Mechanical Behaviour of Materials - I
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Module No # 02
Lecture No # 10
Flow Stress

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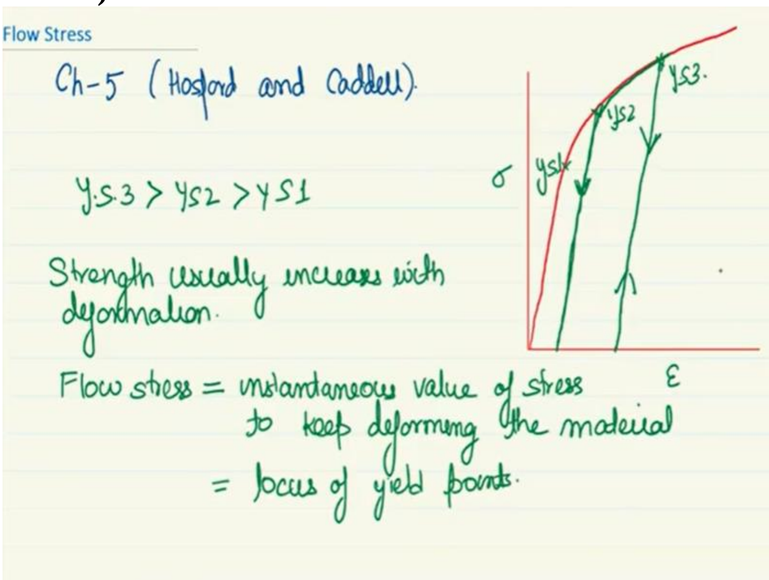

Welcome back students so we will continue our transition to theplasticityofmaterial this part of the content. So far, we have seen the Elastic behaviour we have seen the Tensile properties, Tensile testing, the UTM now we will move on to the plastic behaviour of the material. So, with respect to that first let us get acquainted with the concept of flow stress this content has been covered in chapter 5 by Hosford and Caddell.

Now if you look at typical stress strain curve this is how it will look like, let us draw it**.** So, I have drawn somewhere until before the UTS and to be precise when we are talking about flow stress usually it would be effective stress that we are talking about. We will learn more about it as we go around. So here on the x-axis you have stress on the y-axis you have stress and, on the x-axis, you have strain.

Somewhere here you have the yield strength of the material though let us call it yield strength one because this is the original yield strength of the material. Now let us say you deform the material all the way up to this point and then you take away the load. So the material will come back like this somewhere parallel to this you know the curve that will follow would be something like this.

Now if you start to deform the material again apply the stress again then you would see that it will follow the same curve back and somewhere over here you would have yield strength 2. Now let us say here this is again a place where you drop the load so it will come back like this parallel to the elastic modulus. And then again if you want to load it up so it will follow the same curve and this times its yield strength would be higher so yield strength 3.

So, what you would observe is that yield strength 3 is greater than yield strength 2 is greater than yield strength 1, meaning strength usually increases with deformation. And as you keep increasing the strain the yield strength is increasing. So if you could find a locus of the yield strength with the increasing strain then that would be called flow stress. Or in other word flow stress is also the instantaneous value of stress required to; keep deforming the material or like I said it is also the locus of yield points.

This locus of yield points or the instantaneous value of stresses can be or is usually modelled by a very simple relation which is called power law.

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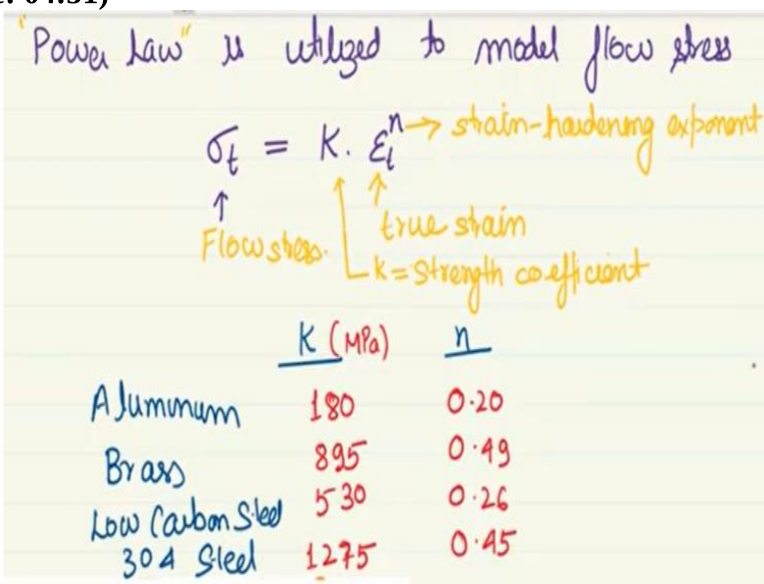

So, model meaning to give a relation or the value of low stress as a function of string. So, this is sigma t which means sigma true stress = k times epsilon t which is again true strain to the power n. Where sigma t is the flow stress epsilon t is the true strain and k value is called strength coefficient and this n is called strain hardening exponent. Keep this term in mind because there is a similar term which people do students do get confused with and that is strain hardening rate.

So, that is a different thing this strain hardening exponent for which is the n when we are using this parabola model. To give you an idea the value of k and n let us put the values somewhere here k n for aluminium, brass, low carbon steel and 304 steel. So, the values are 180 this is in Megapascal for brass it is 8.95 for low carbon steel it is 530 and for 304 steel it is 1275.

n is 0.20 this is 0.49, 0.26, 0.45 so roughly what you would find is that k increases as the strength of the material increases yield strength or the UTS and n increases as the ductility of the material increases. You can also plot this equation in online mathematical.

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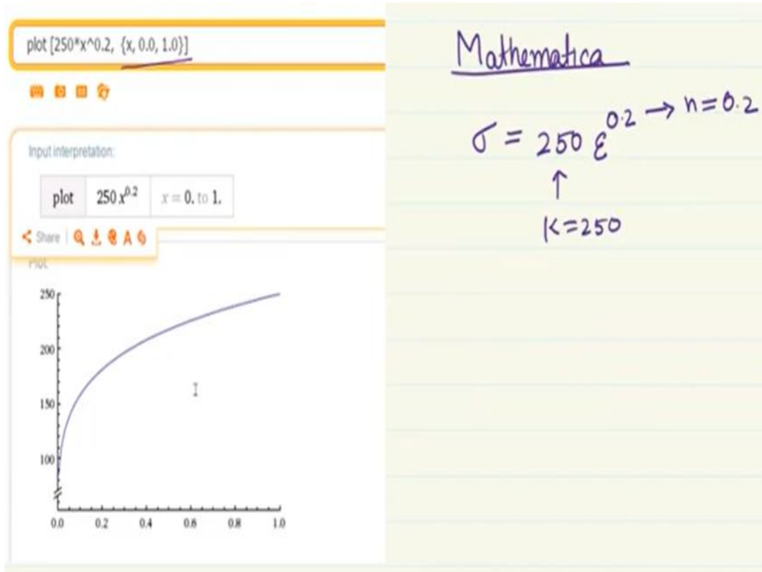

So, you can utilize the site mathematical which allows you top lot online and you can so here the equation which was plotted was sigma = 250 into epsilon to the power 0.2. So k was taken as 250 and n was taken as 0.2 and strain value was as you can see here varied from 0.0 to 1.0 and the plot you get is something like this. So, this as you can see truly represent the stress strain behaviour for the plastic part.

So the flow stress or you can say the yield point locus with increasing strain and on the x axis you have the strain. Now there are different variations of this but before that let me I was talking about the strain hardening exponent

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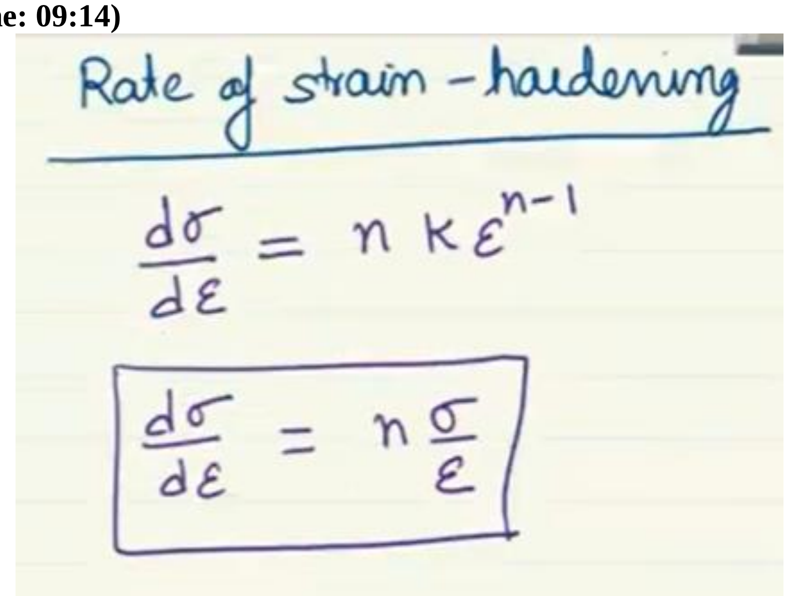

So, let me just make it clear rate of strain hardening. How is it defined? so this is defined as

$$\frac{d\sigma}{d\varepsilon} = nk\varepsilon^{n-1}$$

so, the change in flow stress with increase in strain how quickly it is increasing or how slowly it is increasing that is what is defined by strain hardening rate or rate of strain hardening. And if you solve it comes out to be

$$\frac{d\sigma}{d\varepsilon} = \frac{n\sigma}{\varepsilon}$$

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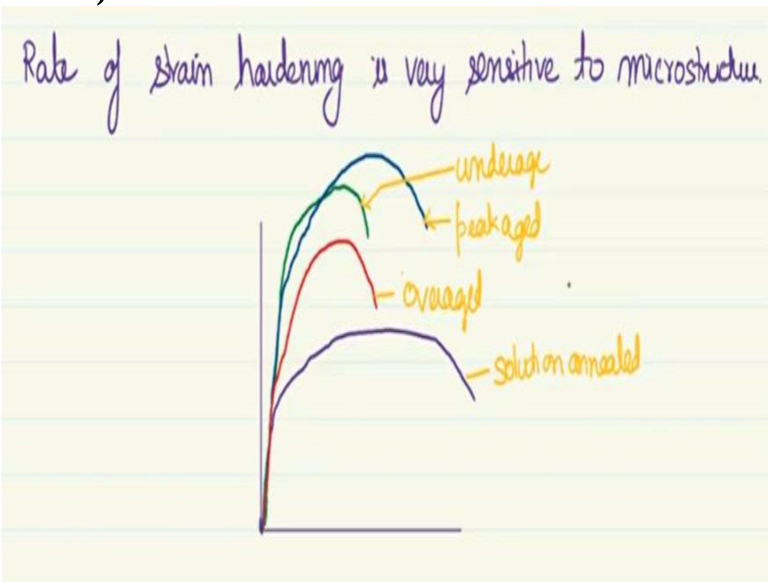

Now this strain hardening rate is very sensitive to the micro structure so even one material under different micro structural condition can give you different strain hardening rates. So, rate of is very sensitive to micro structure again let me and let me draw it here. So, if it were annealed let us say we will take the example of aluminium alloys so if it were aluminium alloy which is age hard enabled but if it were in a annealed condition.

Then you would see a very low strain hardening rate meaning the stress is increasing at a very low rate. On the other hand, if it were aged a little bit but under aged so in that case you do not see very large improvement in strain hardening rate but what do you but you do see is improvement in strength. So, you would get a stress strain curve something like this so here so let me keep writing. So, this is I will write it at the end so that it does not get too crowded.

So, now we will age it edge so it will have not only improvement in UTS but also improvement in strain hardening rate and you would seesomething like this. On the other hand, if you keep moving ahead and you over age it then it may have a very high strain hardening rate but unfortunately it will have it will not have very high strength. So now let me give it all a name so now let me write down the condition names.

So, this is like solution yield this is under aged this is peak-aged and this is over aged I am this one was the under age for the confusion again this is over aged and this is peak-aged. So, againto remove the confusion let me describe this is solution anneal where you have low strength very but very good ductility then you keep aging the material then you see not much improvement in strain hardening rate but you do see improvement in yield strength and UTS.

That is the underage condition in peak-aged condition you see improvement in strain hardening rate as well as improvement in UTS so this is the peak-aged condition. And over aged condition you see improvement in the strain hardening rate but the overall strength drops so this is the over aged condition. And mind you that this is the same material only different microstructure and how do we obtain different micro structure by heating it for different amount of time. So, thus what we clearly see that the rate of strain hardening is very sensitive to micro structure.

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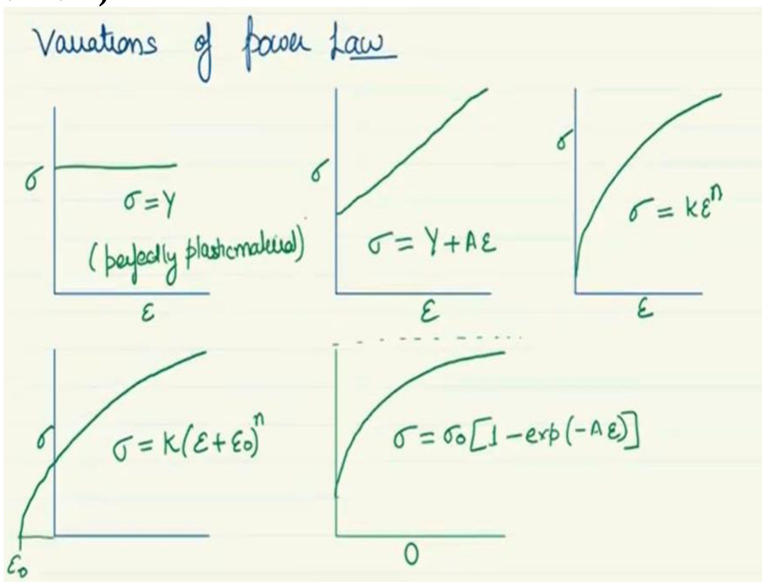

Now let us look at the variations of this Power Law model that we have seen so the one of the simplest or you can say alternative for so let me draw 2, 3 of these because we well need that. So a little bit of confusion here because I am still getting used to this drawing part now let me draw the simplest alternative to power law and it is something like this. So, on the xy axis you have stress and on the x axis of strain for all of these.

And here you are equation is given by $\sigma = y$. So, whatever is the original yield strength it remains the same and this is called also called a Perfectly Plastic material. The other alternative to the power law is something like this where it does not it varies linearly the yield strength or the flow stress varies linearly and therefore your equation is given by something like this a much simpler and cleaner equation.

The third alternative is what we already have which is the Power law equation which we saw when we drew this is how it came out. Now in the power law equation itself we can have a little bit of variation so for example let us say that there is some amount of Pre-Straining in the sample so this is the amount of Pre-Straining. Then actually the stress starts from here but you would be seeing because this much amount of strain is already given here.

So you would see the changes from over here and in that case your equation because you have received the material at this stage can be transformed into this form. So whatever strain you are giving you should include epsilon not in that and then it can be written like this. The other equation is alternative is also something you may have seen behaviour like this somewhere and this is exponential type behaviour so it reaches a peak value and this is given by

$$\sigma = \sigma_{\circ}[1 - \exp(-A\varepsilon)]$$

So, these are some of the alternatives or variations of the parabola behaviour to represent this stress strain behaviour or the flow stress behaviour.

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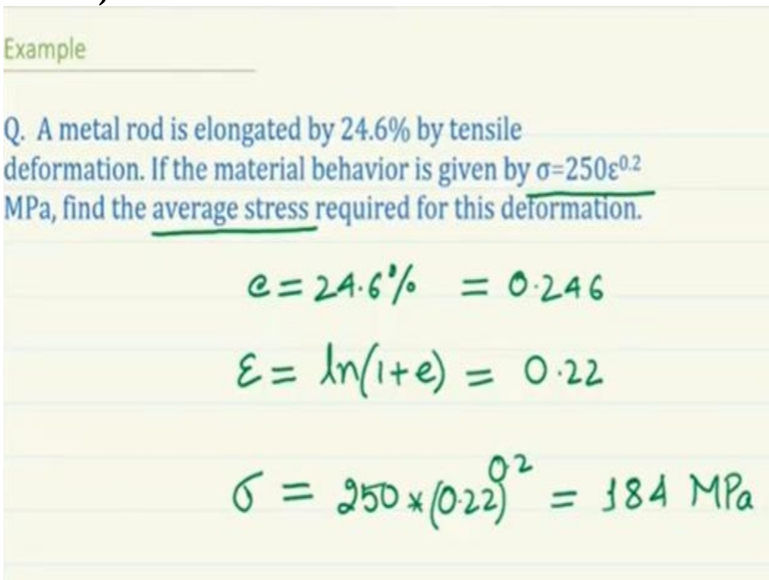

So in this respect let us try to solve one example so the question is a metal rod is elongated by 24.6% by tensile deformation if the material behaviour is given by $\sigma = 250\varepsilon^{0.2}$. Megapascal find the average stress required for this deformation. So we what we are given let us take a look at that so you are given engineering string so you are given e which is equal to 24.6% and if you translate it, becomes 0.246.

So, you will have to translate this to true strain which is equal to ln(1 + e) and when you put in the values you would find that this translates to value of 0.22. Now when we are also given this equation $\sigma = 250 \times 0.22^{0.2}$=184 MPa

So is this what they are asking for and the answer is no. What they are asking is average stress this is the flow stress at the end of this deformation so this is not the stress that you were applying throughout. What it is asking is on an average how much stress was being applied onto the sample.

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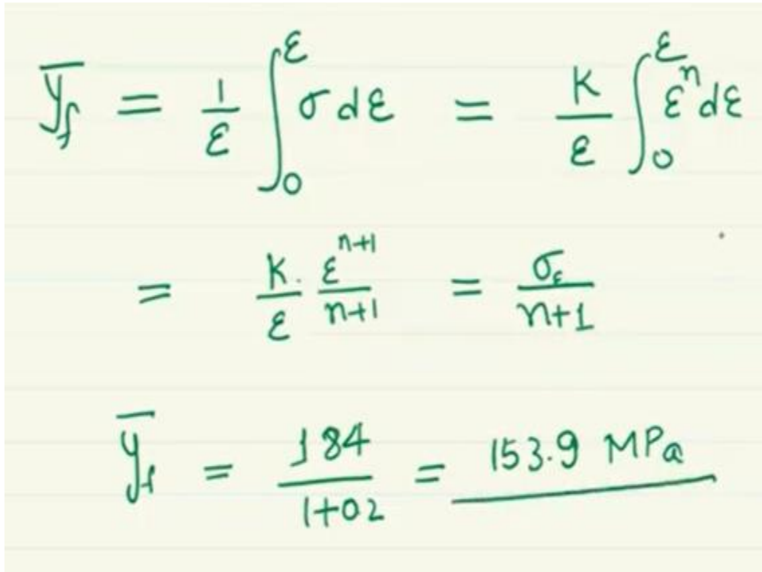

So, how do we obtain that the relation or to obtain that relation let us call it y_f average which is the average flow stress this is basically if the overall strain was epsilon, then this is given by sigma d epsilon. So, you are whatever stress you are applying into integral divided by the over by the total strain. And this will give you all roughly average amount of flow stress that is required is not giving you the maximum stress or the minimum stress this is on an average this much flow stress is required throughout the process.

So, in other words you can say that this flow stress times epsilon would be the same amount of energy as the total amount of energy which you can see here

$$\overline{y_f} = \frac{1}{\varepsilon}\int_0^{\varepsilon} \sigma d\varepsilon$$

So this energy and this energy would be equal so that is where the concept of average flow stress is coming from. Now putting sigma and rearrange it

$$\overline{y_f} = \frac{\sigma_\varepsilon}{n+1}$$

so our solution for this equivalent question becomes very simple for this particular one what we need is 184 /1+0.2. And therefore this turns out to be 153.9 Megapascal so this is the answer that we are seeking.

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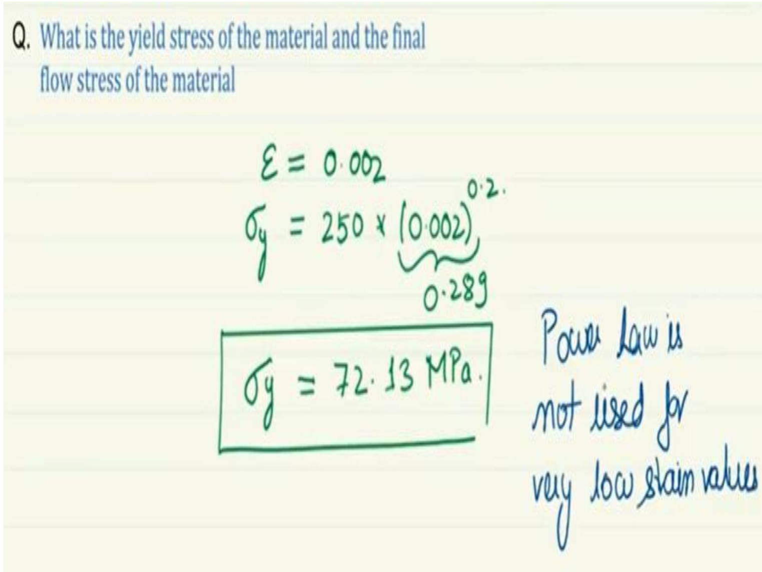

Now taking this equation further ahead and you are asked what is the yield stress of the material and the final flow stress of the material. So, final flow stress we have already obtained which was the 184 Megapascal. Now we have to find the original yield stress of the material so we know that for original distress strain is equal to 0.002 therefore all we need to do is insert this to find original yield strength. So this is $250 \times 0.002^{0.2}$ and this is 0.289×250 so this turns out to; 72.13 Megapascal.

This is the yield stress of the material of the virgin material meaning when it had no deformation. But then you have to be careful that this power law behaviour cannot be applied for usually not applied for very few very low strain values. So, this is not the only limitation there are some other limitations also. So, let us look at what are some other limitations of the power law behaviour.

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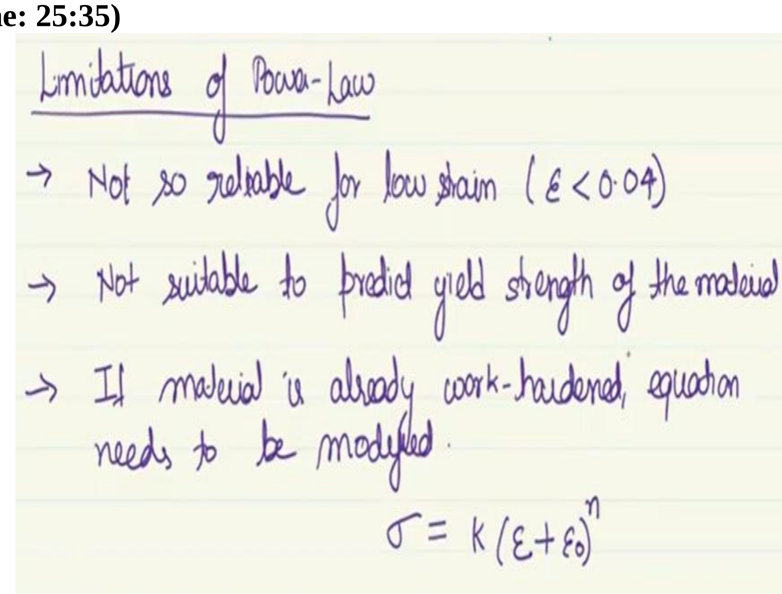

So, this is quite reliable for higher strain but not so reliable when strain values are small. This is something we just emphasized and usually what is that low strain? So that low strain is less than 0.04 and hence not suitable to predict yield strength of the material. Another limitation is that if the material is already work hardened the equation needs to be modified which we have already seen over here when we are looking so that was earlier.

So, if equation needs to be modified what is that modified equation $\sigma = k(\varepsilon + \varepsilon_{\circ})^{n}$. So, this is your pre-strain value so this gives us a very good idea about the flow stress there can be good number of numerical questions that can be set up on this. So it will be good for you if you can do some practice on this problem on these equations on how to translate from engineering strain to true strain.

And also to engineering stress to true stress and then use it in the equation to calculate the flow stress behaviour. Sometimes you may be given a pre-strain what will be the flow stress or what will be the average flow stress and so on. So, these kinds of problems can be setup on this particular topic. So we will close this topic chapter of power law behaviour or basically the flow stress behaviour and next time when we meet, we will talk about the yield criterion thanks.