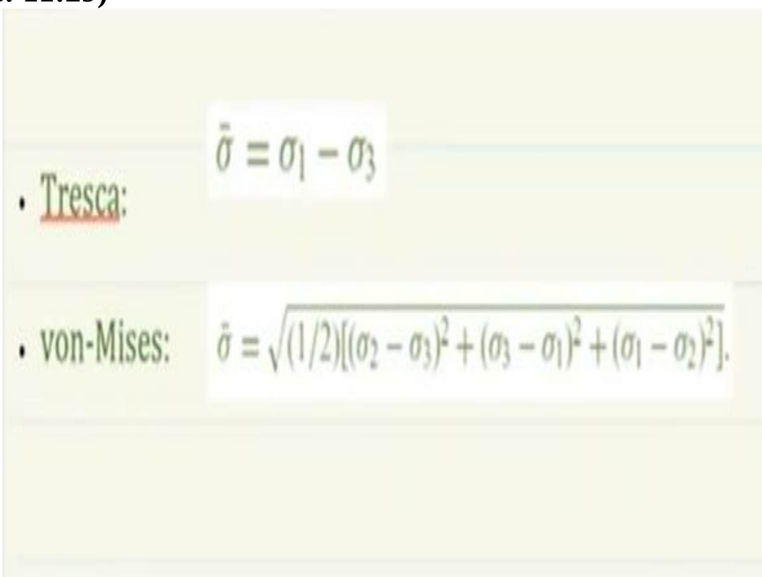

Now that is your equation so this is what would what is meaningful? If we use define Tresca effective stress as $\bar{\sigma} = \sigma_1 - \sigma_3$ and Von - Mises effective stress as 1 by 2 factor and the whole term that we had earlier so that is the effective stress.

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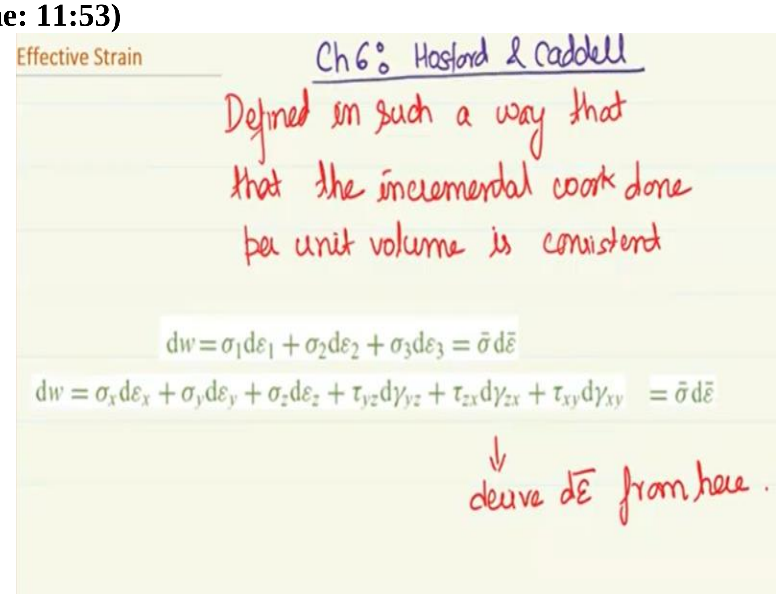

Now let us move on to effective strain. Now effective Strain can be defined as a parameter which is a function of the applied stress of the let mere phrase that. Effective strain is not something that can be defined in the same way as the effective stress. Effective strain is path dependent that is one problem. Second is that you cannot have an independent definition of effective strain because in the end the total work done must be same.

And therefore, the equation from where we will get the value for effective strain is the total work done which is over here. So, it needs to be defined in such a way such that the incremental work done per unit volume is consistent. Consistent meaning if you take the individual direction principle stress principle strain in the 3 direction and that total stress total strain in the incremental format then these 2 should match.

So, the relation for effective strain must come from this and this is the same equation written when you have the more general format for the stresses. So here it was in principle stresses here it is written in general format and in both the cases the total work done assuming an incremental word incremental strain should come to come out to be consistent with each other.

And this is where this is the relation from where we will have to define or derive so that is the equation that is used for deriving the d epsilon bar. However, we will not go through the derivation and you can look at chapter 6 in again Hosford and Caddell. Which deals with effective stress and effective strain and there you would be able to look at the more detailed derivation.

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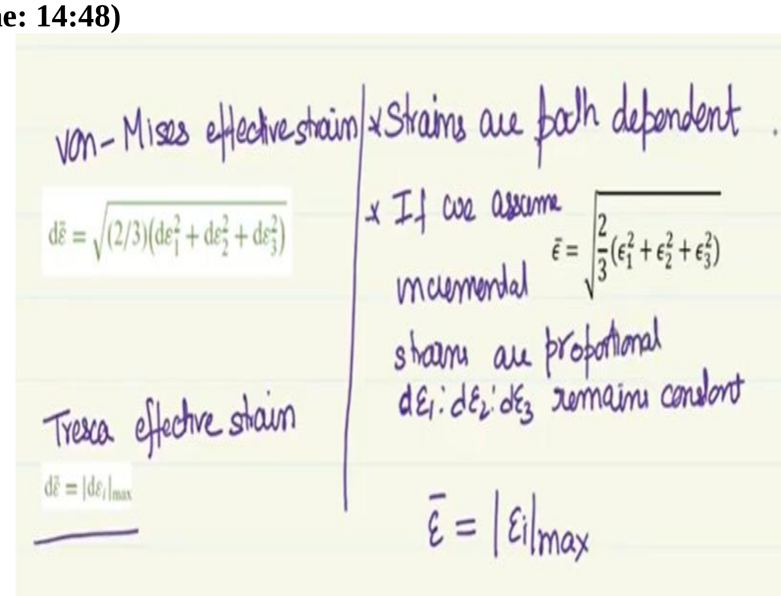

But for now it is sufficient to understand that the Von -Mises Effective strain is given by this relation. And similarly, if you look at the Tresca effective strain this is what the equation comes out to. So, you remember we were looking at the maximum shear strain so the effective strain that we were looking at maximum shear stress in the case of Tresca criterion.

And therefore, the effective strain incremental effective strain for the Tresca criterion turns out to bed epsilon i max. So whichever principal strains is the maximum you take the magnitude of that and you take the and that becomes your effective strain for the Tresca criterion that is what we will derive when we use this constraint the equation given over here.

On the other hand, for Von – Mises we will get this kind of relation so this is again given in terms of principal strains $\frac{2}{3}(d\varepsilon_1{}^2 + d\varepsilon_2{}^2 + d\varepsilon_3{}^2)$. So there is a factor 2 by 3 and this is the effective strain for the Von-Mises criterion. And now one more thing that I have already mentioned to you is that strains are path dependent**.** So that is why all these equations have been written in incremental form.

Because at the small Incremental stage this relation holds but when the deformation becomes very large then this relation will not hold because at each stage the amount of deformation occurring in different principle strains would be different. However, if we assume Proportional strains what does proportional strain mean what it means is that $d\varepsilon_1 : d\varepsilon_2 : d\varepsilon_3$ remains constant.

So, you are deforming in a way that at each stage the ratio of $d\varepsilon_1 : d\varepsilon_2 : d\varepsilon_3$ remains constant. Then you can extend this relation to this one and then this becomes your effective strain relation. So here the $d\varepsilon$ is now replaced with the strains so $\bar{\varepsilon} = \frac{2}{3}(\varepsilon_1{}^2 + \varepsilon_2{}^2 + \varepsilon_3{}^2)$ and for the case of Tresca this epsilon bar translates to ε_i max.

So, this is these are the 2 definitions of Effective stress and Effective strain for Von- Mises and for the Tresca. So, this is the original equation and assuming that we are keeping d epsilon 1 to the epsilon 2 to the epsilon 3 constant we get this. Here too if we this is the accurate relation but if we assume that proportionate that are strains the incremental strains are proportionate so yes this is the term I missed here.

If we assume incremental strains are proportional this is a better phrasing that the proportional strains the incremental strains are proportional which means $d\varepsilon_1 : d\varepsilon_2 : d\varepsilon_3$ a remains constant then this is what we get**.** So now let us look at a uniaxial stress condition what will happen in the case of uniaxial stress condition we have already seen for we have already seen for the stress part now let us look at the strain part what will happen to the uniaxial strain if we are looking at uniaxial stress strain curve.

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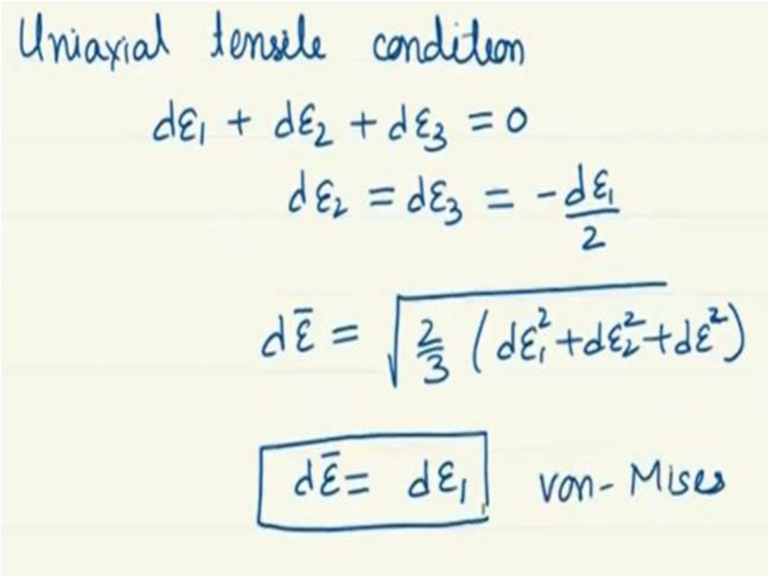

Uniaxial tensile condition so here we remember that the primary strain is happening in longer direction in which you are pulling the dog bone or your sample and 2 lateral strains are taking place in perpendicular directions. So, one thing we have to first keep in mind is that since it is plastic deformation so $d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3$=0. But at the same time $d\varepsilon_2$ and $d\varepsilon_3$ are same.

And therefore, if you put it here what you will get is that it is half of the $d\varepsilon_1$. And now if you; put this relation into our relation for the effective strain. Then we have 2 by 3 and when you put it here so ${d\varepsilon_1}^2$ instead of $d\varepsilon_2$, we will put ${d\varepsilon_1}^2/4$. So, this is also ${d\varepsilon_1}^2/4$ so these 2 become ${d\varepsilon_1}^2/2$ and therefore the whole thing is $3/2{d\varepsilon_1}^2$ and this 2 by 3 gets cancelled out and what we are left with is $d\varepsilon_1$ so $\overline{d\varepsilon}$ = $d\varepsilon_1$ for Von - misses.

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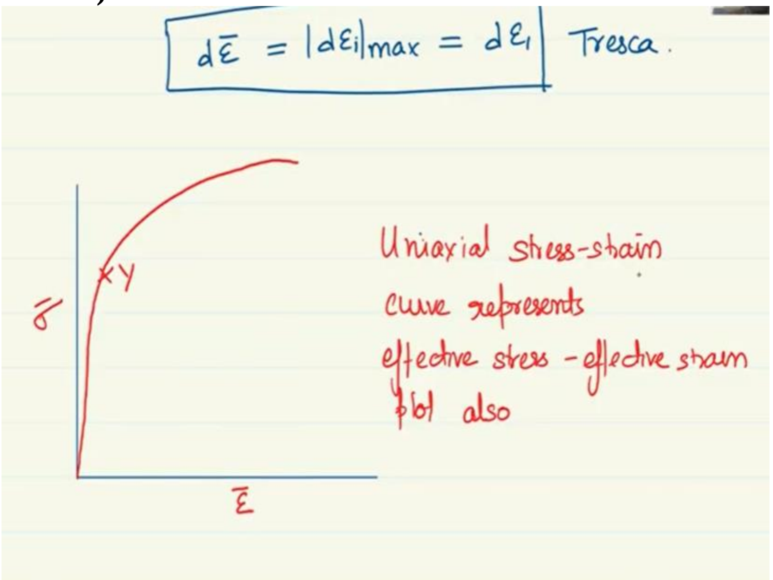

And if you were using the Tresca condition then also we know that d epsilon bar = d epsilon i max which would have been d epsilon 1. So even in the Tresca condition we get so, this is giving us a very interesting insight about stress strain. Let us say we have the stress strain curve drawn like this for a uniaxial tensile condition. Now how would the effective stress strain look like so the way we have defined this also becomes the effective stress and this also becomes the effective strain.

Because we have already seen the $\bar{\varepsilon}$ is equal to $d\varepsilon_1$ and sigma bar is equal to so in the effective stress only one stress is acting which is the sigma 1. And it is equal to Y at the point of yielding by our definition and therefore this whole curve is now also representing the effective stress strain curve. So, what we understand here is that Uniaxial stress strain curve represents Effective stress Effective strain plot also.

So, that is a very powerful and insightful statement let us see how. So, you have done a simple uniaxial stress strain test and you have plotted it like this now if you do some other multi-axial deformation and you plot or you are able to obtain the effective stress and effective strain for this how would it look like according to this yes this is how it should look like you do forging you do rolling you do any other deformation.

And the effective stress and effective strain plot that you would obtain from there would look like this or in other words you can if you have obtained the uniaxial stress strain plot you are in other words also in possession of the effective stress strain plot for all other conditions. So that is the usefulness or utility of this particular statement. So now we have the stress strain colour for all different multi-axial loading condition that is what we have now**.** So, now that we have understood the effective stress strain properly.

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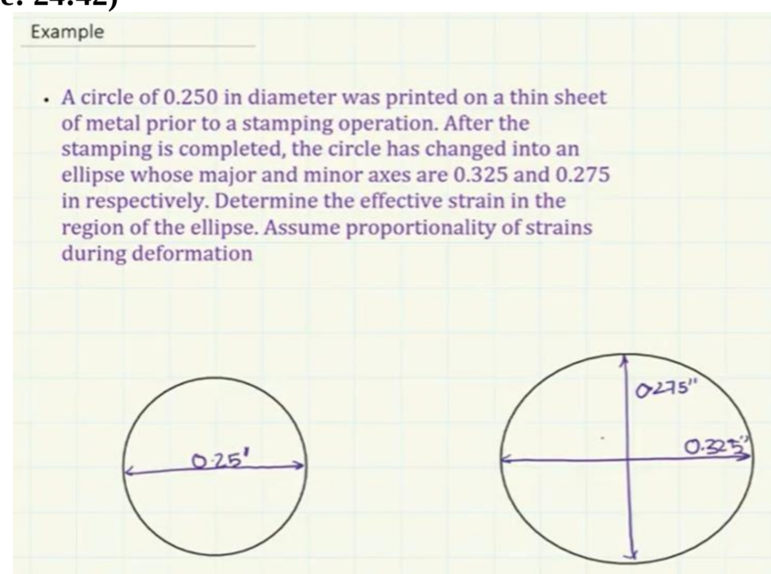

Let us look at one example the example is a circle of 0.25 inch diameter. So, this is the diameter given to you was printed on a thin sheet of metal prior to stamping operation. So, this thin sheet was stamped out by operation and this is the diameter 0.25 inch after the stamping is completed the circle has changed into an ellipse**.** So, before this is before stamping and after stamping this is how the circle looks like now the major axis is 0.325 inch and the minor axis is 0.275 inch**.**

So, both the axis have got elongated a little bit only different amounts. determine the effective strain in there gi on of the ellipse assume proportionality of strains during deformation so here you are already given that can assume proportionality of strains so instead of calculating the incremental effective strain we can directly calculate the total effective strain.

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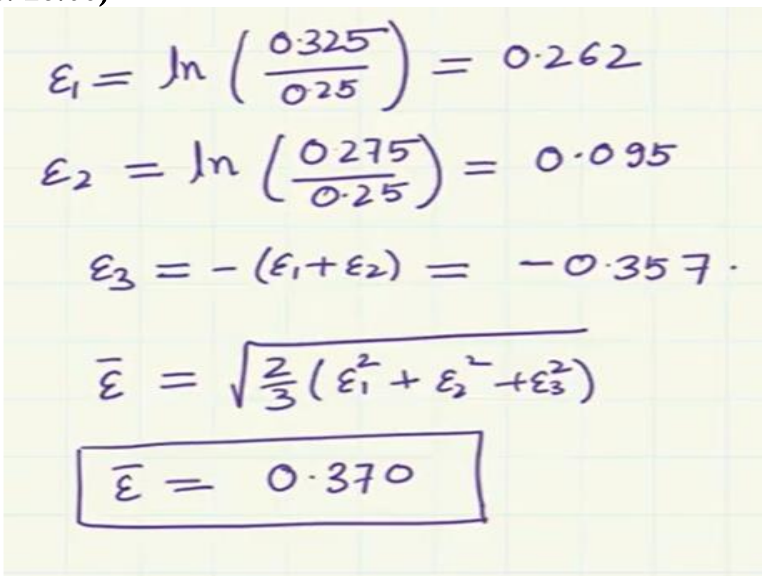

So let us first calculate

$$\varepsilon_1 = \ln\left(\frac{0.325}{0.25}\right) = 0.262$$
$$\varepsilon_2 = \ln\left(\frac{0.275}{0.25}\right) = 0.095$$

So, we know the true strain in this direction we know the true strain in that direction and therefore epsilon 3

$$\varepsilon_3 = -(\varepsilon_1 + \varepsilon_2) = -0.357$$

which because it is a plastic strain is equal to -0.357.

And now that we have epsilon 1 2 3, we can very easily calculate the total effective strain

$$\bar{\varepsilon} = \frac{2}{3}(\varepsilon_1{}^2 + \varepsilon_2{}^2 + \varepsilon_3{}^2) = 0.370$$

So, what we see is that the effective strain is equal to 0.370.

And we are assuming that once when this original diameter circle of the thin sheet was getting deformed then it was getting deformed proportionately it is not that it was deformed completely in one direction and then the other direction. Then this relation does not hold it means that at every stage the amount of deformation happening on this axis was proportional to the axis the strain happening on this axis so that is one condition that has been assumed.

So as long as during your deformation process this particular condition is maintained you can use this relation and calculate the effective strain**.** So, this was a small topic and I hope I have conveyed to you the concepts of effective stress effective strain and also the importance of the way we have defined it. Because then we are able to utilize the true stress through strain curve from the uniaxial tensile condition as effective stress effective strain plot so with that, we will close this session thank you.