Mechanical Behaviour of Materials - I
Prof. Shashank Shekar
Department of Material Science and Engineering
Indian Institute of Technology - Kanpur

Module No # 03
Lecture No # 11
Yield Criterion Basics

Welcome back students So, today in this lecture we will start talking about yield criterion. So, if you remember uniaxial tensile test, you would know that we have something called as yield strength or yield stress. So, at that particular stress value the material transits from elastic region to plastic region. What would happen if the stress state was not uniaxial in nature? It would be actually a lot more difficult it is not straight forward to understand when the plastic deformation or the plasticity in the material would begin.

We will show that with an example and hence we need some criterion which would include the overall stress state to define whether the material is still in the elastic region or if it has transited into the plastic deformation zone.

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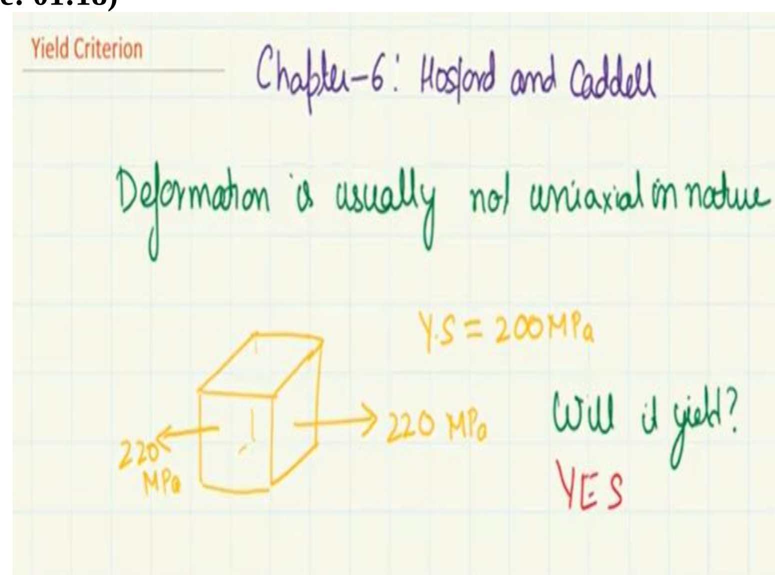

So, let us begin so this yield criterion you would see is covered in chapter 6 by Horsford and Caddell. So, the first important point to understand is that deformation is usually not uniaxial in nature. So, let us say we have a body like this where whatever stress state we define remains uniform throughout**.** So, let us say it is initially under uniaxial condition and it is given that the yield strength of this material is 200 Megapascal.

Now if you are applying a uniaxial stress like this and let us say it is 220 Megapascal. The question is will the material yield and emphatic answer is yes, it will yield in uniaxial we know the overall stress of 200 Megapascal has been exceeded.

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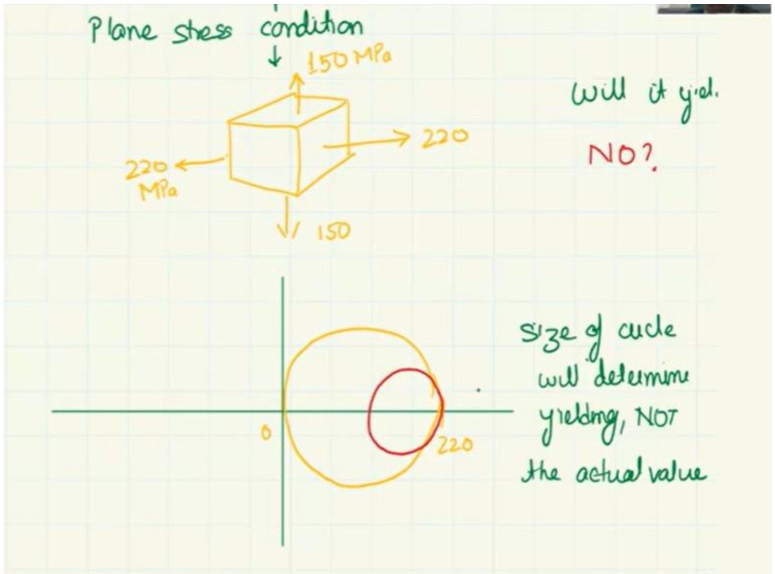

Now let us look at a little different example again we have the same body but now it is not under uniaxial stress state. So, but this direction we would still apply 220 Megapascal. But so the drawing may be a little misleading so when I say it is basically on the opposite faces so that much should be clear to you. Now we are also, applying yield stress of, 150 Megapascal along this and yes, I did not mention it explicitly but you can assume plane stress condition for both these.

Now again the question it is will it yield so it looks like it should because we have applied the same stress in fact, we are applying more than that stress. So, the first instinct would be to say yes but the answer is no. Actually, and when you draw the Mohr circle this will become even more clear but for now, I will just draw it like this. So, for the first state where we have uniaxial under this condition which means that on the other side again assuming plane stress condition here it is 0.

So, the overall stress state can be defined like this by using this Mohr circle. So, here it was 220 here it was 0 and therefore we know it will yield. Now in the second condition what is happening is that the stress state on this direction is still 220 but on the other direction it is now 150. Now you see that the size of the circle has grown smaller although its position is still the right most position is still at 220 but the overall size of the circle has become smaller.

And only later we would see that it is the size of the circle that determines whether yielding will take place or not the actual value. So, you see it is it must have come as a surprise to most of you because what this means theoretically that even if you make it 500 or 1000 but as long as the size of the circle meaning the other side you are still applying 900. So, 900 and 1000 these are the 2 stresses you are applying then the material will not yield.

That is what this condition or this will something condition that we will define later on but in general this is what the yielding means. And that is how multi-axial stress condition can change the overall scenario. And that is why we need to have a good understanding of overall stress state and not just one or unidirectional stress state. Having said that now let us look at what are the places where we see multi-axial stress in nature so, not in nature in a process.

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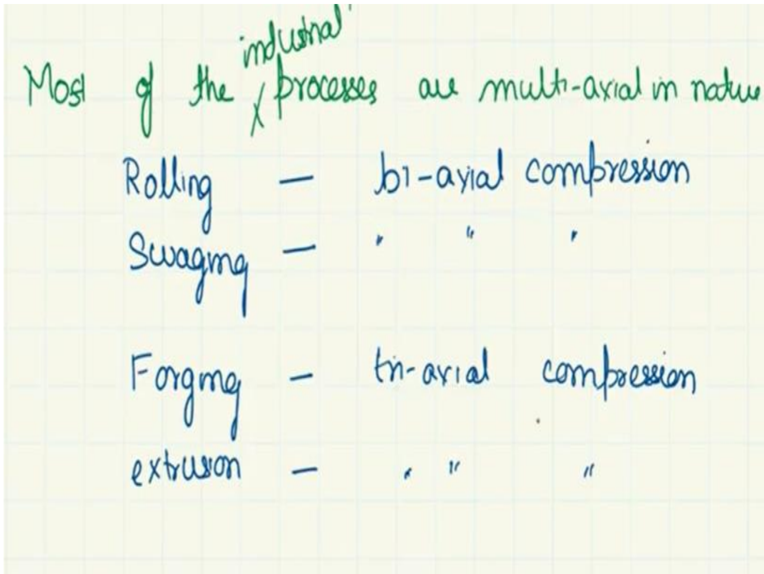

So, let us look at some of these processes most of the actually I should say industrial process are multi-axial in nature. For example, if we talk about Rolling then what is the strait of stress it is bi-axial compression. If we talk about swaging this is also, bi-axial compression. If we talk about forging then it is tri-axial compression. If we talk about extrusion then again it is tri-axial compression.

And these are some of the most common ones but if you keep on listing even other industrial forming processes you would see that all of them are multi-axial in nature. So, what is it that we understand from this that we need to have a formulation which can take into account this multi-axial state of stress?

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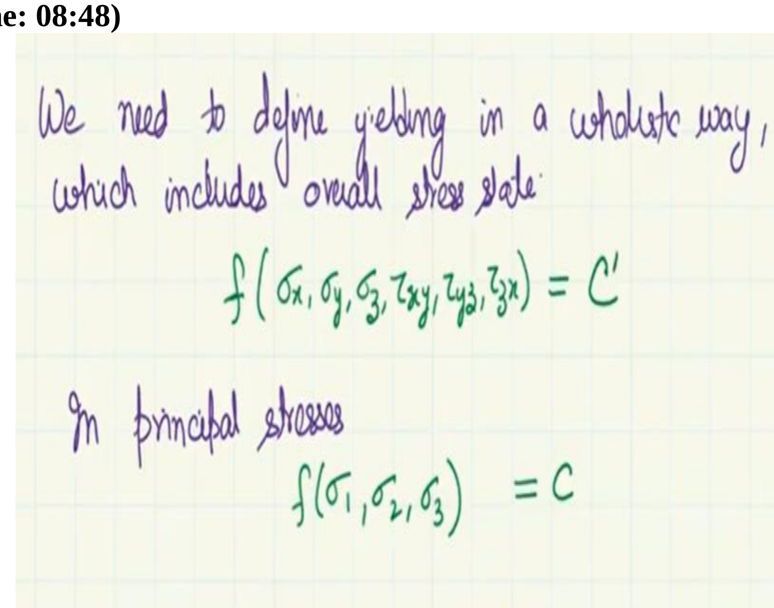

We need to define yielding in a holistic way that is the first thing what do we mean by holistic way? Which includes overall stress state**,** so if we were to put it mathematically then we can write it like this, that

$$f(\sigma_x\sigma_y\sigma_z\tau_{xy}\tau_{yz}\tau_{zx}) = C'$$

Some function of this when this reaches some constant value that is when the material starts to yield.

So, that is our objective or that is what we understand gate we get understanding from the above example that this is what will be needed and it looks like it would be a very gigantic or humongous task. But it is not really so as you would see it turns out to be a lot simpler than what this equation states. First of all, if you look at the same thing if it were to be written in terms of principle stresses then in terms of principle stresses, we would write as.

So when we have principal stresses obviously, we do not have shear stresses. And therefore, it is just a function of sigma one sigma 2 sigma 3 and since these 2 constants would not be same. So, let me put a prime over here to just to distinguish that this is a different constant and this is a different constant unless someone gets confused that both the equations will yield the same question that is not the case.

Now what are some certain assumptions that we need to make particularly from the point of view that we have isotropic material and what are our observations which must be fulfilled if we define any yielding criterion. So, what are those assumptions so let us list out those.

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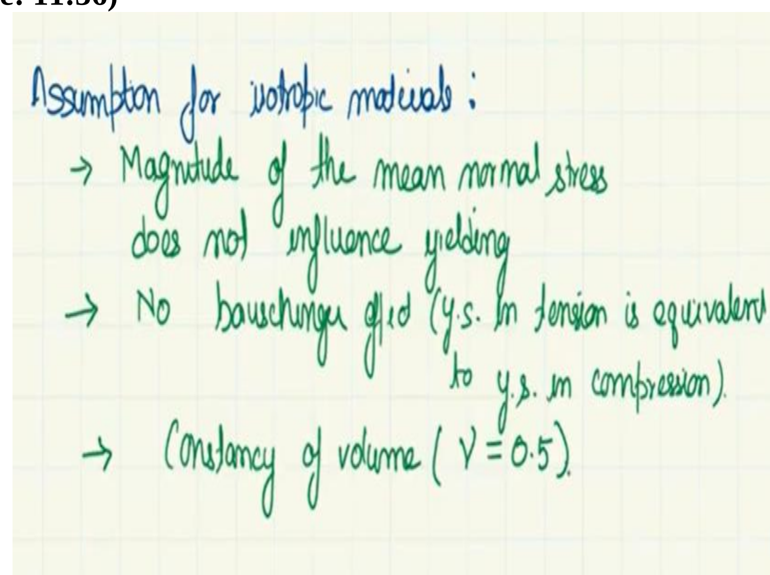

So,we are assuming isotropic material meaning a polycrystalline material which is which has uniform properties in all direction. First of all magnitude of mean normal stress does not influence yielding. This is an observation which is very valid and well known that if you increase the magnitude of mean stress then the yielding the yield strength of the material does not change.

For example, if you are doing the yield test in air versus you take the whole thing the whole setup inside water where there is hydrostatic stress and you keep going deeper and deeper. So, hydrostatic stressing keeps increasing. But the yield strength of the material does not change. This is, understanding and that therefore you are criterion must be able to fulfil this. And if you take the stress tensor then what it means is that?

You can take away the sigma m which is $\frac{(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})}{3}$ from this factor this quantity away from this might deduct this from the stress tensor. And the quantity that is left behind would still define the same yield criterion or the yield strength of the material. No passenger effect now in usual material you would see that there is very little or negligible difference whether you are applying tension or compression.

If you go to the bonds property then we know that it is a little bit asymmetric. But the amount of strength that we are applying in the range that we are working on in that range it is almost u shape which is symmetric. And hence the Bauschinger what it says is that there should be no Bauschinger effect meaning the stress in the tension should produce the same amount of strain as the stress in the compression.

Yield strength intention is to yield strength in another important criterion is that, the volume of the material should not change. So, there should be a constancy of volume because it is plastic deformation so the overall volume must not change. And it is expressed as constancy of volume which in other words means that the Poisson ratio ν should be equal to 0.5. So, these are some important assumptions that must be satisfied if we are to set up an equation or set up our model mathematical formulation to define the yielding of the material.

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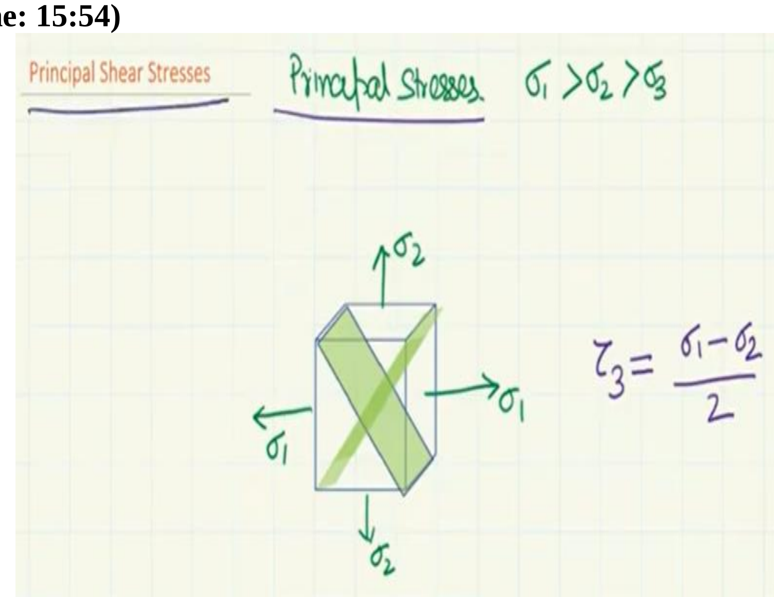

Now in this respect let us visit one topic that would be what will be very helpful for us to understand the overall yielding condition and that is principle shears tress So, you remember the principal stresses we had 3 principal stresses $\sigma_1, \sigma_2, \sigma_3$. And usually, we know that $\sigma_1 > \sigma_2 > \sigma_3$. Now let us say this is a body where sigma 1 and sigma 2, are acting.

So, this is σ_1 like this and this is σ_2. Then you can easily show that the plane at 45 degrees would be the planes where maximum shear stress in this particular plane. So, if you are assuming a plane stress condition in this particular plane that would be the maximum. So, plane that particular plane would have maximum shear stress and that would be given by. I am giving the name τ_3 right now because as you would see it would be the lower one of the lower ones.

So, we will say $\sigma_1$and you can easily show that it will be $(\sigma_1 - \sigma_2)/2$.
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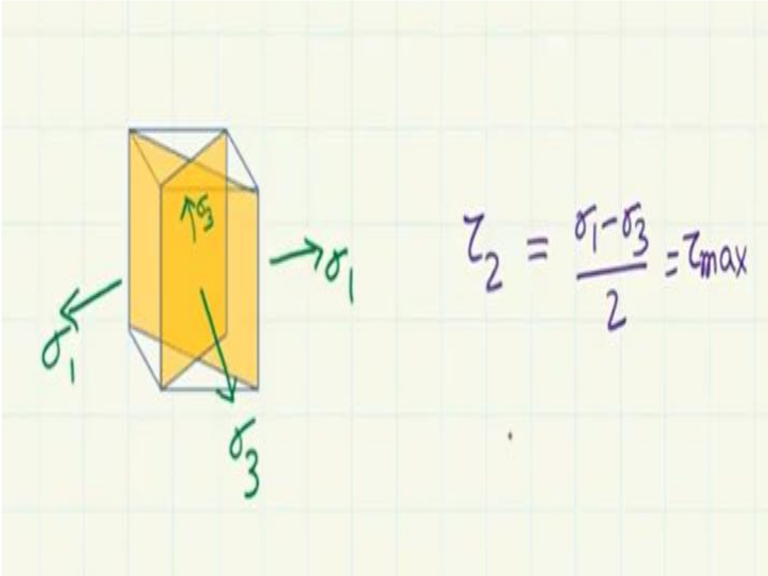

The other condition that we can have where this is sigma 1and now let us in this particular plane. So, this is so there is a sigma 3 in this direction. So, sigma 1 is in this direction and σ_3 is in this direction. So, again for this particular plane the maximum shear stress would be along these orange planes which are at 45 degrees and since σ_3 and sigma 1are the maxima minima and maximum.

So, σ_1 is the maximum σ_3 is the minimum so, whatever τ we get here. So just put τ and there is a $(\sigma_1 - \sigma_3)/2$. But since these 2 are the extrema therefore these will also, be the largest value and therefore this will be τ_1 or equal to τ max. Now so, basically this is not in the terms of shear stresses it is not termed according to the highest and lowest this is 1 and 3 so, this is τ_2.
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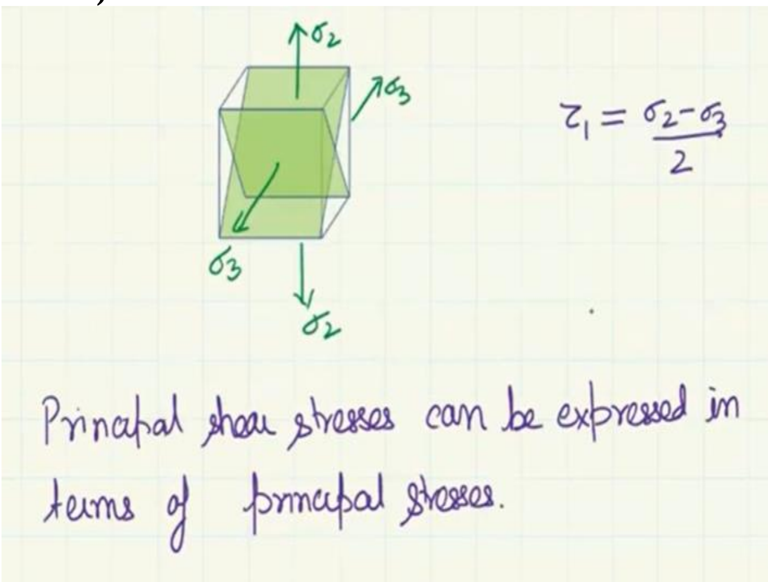

And now over here we have sigma 3 sigma 2. So, here we have τ_1 =$(\sigma_2 - \sigma_3)/2$. And these are the maximum shear stresses in that particular plane so, this is the maximum shear stress in the plane of σ_{12} which is given by τ_3 is σ_{12} .If you take at any other plane then it would be it would be lower than this value. Similarly for the plane in the σ_1 , σ_3 plane this is the maximum shear stress in fact this happens to be the highest shear stress that you can get for this stress state.

And for this plane σ_1 and σ_3 plane τ_1 this is the highest shear stress. And therefore, these are called the principal shear stresses. And what we see is that principle shear stresses can be expressed in terms of principal stresses, which wasσ_1 *and* σ_2 *and* σ_3 . So, now that we are comfortable with this concept of principle shear stresses and we know the relation and how to get the principal shear stresses which is in terms of the principal stresses $\sigma_1, \sigma_2, \sigma_3$.

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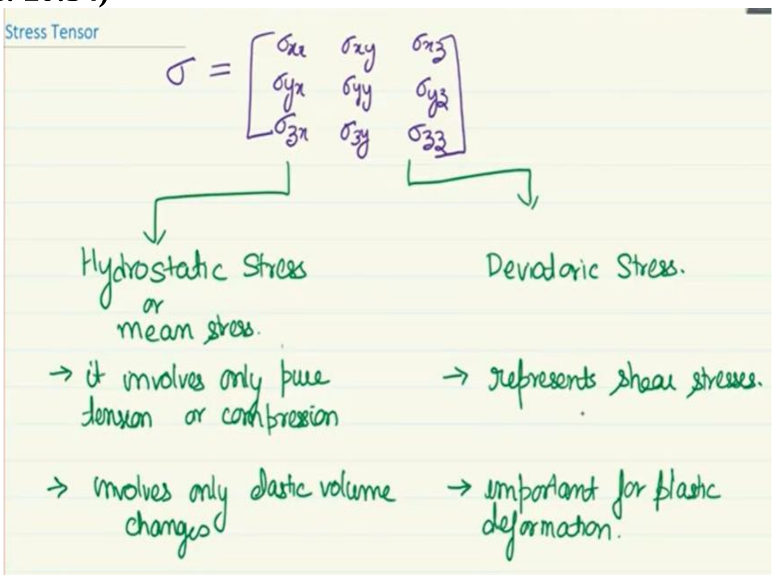

Now let us move on to another important or basically look let us now look at the Stress Tensor quantity and then you would be able to get start getting an idea on how to define that criterion. Now we know that stress is a tensor so, it has 9 quantities and it is $\sigma_{xx}, \sigma_{xy}, \sigma_{xz}$. Now, this stress tensor or the overall components of the stress can actually be divided into 2 major components.

What are those 2 major components? Those 2 major components are hydrostatic stress and deviatoric or deviator stress. Hydrostatic stress is basically nothing but the mean stress so, this is also called as mean stress it involves only pure tension or compression. On the other hand, Deviatoric stress it represents shear stress we will be able to dissociate the 2 components very soon.

First just let us go through the terminologies and what they mean for them mean so, this represents shear stresses. Now if you remember in the bond model when you are doing just tension or compression you are basically only what it heels only the elastic deformation of the material. So, hydrostatic stress involves only elastic changes.

On the other hand, shear which is basically shearing of the plane of atoms and this stress is what leads to breaking of the bond and hence generation or the dislocations as something a concept that we will visit very soon. So, this one this is the one which actually leads to or causes plastic deformation so, this is important for plastic deformation.

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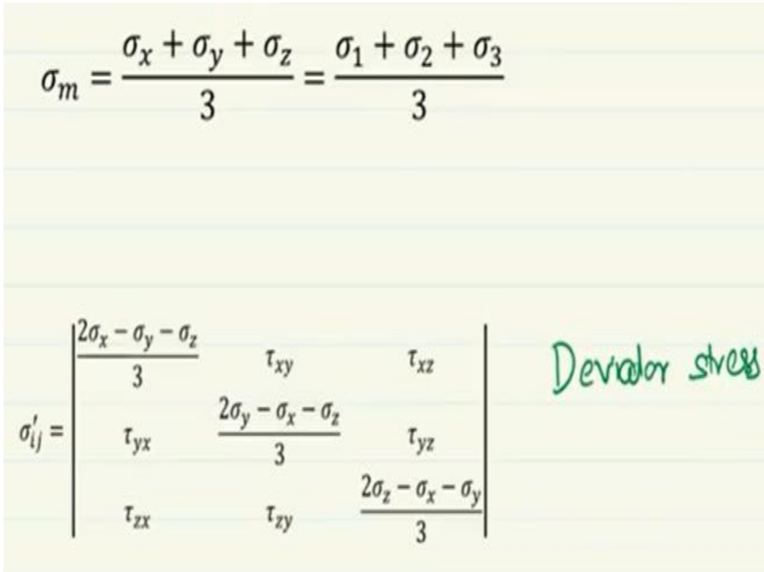

Now coming to the definition so, sigma m or the mean stress which is also, the hydrostatic stress like we said is nothing but $(\sigma_x + \sigma_y + \sigma_z)/3$. And if it were to be expressed in principle stresses then it will be $(\sigma_1 + \sigma_2 + \sigma_3)/3$. So, this is the hydrostatic stress component. Now if you take away this component which is from the diagonal component $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$.

If we take away this component from general stress matrix so, this is what this is the stress general stress matrix and these are the $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$. So if you subtract then this is where these quantities will get subtracted and what you are left with is this one. So, this is the Deviator stress.

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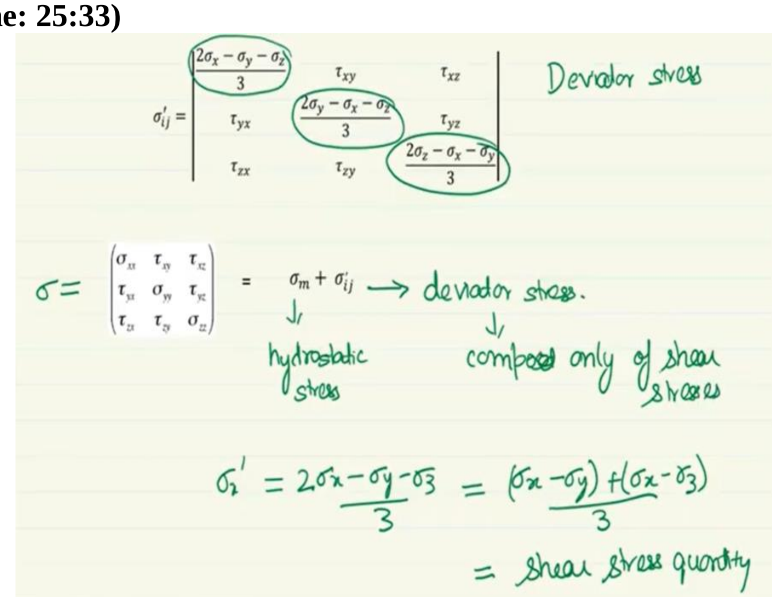

So clearly this one was obtained by subtracting this and therefore the overall stress is nothing but sum of $\sigma_m + \sigma_{ij'}$, $\sigma_{ij'}$ is what our deviator stress. So the Hydrostatic stress plus the Deviator stress. So, this stress quantity is clearly composed of hydrostatic stress and deviator stress. But that is not all what we will what we will see again now is that this deviatoric stress is actually composed only of shear stresses how let us see.

Now we have this quantity. So, these are your these are purely shear quantities τ now the only ones that we are notable to recognize as shear are these quantities sigma x, 2 sigma x – sigma y let us call its

$$\sigma_{x'} = \frac{2\sigma_x - \sigma_y - \sigma_z}{3}$$

but this can also, be written as

$$\sigma_{x'} = \frac{(\sigma_x - \sigma_y) + (\sigma_x - \sigma_z)}{3}$$

So, this is clearly a shear stress quantity.

And thus if you look at all these terms this one this one and this one all of these are basically nothing but shear stress quantity. And therefore, deviator stress is composed only off.

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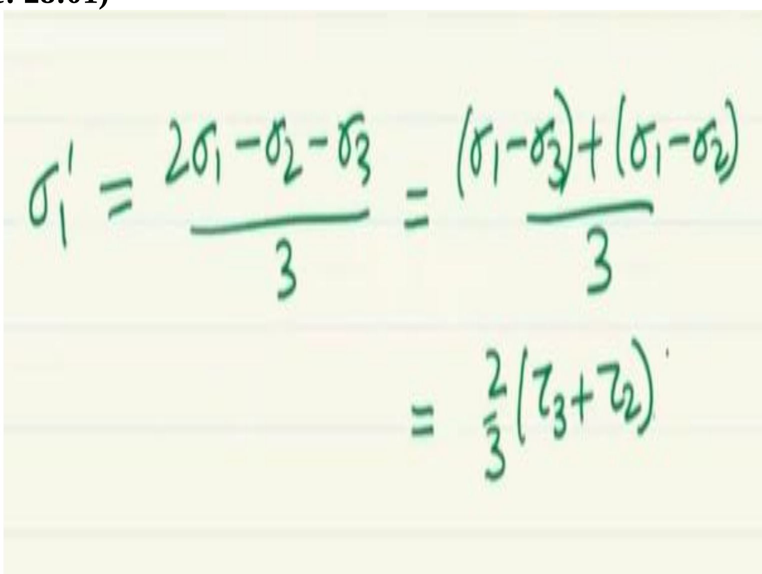

So, if we were to write it in terms of principle stresses then this would become like this which is equivalent to by 3 this is τ_2 this is τ_3 this is $\tau_2 + \tau_3$. Therefore, if you were to write in terms of principle stresses again you would see that it will translate to principle shear stresses. That deviate stress can will the deviator stress will transform into a stress matrix with only the principal shear stresses.

And therefore, it is now clearly established that the deviator stress is composed only of the shear stresses. Now with this understanding we are now clear that the plastic deformation that takes place in the material takes place only when there is a shear stress and the shear stress is basically some function has to be some function of shear stress. So, the shear stress must the combination of the shear stresses must reach some value when the deformation of plastic deformation can be said to have begun.

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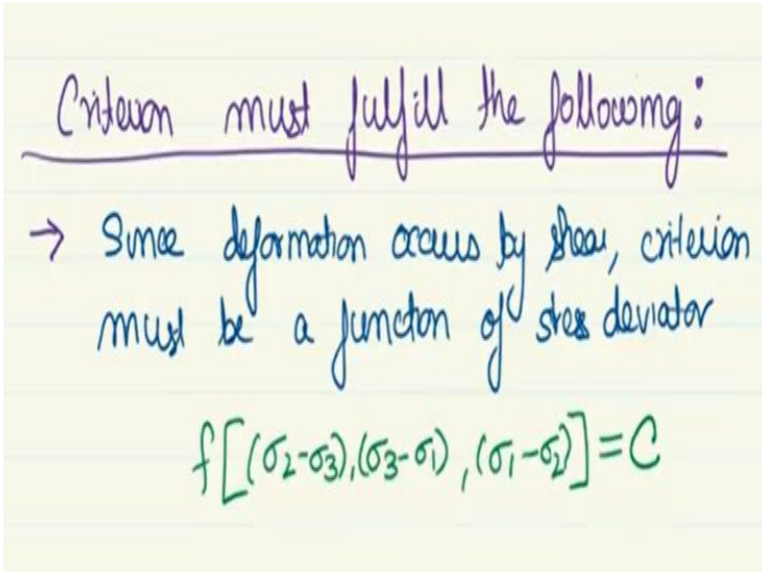

And therefore let us based on our understanding we can say that the Criterions must fulfil the following requirements. What are these requirements let us see, since deformation occurs by shear stress? So, earlier we said that it should be some function of$\sigma_1, \sigma_2, \sigma_3$ or $\sigma_x, \sigma_y, \sigma_z$ but now we can refine it further. Now we can say since deformation occurs by shear criterion must be a function of stress deviator.

So, stress deviator represents the overall stress state in terms of shear stresses alone and we know that this is what will cause the plastic deformation. Not the mean stresses in fact we have given that example that if you change the mean stress either plus or minus it does not change the yielding condition. So, this is the only stress state the only component of the stress state which will define the yielding is the deviator.

Therefore, we can now say that f as a function of$((\sigma_2 - \sigma_3), (\sigma_3 - \sigma_1), (\sigma_1 - \sigma_2)$ this is in terms of principle stresses. Similarly, you can also, get it in terms of other or normal stresses this should reach some value C. So, this is our more fine-tuned equation this is the formalism of the mathematical equation that we should obtain.

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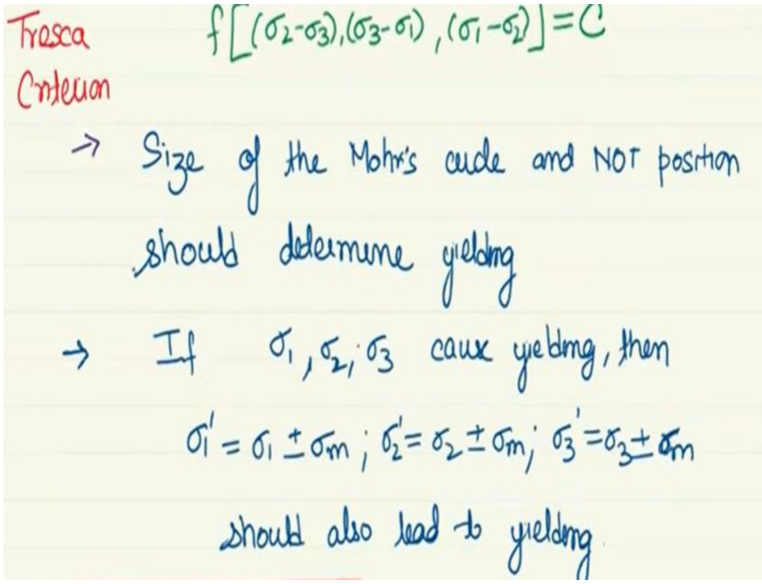

And now based on this we are now confirmed what I said earlier with that example that it is size of the Mohr circle does not matter and not position should determine yielding. Now that we are saying that the mean stresses should not cause change in the yielding then mathematically how do we say that so if we are if the condition that $\sigma_1, \sigma_2, \sigma_3$ it is a combinational stress state that causes yielding.

Then σ_1' equal to σ_1 plus or maybe minus whatever you do a mean stress value or any other stress value. In fact σ_2' equal to σ_2 + - sigma m this should also, lead to yielding. So, this is another condition that it must satisfy if the above what we have said what we have derived earlier is true then this is this must also, be true. Now if we want to get a bit little bit more understanding about how the criterion should look like then we can also, look at our isotropic material

Now if you are looking at isotropic material and change the orientation and then you get some stress state and then you derive the yielding condition then whether it is yielding or not that; condition should not change just because you have changed the axes.

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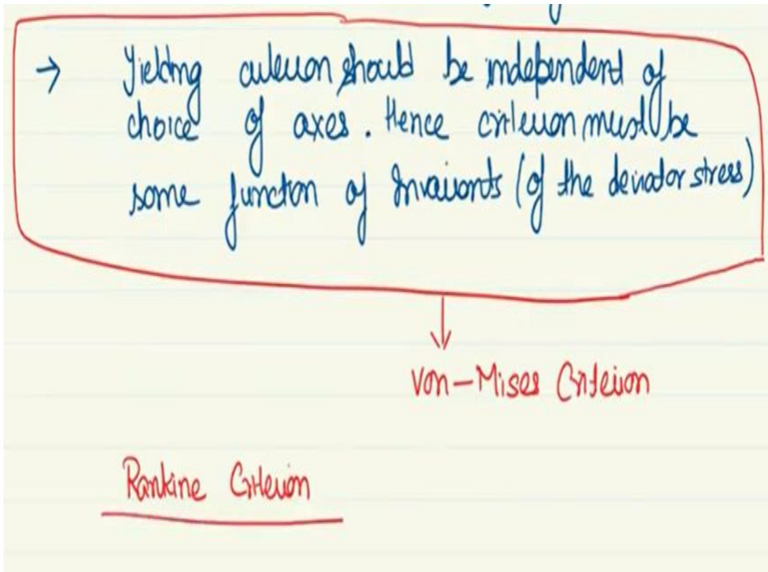

Therefore, the yielding criterion should be independent of choice of axes and hence this something which is independent of the choice of axes is invariant. And therefore, we can also say that this criterion must be some function of invariance and this time not the invariant of the original stress tensor. But now that also, we have described that only the deviator contains the shear stresses so, invariance of the deviator stress component.

So, this part has helped us jot down or finalize our criterion to a great deal as you would see that this particular statement can help us get down to a very meaningful yielding criterion. That is just one of the link criterion sand the yielding criterions are not universal it can change from material to material only that it must satisfy those conditions that we have described earlier.

And the one that is based on this one just based on the overall which is able to satisfy all of these are what is called as Von- Mises criterion. And this one is what helps us boil this well down to the final equation for the von - Mises criterion even otherwise. There is still another one which satisfies up to this one all, these one it need not be directly a relation of the invariance but it does satisfy all the other conditions and that is a Tresca criterion.

Actually, it will also, satisfy the last one only that it is a much simpler form of the equation and this one will be directly utilized from the invariance of the deviatoric stress. And this one is a lot simpler one although it satisfies all the condition in fact any valid yielding criterion must satisfy all these conditions. So, we will look at the derivation of not the derivation but now that we have this equation, we will be able to jot down the Von-mises criterion.

And also, based on our primary understanding we will be able to jot down the criterion and then we will see whether they satisfy all the condition and how they relate with the yield strength and the shear strength of the material. And along with that we will also, look at another yielding criterion which is the ranking criterion although it is it does not satisfy all this and we will see which of the one sit does not satisfy.

So, with the introduction of these 3 yielding criterions and the conditions we will end this lecture over here. And we will in the next few lectures we will go through these different criterions thank you.