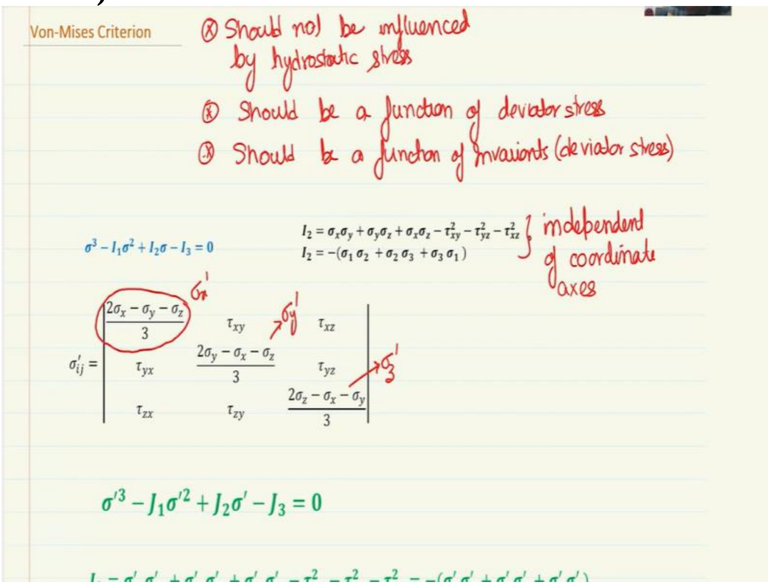
Mechanical Behaviour of Material - I
Prof. Shashank Shekhar
Department of Materials science and Engineering
Indian Institute of Technology, Kanpur

Module No # 03
Lecture No # 12
Yield Criterion's Tresca, Von-Mises

Welcome back students we were talking about yield criterions and 2 conditions or actually there is one condition that any yield criterion must satisfy is that is should not depend upon on hydrostatic stresses. So that is something that or we looked at we realized last time.

(Refer Slide Time: 00:38)



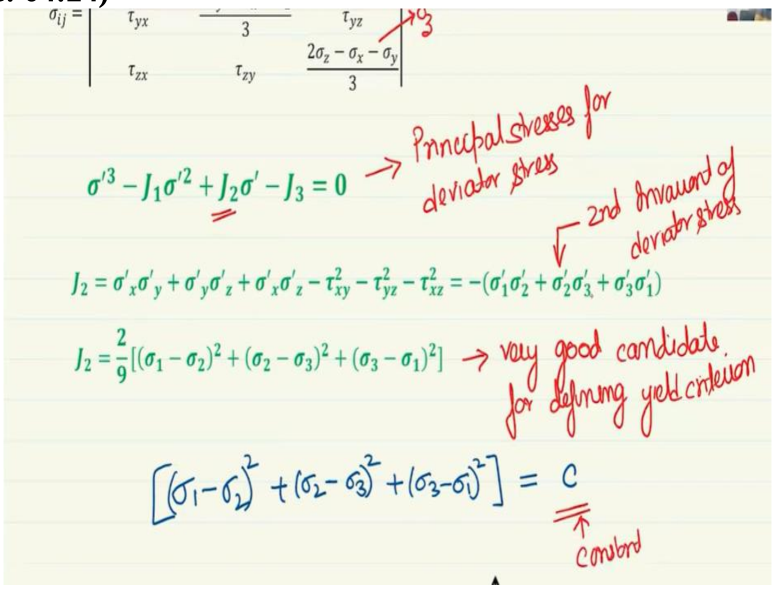
So the criterion should not be influenced by second thing that we noted was that if we want to arrive at such criterion then it should be some function of deviation stress. So with this understanding let us look at how to get to one of those criterions and it is one messaged called one message criterion. Now you remember that if you want to get a principal stresses then the principle stresses are given from this equation where I $_1$, I $_2$, I $_3$ are invariance.

Again we discussed with respect to how to derive a criterion there we discussed that the criterion should also be independent of axis. So it must be some function of the invariance of a deviator stress. So this is how our equation looked like for finding the principle stresses the roots of this equation would give you the principal stresses where I $_1$, 1 $_2$, I $_3$ are invariants and one of these invariants I $_2$ looks like this.

If you write it in general terms it is like this and if you write it in terms of principle stresses this is how it looks like. And this particular invariant happens to be independent of coordinate access actually all of them would be and this one is even more interesting by the very definition the invariant should be independent of invariant axis. Now this is how our deviator stress looks like $\sigma_{ij'}$ where now we have taken away the hydrostatic stress from the diagonal component.

So this is how it looks like so let us call it sigma x prime this becomes now let us try to find the second invariant for this deviator stress and this is $\sigma_{x'}, \sigma_{y'}, \sigma_{z'}$. Similarly we can also write this in terms of principle stresses then it will become $\sigma_{1'}, \sigma_{2'}, \sigma_{3'}$. And in that case your invariant for the stresses deviator would be obtained from this equation.

(Refer Slide Time: 04:24)



So this would give you the principle stresses for deviator stress where J_2 is the one that. We are interested in so J_2 going from this previous equation would look like would have this form and I have already written the equation. So that there is no error in when I am writing it down so J_2 would look like this in terms of σ (x, y , z) or in terms of principle stresses. It would look like this where remember

$$\sigma_{1'} = \frac{2\sigma_1 - \sigma_2 - \sigma_3}{3}$$

$$\sigma_{2'} = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{3}$$

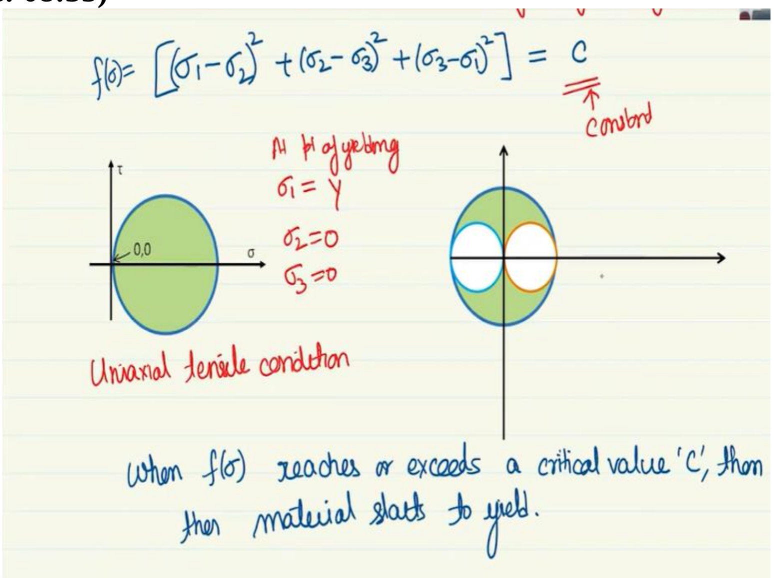
So if you take this as your now this not if but this is now your J 2 or the second invariant of the deviator stress. And based on our understanding from the last class it is this particular function is this is a function of deviator stress. And it is a function of invariance of the deviator stress and what is more is that if you expand it you write $\sigma_{1'}, \sigma_{2'}, \sigma_{3'}$ in terms of sigma 1, 2, 3.

Then it translates into this form and here you can clearly see now this form shows that it is clearly a function of shear stress. So it is a function of shear stress satisfies the first condition it is the function of deviator stress is satisfies the second condition. And it is a function of the invariants so it satisfies the third condition and therefore this looks like a very good candidate for defining a yield criterion.

In other words we can write the one message criterion can now, based on this can be written as we can get rid of the factor here and we can write. When this function reaches some critical value let us call it c so this c would be a constant so when this function reaches some critical value then this particular material which is in this stress state would yield. Now if you take this as the definition of yielding then you can see it is taking into account the overall stress state it is taking into account only the shear stresses.

As we expect that only shear stresses cause plastic deformation and it is a function of deviator stress and it is a function of the invariance of the deviatoric stress. So certainly it does define some mechanics which will describe the yielding of the material. And therefore this will be a very good criterion to define yielding and now how to obtain the next question is how to obtain this constant C. So this is a constant that we know and let me to be precise.

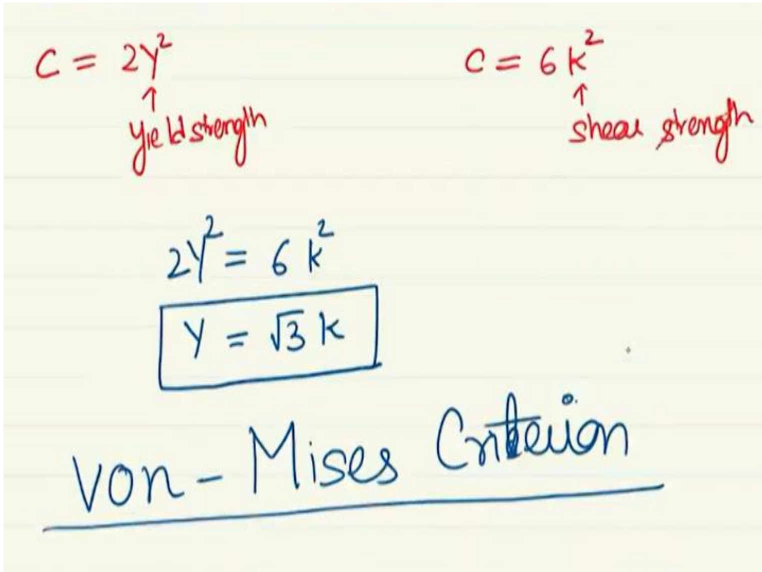
(Refer Slide Time: 08:35)



So that we understand when this function which is on the left so when f(σ) reaches or exceeds critical value C. Then the material starts to yield so we have now a criterion and it looks a lot more simple than we initially thought it would be. Now we are what we need to do is find out the value of this constant C. So how do we find the value of constant C for that we know that this particular equation should be valid for all conditions and that would mean it should also be valid for uniaxial tensile conditions.

Now when we are talking about a uniaxial tensile condition then we know that it can be represented by a Mohr circle like this. So this is a uniaxial tensile condition and what happens in a uniaxial tensile condition we have sigma 1 at the point of yielding sigma 1 = y, sigma 2 = 0, sigma 3 = 0. So if we insert these values $\sigma_1, \sigma_2, \sigma_3$ then we will have the value of C in terms of y.

(Refer Slide Time: 10:42)



So let us see how does, it look like and if you put it all together then would see that basically here sigma 1 all other terms goes to 0 only σ_1 and remains non zero and it is y. So this $2y^2 = C$ and therefore constant is equal to $2y^2$ on the other hand we can also find this constant in terms of the shear strength and for shearing. We know that this is a Mohr circle that represents the shearing condition.

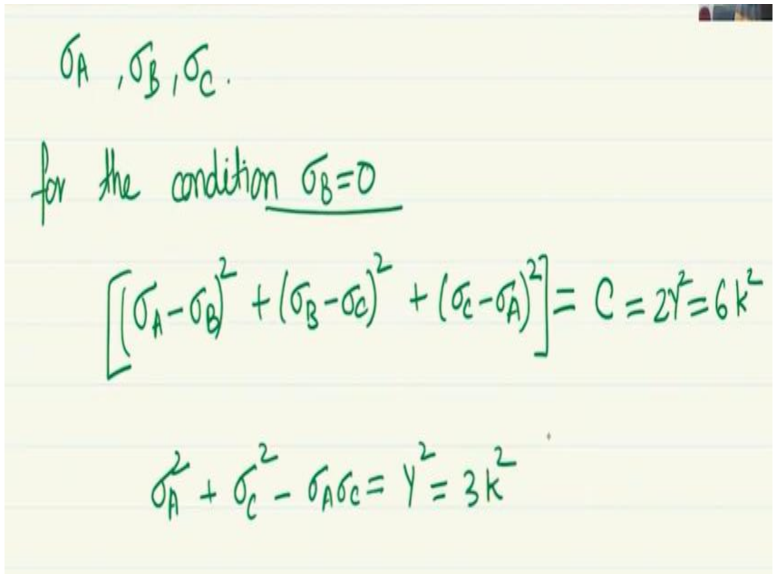
So this represents and under shearing we know that σ_1 = +k , $\sigma_3 = -k$ and σ_2 = 0. So now when we put σ_1 = +k and σ_2 = 0 what we get is c = 6 k^2 . you can check that out it will be a good exercise for you and a good hands-on experience. If you do this on your own it is a one step process so C = 2 y^2in terms of yield strength, where y is yield strength of the given material and k is the shear strength of the material which means that based on this criterion.

We get a relation between y and k and what is that relation what we get is that$y = \sqrt{3k}$. So if you have shear strength of the given of a material you can find from that you can find what should be the wheeled strength or if you have the yield strength given. Then we know that $y/\sqrt{3}$ would be the shear strength of the material.

So this is our von mises criterion let me write it and it would be written something like this would be the formal way to write this one criterion that this should value should reach C or above for yielding to occur. Now here we have $\sigma_1, \sigma_2, \sigma_3$ and if you want to see how the σ_1 varies with σ_3 at the point of yielding. So we want let us we want to draw the locus of how the σ_1, σ_2 are related at the point of yielding then what we can do is we can take σ 1, 2, 3 as constant.

Let us say 0 so and now since we are taking one of them 0 and keeping it constant therefore we are not following the convention of sigma 1 sigma 2 sigma, sigma 2 greater than sigma 3 and therefore let us now use the term sigma 1, 2, 3.

(Refer Slide Time: 14:18)



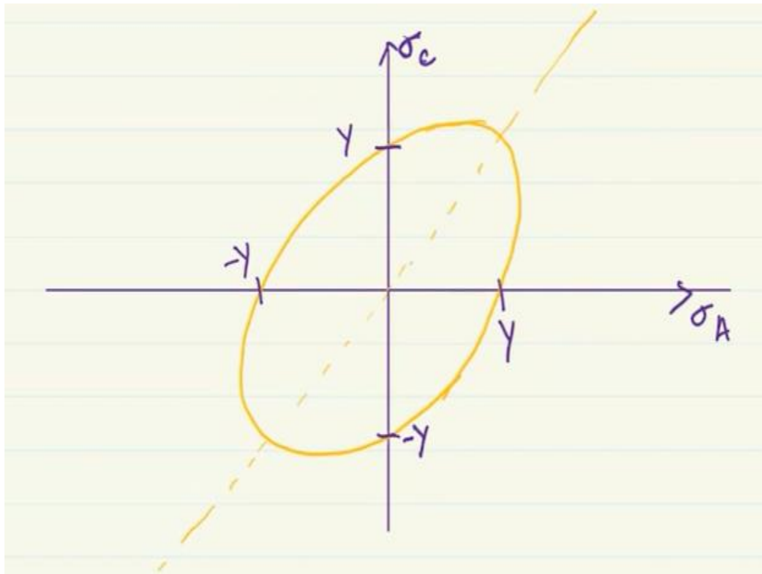
Let us use the term sigma a sigma b and sigma c and here let us say for the condition sigma b = 0. How does the plot look like so if you write the so all you have to do is in the sigma equation that we have over here in terms of sigma 1, 2, 3 you can see it is symmetric with respect to σ 1, 2, 3 as long as it is in this form of the equation here. It is say that sigma we will now convert it to σ - a, b and c.

So we will so here we are taking this equation and converting 1 to 3 to a, b, c and this constant is c which we know is equal to either 2y square or equal to 6 k square. Now here we are saying that we want to draw locus at the point of yielding and to simplify the equation because it will be a 3 dimensional equation we will keep sigma b = 0 and therefore now we have 2 axis sigma a and sigma c.

So the equation would be in terms of

$$\sigma_A{}^2 + \sigma_c{}^2 - 2\sigma_A\sigma_C = y^2 = 3k^2$$

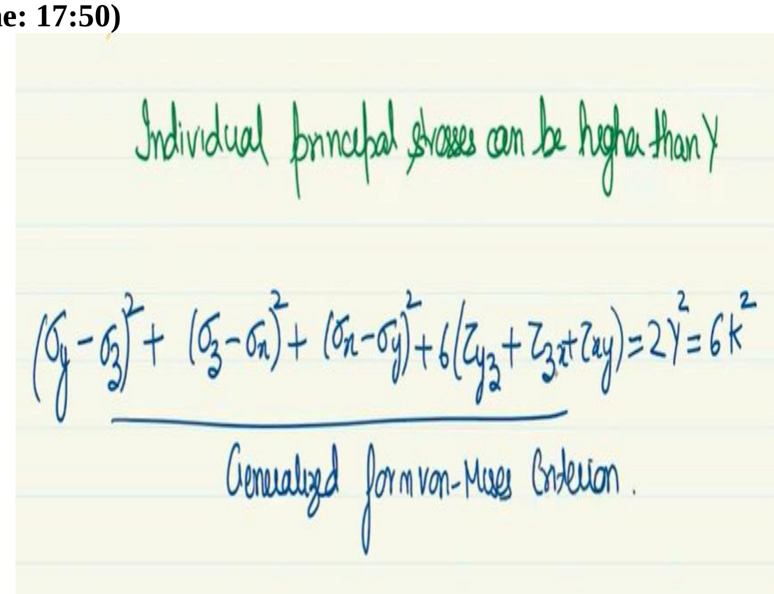
(Refer Slide Time: 16:17)



If you are draw this how it would look like so it is an ellipse remember and the axes are as I said one is σ_A, and the other is σ_C. So there are still the principal stresses only that we do not know which one is one with 1 is 2 and which 1 is 3. So we have named it as σ_A, B and C and now at this particular point here what we have is σ_C is also equal to 0. So if you put it in this equation then what you would find is that this value is equal to y similarly this one will be equal to y this one will be equal to -y.

So although the ellipse may not look symmetric but you will have to take my word for it and this is tilted at 45 degrees. So this is the line which is at x = y angle meaning 45 degrees angle and what we see is that if you go these points. So here σ_A is greater than y here σ_C is greater than y here sigma is less than –y sigma here σ_C is less than –y here both of them are less than –y here both of them are greater than +y.

(Refer Slide Time: 17:50)



So what it is showing is that the individual principle stresses can be higher than you this plot this ellipse so long as you are inside it then no yielding is taking place. So as long as the principal stresses state is such that the principal stresses lie within the circle then no welding is taking place as you are as you reach the circle it is then that you are actually beginning to yield and when you are above this circle ellipse then you are all beyond this line.

Then the yielding has begun so this hatched region let me is no yielding and over here it is everything beyond this is actually yielding and the line describes partition. So from no wielding to yielding so if you were looking at the uniaxial. So it is you are increasing the stress and at this particular point your stress has reached the strength of the material.

So here beings and when you keep increasing the stress then more and more deformation is taking place and as long as you are below this no healing is taking place you are in the elastic regime. But then now we are in a multi-axial stress state and in this particular case 2 axial stress state. So as long as you are inside this circle then you are not doing that you are not leading to any yielding and beyond this you are causing yielding.

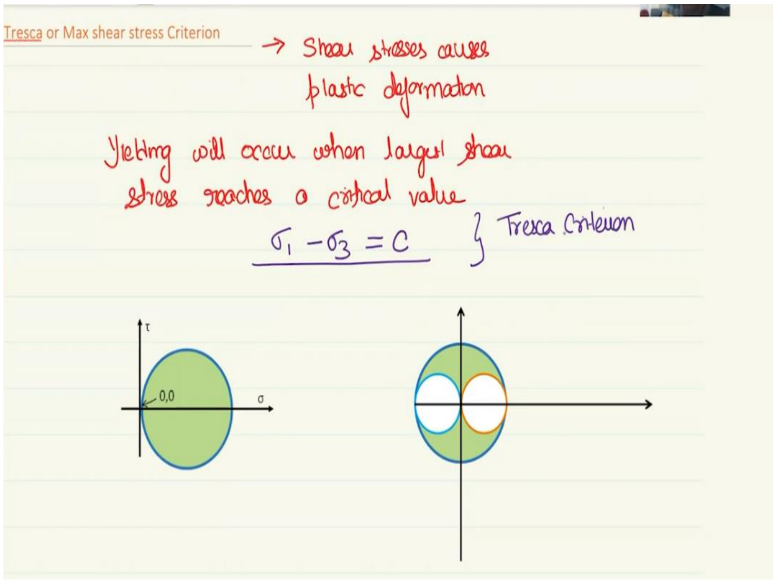
And what we see is that principle stresses even higher than the yield strength may not lead to yielding up to certain value. Of course as you can see so there is a possibility that you may that the principle stresses may be higher than the yield strength of the material but you are still not yielding given the stress state. So this is something that we looked at from the example but now here we can clearly see with the help of the equation that we have derived for the criterion.

So now also I wanted to end this particular I would also like to give you the generalized equation meaning in so far we have looked at in terms of principle stresses how would this equation look if we had general stress state meaning σ (x, y, z) and τ (x, y, z). So in that particular way it will be written like this so this is the generalized form of the one message equation.

So in this particular section we have looked at we started from our requirements that there should not be the criterion should not be influenced by hydrostatic stress should be a function of deviatoric stress should be a function of invariance. From there we come to this J_2 and therefore we can say that when J_2 reaches certain value it will start to yield and we can if we remove the factor then is what we are left with the $(\sigma_1 - \sigma_2)^2) + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = c$

Then we moved on to find the value of C and we found that C = $2y^2$ square in terms of yield strength of the material or equal to $6k^2$ where k is the shear strength of the material. Then we looked at the locus in, 2 axis σ_A and assuming σ_B is 0. This is how it is looks like and in fact we also looked at the generalized of the equations this is how looks like. If we have the stresses in the general form and now; just the last point regarding the one message.

(Refer Slide Time: 22:41)



If you were to draw the surface in the 3 dimension where the yield begins then it would be a surface and that surface would actually look like a tube. So let me just draw this so again I will not use σ (1 2) I would use σ_A, σ_B and σ_C and this would be tilted at a equi-axis angle from all the 3 axis and the tube would be. So this is what the equation that we have describes this is the surface that you will get for the yield in the 3 principal axis directions.

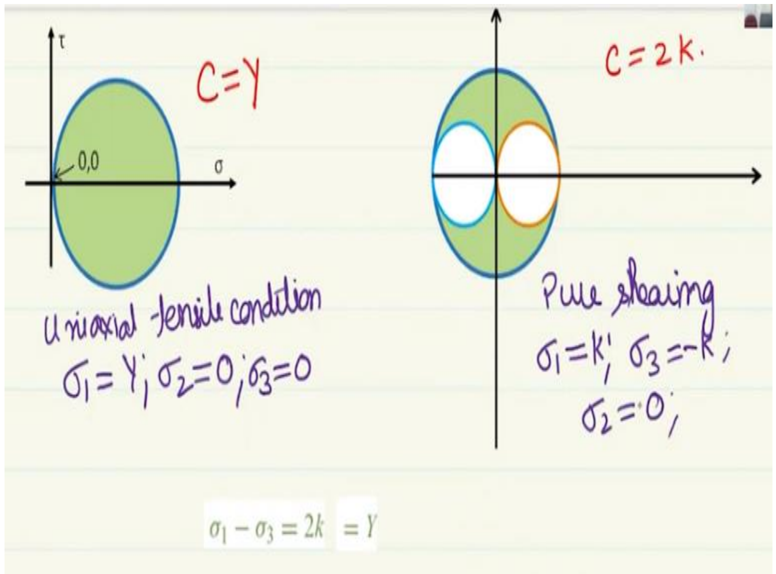
And the radius of the circle can be shown to be not the circle the cylinder and this would be called the yield surface. So again what it means is that as long as the principle stresses lie within this surface then yielding does not take place. But if you are at the line yielding just begins and when you cross this line then the formation is continuously taking place. So we will now move on to so this is the phone message criterion.

Now let us move on to still another criterion and the name of this criterion is tresca or max shear stress criterion you remember. We said that shear stresses are what cause the yielding or plastic deformation. So one of the criterion directly picks up from here and says that the yielding will occur when the largest shear stress will reach a critical value when largest shear stress reaches a critical value.

And if you remember one place we showed that the principal shear stresses can be expressed in terms of principle stresses and of these 3 principles shear stresses the one that is the maximum is $(\sigma_1 - \sigma_3)/2$. So basically $(\sigma_1 - \sigma_3)/2$ should reach some critical value or in other words we can write it like this that $(\sigma_1 - \sigma_3)$ should reach some critical value C this becomes our max shear stress criterion or the tresca criterion.

Now these again we have a constant C therefore what we want is to f define the value of or find the value of C.

(Refer Slide Time: 26:44)



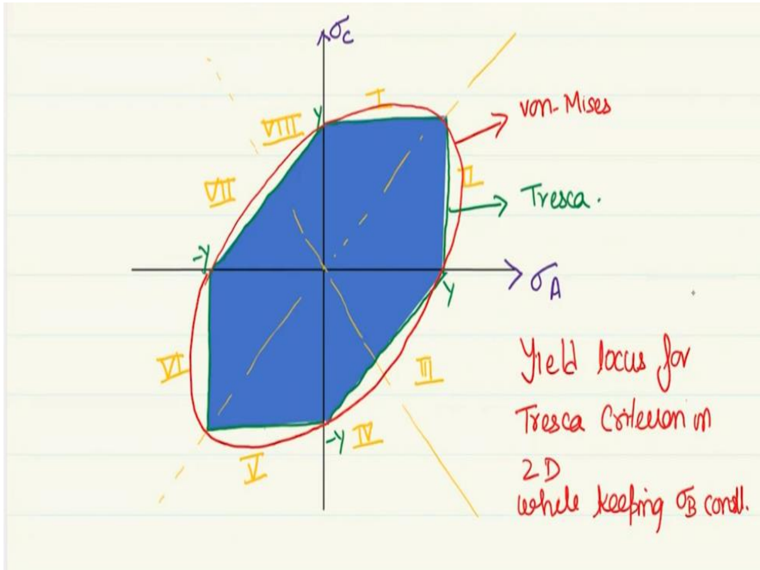
So one of the conditions that we can utilize is again the uniaxial tensile condition and we know that in the uniaxial tensile condition σ_1 = y , σ_2= 0 σ_3 = 0. Therefore when you put it over here we get y – 0 = C and therefore the value of C = y. Now like previous time we will also try to find this constant in terms of the shear strength of the material.

So we know that the pure shearing takes place then under so this is the pure shearing condition and this is the Mohr circle that represents the pure shearing condition under this we have sigma 1 = y sigma 3 equal to minus this should be k , σ_3 = –k and σ_2 = 0. Therefore what we have now is that k + k = c or c = 2 k therefore now we can write down the equation like this is what the criterion translates to σ_1- σ_3 = 2 k = y.

And this also gives us another relation between the yield strength and shear strength. So here we see that y = 2 k on the other hand in the previous case we had y = $\sqrt{3}$k. So there is a slight difference and that is dependent upon how we have derived the 2 equations. So now that we understand the other criteria in the Tresca criterion. Let us move on to finding the locus of the yield point in terms of the principal stresses.

So again what we want is to derive this equation or to write draw a profile or the locus in terms of the principle stresses. But again we do not know which one is $\sigma_1$and 2 and 3 so we will write this in terms of σ (A, B and C).

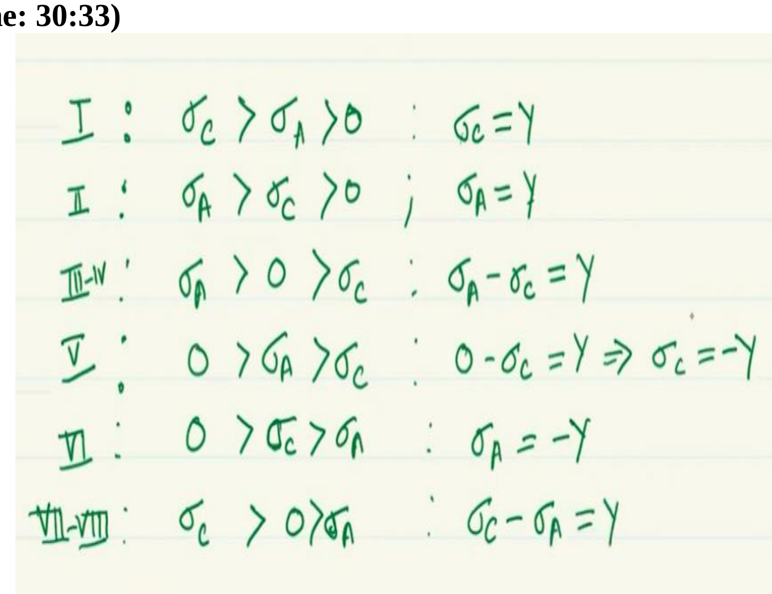
(Refer Slide Time: 29:26)



So let us say this becomes your sigma a this becomes your sigma C now we will also draw 45 degree line that is x equal to which will be at equal slope x = y. So it will look like this so anywhere over here sigma C is greater than sigma A and over here sigma A is greater than sigma C. Similarly we will draw a line here because these would define different zones we will get different values or different forms of the same equation.

So we will call this as zone 1 will call this as zone 2 call this as zone 3 call this as zone 4, 5, 6, 7 and 8.

(Refer Slide Time: 30:33)



Now let us start from zone 1 in zone 1 what we see is that sigma C would always be greater than sigma A and both of them would be greater than 0. So C would be greater than sigma A would be greater than 0 which means that the when you apply the tresca criterion which is maximum minus minimum principle stress is equal to the yield strength then maximum here is sigma C and minimum is 0.

Therefore yield strength in this one would be constant which will be equal to sigma C. In zone 2 what we will have so before I go to zone 2 so now if sigma C = y which is a constant there what we have is a constant like this. So this is how the profile would look like this is a straight line from here to here horizontal parallel to the x-axis. Now coming to zone 2 where we know that sigma A is greater than sigma C is greater than 0 which means this is the highest this is the lowest.

So yield stress is equal to sigma A therefore y is equal to sigma A or in fact I should say sigma C = y and sigma A = y. So I will write it like this because C and A are now variable while y is a constant. And therefore this is becomes another straight line and this is what we have over here. So this is a straight line this is value is y now in zone 3 what we have is that sigma A is greater than 0 but sigma C is in the negative side.

So sigma C it is greater than sigma C and therefore yield strength would be defined by this minus this sigma A – sigma C or x – y = constant. Therefore this will become a straight line like this and this is what we have over here. Now in zone 4 what we have is that sigma A is also less than 0 but it is still greater than sigma C. So this it is less negative than this one therefore sigma is greater than sigma C.

And now in this case what we will get is that this is the maximum this is the minimum and therefore 0 minus sigma C will be equal to y implies sigma C = -y. And therefore what we get is a horizontal line so this becomes our profile or locus for zone actually this is 3 and 4 and this is zone 5. Now, coming to zone 6 so here sigma C is greater than sigma A but both them are less than 0.

Therefore what similar to this is here we will get sigma A = -y and therefore again we have a straight line here this is the profile. Now in zone 8 what we have is sigma C is greater than 0 is greater than sigma A. So you can see that here sigma A is in the negative but sigma C is in the positive so sigma C is in the greater than 0 but sigma A is less than 0. And therefore the focus of yielding would be given by sigma C by sigma A = y.

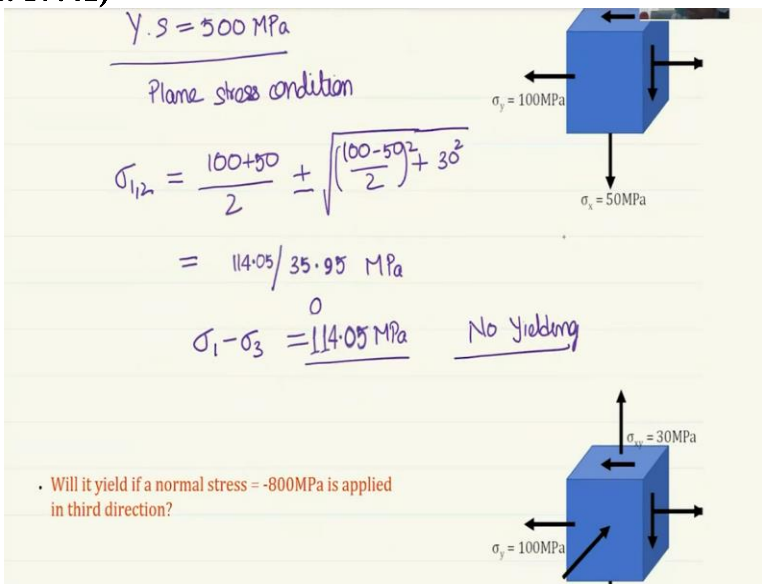
And therefore this is a straight line like this therefore this becomes our this hexagon type of shape is what defines the yield condition under the Tresca criterion. And here what we see is that the maximum value that any of these are talking is equal to y. So this is +y this is –y so this is a little different from the previous locus that we obtained and again this is for just one plane.

So here we are assuming that sigma B = 0 so overall this is how the yield locus of Tresca criterion would look like in a 2 dimensional plot. So let me write it over here in 2 dimensions while keeping third one constant which is sigma B. And remember that sigma A, B, C also represent the principle stresses we have not used 1, 2, 3 because at least to being with we do not know which one is greater.

So any of them can become greater than therefore we cannot switch the names in between so what we have done is we named it as A, B and C. Now how do the yield locus for the Tresca criterion compare with that of the one message criterion. So if you could draw it then what you would see is that so this is how through 2 compares. So the red one is for the Von-Mises and the green one is for the Tresca.

So the greatest difference occurs along this line and along this line; and they are same values at 1, 2, 3, 4, 5, 6 different points which clearly shows that both the criterions are not very far off. So both of them are very close to each other which means both them are fairly accurate description of yielding condition. Now let us look at one example to understand this.

(Refer Slide Time: 37:41)



Let us say you are given the stress analysis of a space one of the particular component in a stress graph is given and you are given that the yield stress of the material is 500 Megapascal. So first let us write it yield strength = 500 MPa now you are given the stress state for that component and assume that it is the same everywhere like this. So you have 100 MPa acting here in the x-axis and 50 MPa acting normal stresses along the y axis and there is a shear stress of 30 MPa.

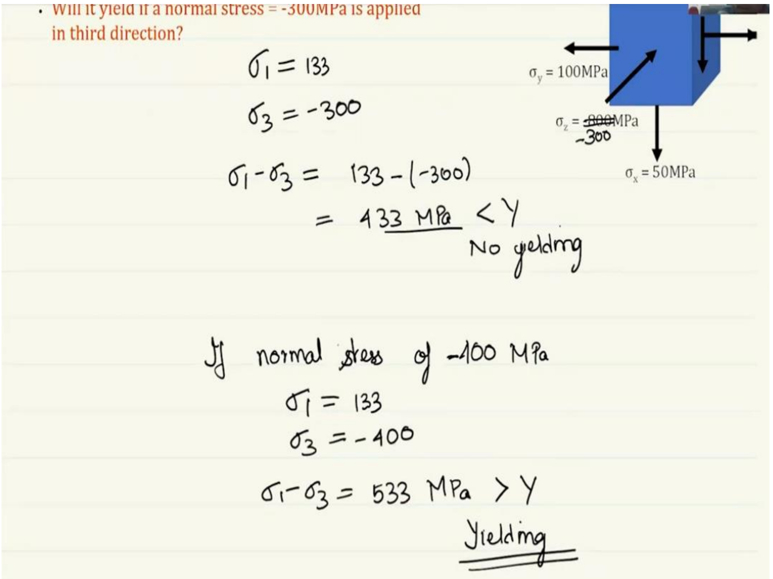
So 30 MPa and therefore 30 MPa over here therefore this is a plane stress condition now if we wanted to find the principle stresses over here then we know that we can use these values and put it in the Mohr circle and the Mohr circle using the Mohr circle you would know that we can get. So this will become the centre of the circle which is the x and y stresses so divided by 2 that brings you to the centre of the circle.

And then you find the radius which is equal to $[(100 – 50)/2]^2$ plus this is the shear stress 30^2. And when you calculate it you would see that the true values would come to slash Megapascal. And since it is a plane stress condition which means the third one also exist there and therefore there is a third stress which is 0 Megapascal. So if you were to find the $(\sigma_1 – \sigma_3)$ which should be equal to y for the Tresca condition to satisfy the yielding.

So if you remember $(\sigma_1 – \sigma_3)$ equal to 2 k = y so this is our equation which defines. So if the $(\sigma_1 – \sigma_3)$ = y and greater than y then yielding takes place. If it is less than y then no yielding takes place. So we will go back and look at what is the value of $(\sigma_1 – \sigma_3)$ and this is clearly much lower than yield strength. So no yielding takes place so this I what you are asked exhibit yielding.

So no it will not exhibited so here was supposed to be a change this should be equal to 300 Megapascal. And similarly there should be 300 Megapascal over there.

(Refer Slide Time: 40:50)



So there is a compressive stress so other conditions remain same but additionally you are now applying a 300 Megapascal stress over here the question is will it yield if you are applying now this additional compressive stress of -300 MPa. So again what we need to do is find σ_1, σ_2, σ_3 3 but then this time your task is easier why is it easier? Because σ_1, $\sigma_2$which is a plane stress condition is already obtained and therefore the third direction where no shear stress was acting this will be the sigma the other principle stress.

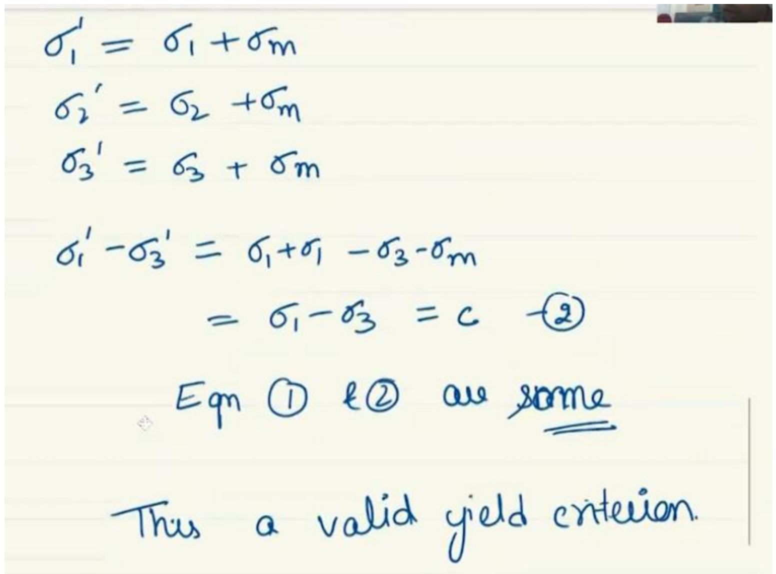
And therefore this becomes since it is minus so this becomes the sigma 3 so in this particular case σ_1 = 133 and σ_3 = -300 implies ($\sigma_1 - \sigma_3$)= 433 Megapascal. So it is still less than yield point and therefore no led but now this will give you an idea that. Now let us say if I applied a if normal stress of 400 MPa was applied. So this is less than the yield stress of the material so on any particular access you are not exceeding the yield stress only you are applying a -400 MPa over here.

Therefore now this time what will happen? σ_1 will be equal to 133 and σ_3 will be equal to -400 and therefore ($\sigma_1 - \sigma_3$) will be equal to 533 Megapascal which means that is greater than yield strength and therefore yielding will take place. So what do we see here? That even though; none of the principle stresses exceeded the yield stress of the material but ($\sigma_1 - \sigma_3$) exceeded the yield strength which is the highest shear stress.

So as long as the highest shear stress is greater than the yield strength then we see yielding that is what this particular criterion Tresca criterion defines and under this criterion under this definition. We see that this will actually yield okay so we have now seen 2 criterions and first one was obviously like valid criterion now what about this Tresca criterion is it a valid criterion or not.

So let us look at or to try to determine whether this is a valid criterion or not how do we determine? One of the things most important things is that when you apply a hydrostatic stress the yielding should not change the condition for a yielding should not change.

(Refer Slide Time: 44:07)



Now here we have said that $(\sigma_{1} - \sigma_{3})$= C this defines our yielding. Now let us make our $\sigma_{1'}$ which is a new stress now we are added some mean stress or hydrostatic stress to all the components. So

$\sigma_{1'} = \sigma_1 + \sigma_m$

$\sigma_{2'} = \sigma_2 + \sigma_m$

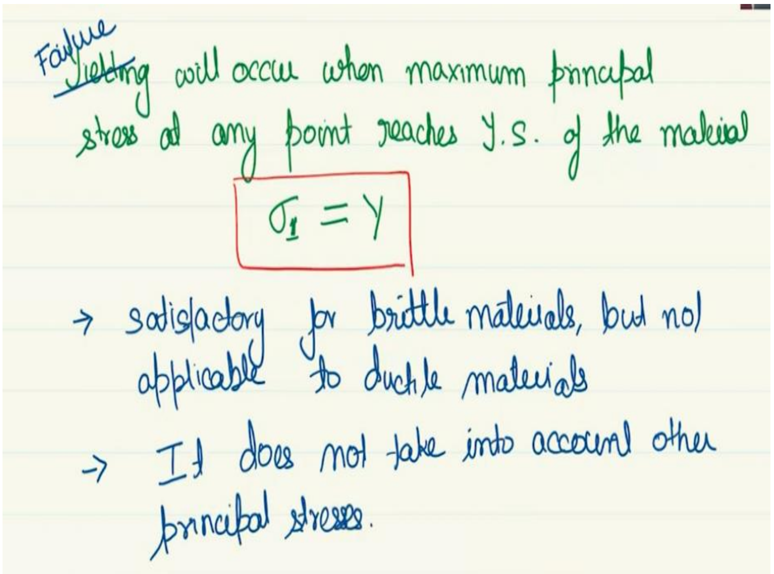
$\sigma_{3'} = \sigma_3 + \sigma_m$.

So now this is the same condition but with an additional mean stress applied on all, the principal or this is on the element and therefore these are the new principal stresses.

So now I will apply the condition on to this so this is $\sigma_{1'} - \sigma_{3'} = \sigma_1 - \sigma_3$ so this is same as equation 1 so this is again translates to equation 1 and 2 are same. Meaning what this means is that applying a hydrostatic stress did not change the yield criterion therefore this is a valid yield criterion so we will do this much test to test to find out whether the criterion given to us is valid or not.

And we clearly see that this is valid you can for your exercise you can try this also on the one message. And I can tell you before in that you would find that this is that one message is also valid criterion. Now let us move on to another criterion which is called the rankine or max stress criterion so one of the criterion we saw talked about maximum shear stress. So why not talk about maximum stress maybe we can define that actually this particular criterion is not about yielding it is about failure.

(Refer Slide Time: 46:49)

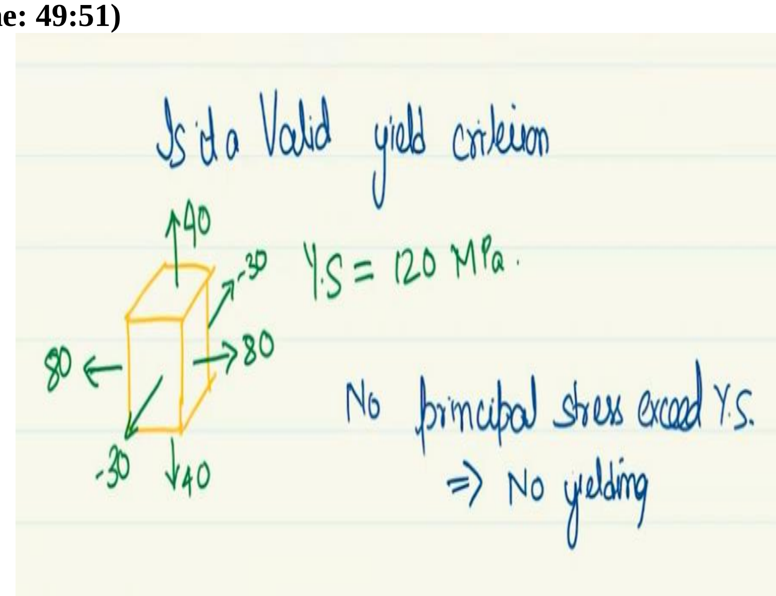


But let us say it is about yielding so yielding will occur so let us say it defines about yielding when maximum principle stress at any point reaches yield strength. So to define it this will equal to sigma 1 because this is the maximum principle stress σ_1 and σ_1 reaches y then yielding takes place that is what we can write mathematically this equation into. So this is how this will be defined but I will show you in with an example that this will turn out to be a invalid criterion.

And that is not surprising because it is not taking into account shear stresses it is only account taking into account normal stresses. But what is more important it is that this criterion is actually not defined for yielding but for failure. And this is criterion which has been found to be satisfactory for brittle materials but not applicable to ductile materials. So when you want to talk about failure of brittle materials then you apply this criterion and it has been found to be fairly satisfactory.

So as is clear one of the limitations of this model even if you take it for failure it does not take into accounts other 2 principle stresses.

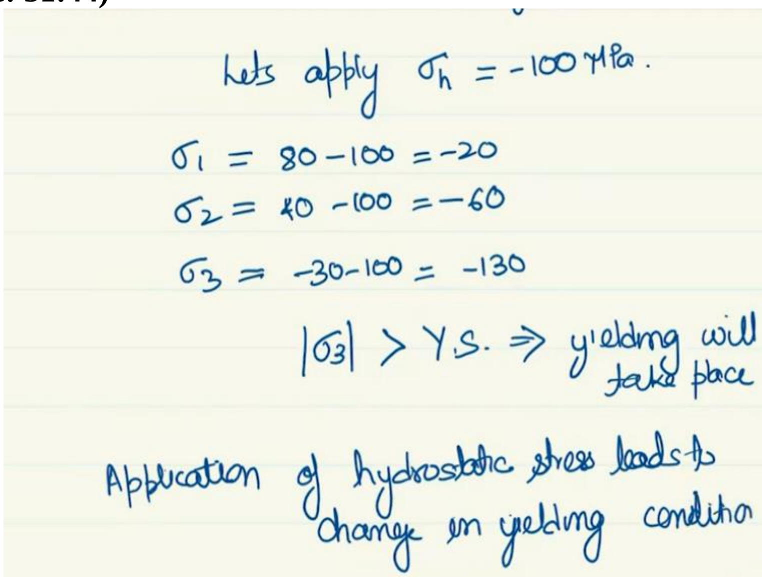
(Refer Slide Time: 49:51)



So now let us take a look at whether it is a valid is it a valid yield criterion we do not know what or how to check for failure criterion we only know about or we have talked about yield criterion. So let us say you are given the stress state like this so the material is given to you like this and it is given that yield strength of this material is equal to 120 MPa. Now you are given the stress state like this 80, – 80, 40 ,– 40 so it happens to be all principle stresses.

So none of the principle stressed are equal or exceed the yield strength therefore in this particular case it is not yielding. Now let us say that we apply hydrostatic stress nothing should change according to our requirement for valid criterion nothing should change. But let us say we are uploading but in this case you will see that something would change.

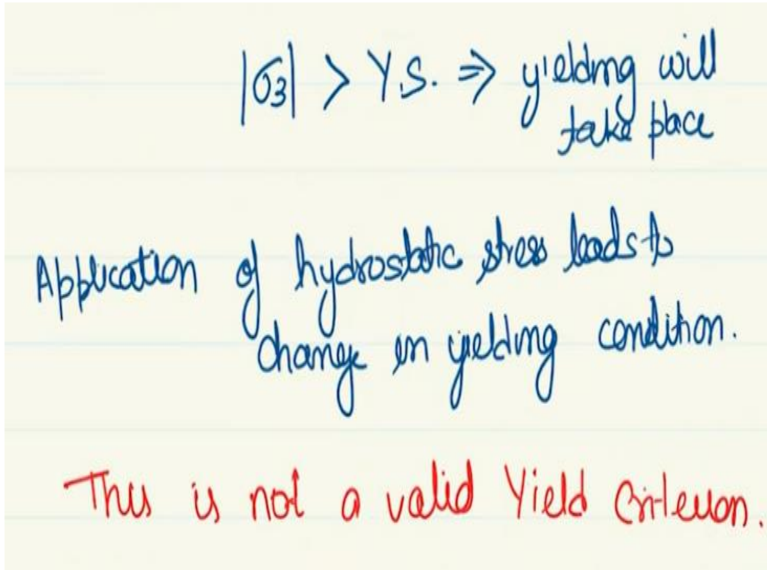
(Refer Slide Time: 51:44)



So let us apply sigma hydrostatic equal to -100 MPa. So now what will happen is σ_1 become 80 – 100 = -20 σ_2 = 40 – 100 = -60 and σ_3 = -30 – 100 = -130. Now this σ_3 in magnitude is greater than yield strength of the material implies yielding we take place. So here what is happened that application of hydrostatic stress leads to change in yielding condition?

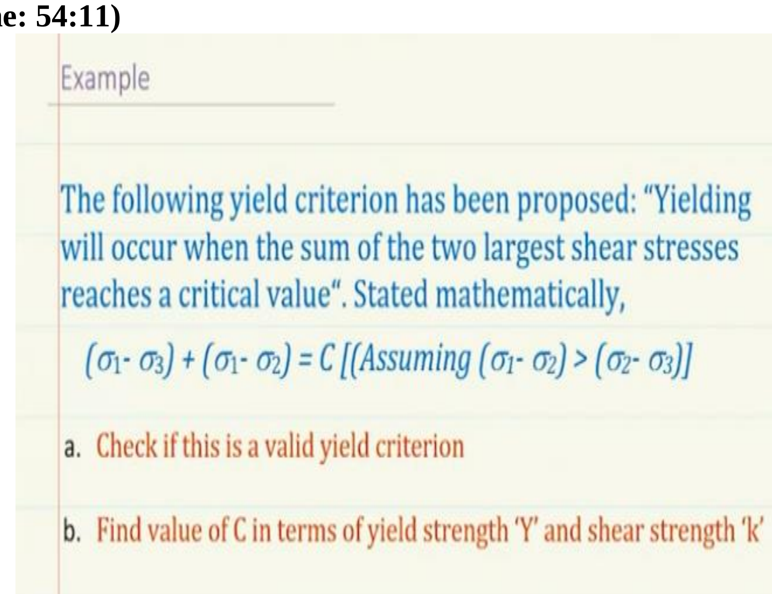
Now this is something that we had clearly mentioned that should not happen for a valid yield criterion.

(Refer Slide Time: 53:18)



Which means that this not a valid yield criterion. And like I mentioned earlier this is actually not even used as a yield criterion it is used as the failure criterion and over there it has been shown or known to be satisfactory for brittle materials. Therefore we should not use it for the understanding or defining the yield condition of the material. Because we can clearly see that applicable of hydrostatic stresses are making a difference over here. So now let us move on to one last example for the day for this particular content.

(Refer Slide Time: 54:11)

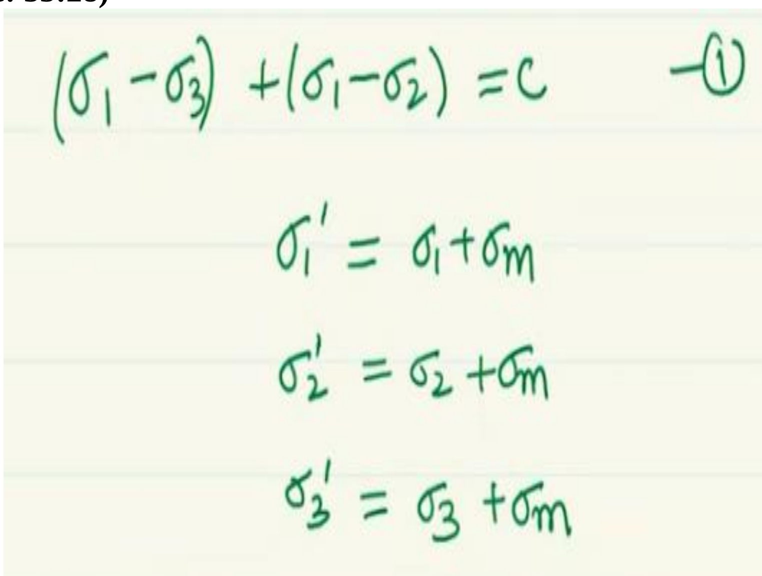


So you are given a new yield criterion so remember that the criterions are not universal which means that it does not mean that all the material will have to follow the same criterion. There are certain basic minimum requirement for the criterion and based on that you have several yield criterion. So one of the yield criterion is given over here the following yield criterion has been proposed yielding will occur when the sum of the 2 largest shear stresses reaches a critical value.

And if you want to state it mathematically you can write it like this $(\sigma_1 - \sigma_3) + (\sigma_1 - \sigma_2) = C$ where C is some critical value that is must reach and it is a constant. And what we are assuming are these are the 2 highest shear stresses meaning that third one which is (σ_2- σ_3) is lower than the (2σ_1- σ_3) to be the highest these 2 are the other tool were one.

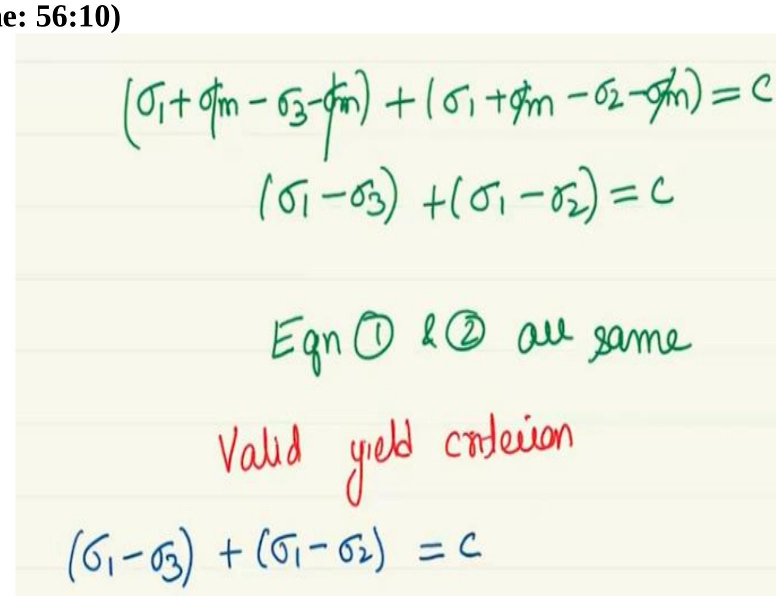
But in these 2 it is known that this one is higher than this and therefore this is your equation. So again if you first thing that we want to do is check if this is valid yield criterion and how do we do that?

(Refer Slide Time: 55:28)



(σ_1- σ_3)+ (σ_1- σ_2)= C. Now this is equation 1 now let us say we have applied the hydrostatic stress therefore we get $\sigma_{1'}$ this is the new principle stresses $\sigma_{1'} = \sigma_1 + \sigma_m$, $\sigma_{2'} = \sigma_2 + \sigma_m$, $\sigma_{3'} = \sigma_3 + \sigma_m$. Now what we will do is we will put this back into this equation.

(Refer Slide Time: 56:10)



On putting all the values in yield criterion equation, it comes out to be

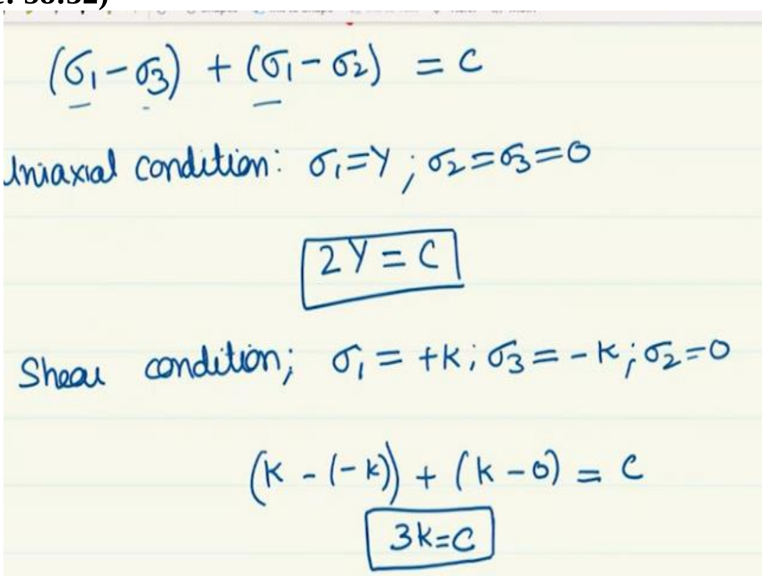
$$(\sigma_1 - \sigma_3) + (\sigma_1 - \sigma_2) = C$$

what we see is that this is again come down to same equation that we started with which means that equation 1 and 2 are same.

Adding the hydrostatic stress did not make any change to this equation and therefore if the original equation derived defines that the material will yield then this equation will also define that the materials yield. If it says no then this will also say no there will no difference and therefore there is no effect of hydrostatic stress and it means that this is a valid yield criterion.

Now the second part of the equation is that you have to find the value of C in terms of yield strength y and shear strength k. Now this is something that will be useful for you to practice with various types of yield criterion. So what we do again here is we are given the equation $(\sigma_1 - \sigma_3) + (\sigma_1 - \sigma_2) = C$ How do we find? How do we what did we do earlier to find the value of c.

(Refer Slide Time: 58:32)



We first use uniaxial condition what do we know about uniaxial condition σ_1 = C , $\sigma_2 = \sigma_3$ =0 .So what we get here is that 2 y because there is σ_1 here this is σ_1 here these 3 and 2 or 0. So 2 y = C so then we get the value of C in terms of y similarly we apply pure shear condition. In pure shear condition σ_1= +k and σ_3= -k and σ_2 = 0.

So we again put this here so we k – k + this part k – 0 = C therefore this is k + k + k. So this is equal to C is 3 k = C. So we have applied this equation on uniaxial condition and shear condition to get the values of C. And we know that this is a valid criterion and these are the C so we have solve this problem you can go through the book on (Hosford)and you would find that there are several examples.

In their where you get new yield criterion you can do the exercise of whether it is a valid criterion that is simple just add mean stress to all $\sigma_1, \sigma_2, \sigma_3$ then put it in the equation and see whether the equation changes. If it changes then probably you will get different values under different hydrostatic condition. If not then it means that it does not change and therefore it will it is a valid yield criterion.

Second to find the value of C in terms of y and k just use the uniaxial condition is σ_1 = y, $\sigma_2 = \sigma_3$

=0. And then put it the equation you will get the value of C in terms of y. For value of C in terms of shear strength for the shear condition we know that σ_1=k and σ_3= -k and σ_2 =0. So put this over there and you would get the value of C in terms of k.

So today in this lecture we looked at different yield criterions we derived the Von-mises then we looked at the Tresca condition. We also looked at a rankine condition which is not a yield criterion but a failure criterion and if you use it as yield criterion. We saw that it will fail it will not be valid criterion we also looked at how to find whether yield criterion is valid or not and we also know now how to find the value of the constant C in terms of yield strength and shear strength k. So with that we come to end of this topic thank you.