**Fundamentals and Applications of Dielectric Ceramics**
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**Lecture No. – 21**
**Losses in Dielectric Materials**

So, welcome to the new lecture of this course, fundamentals and applications of dielectric ceramics.

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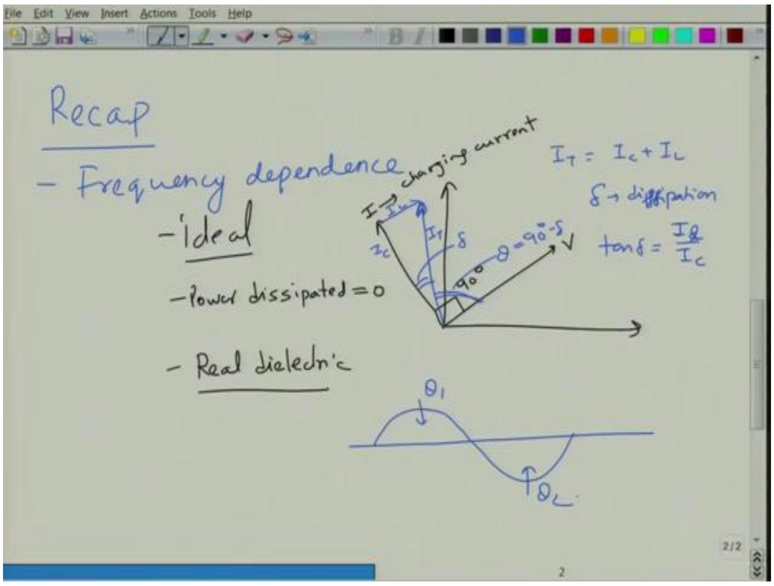


So, let us just briefly do a recap of what we learnt in the last class. In the last class, maybe the last couple of lectures, we started our discussion on frequency dependence of dielectric properties. In that, we looked at first the case of ideal dielectric. In the case of ideal dielectric we said, if you have this real imaginary phasor diagram, and if your voltage is V in this direction, then current leads the voltage by angle of $90^0$ for ideal dielectric.

And, as a result, all the current is basically charging current and hence the power dissipated is equal to zero. So, there is no power loss in ideal dielectric because there are no losses. And, then, we moved on to real dielectric. In the case of real dielectric we just introduced this, that if you apply a voltage V, then you might have a situation that the total current is basically at a certain angle.

So, this is δ angle and this is, let us say θ, and this θ is basically:

$$\theta = 90^0 - \delta$$

And this total current is now sum of two currents; one is charging current and then you have loss current, and this I total. So,

$$I_T = I_C + I_L$$

And this angle δ is basically a representation of power dissipation. So,

$$\tan\delta = \frac{I_L}{I_C}$$

it represents is how much is the loss in a system.

So, basically, what we said is in a real dielectric. If you pump in $Q_E$, you take out $Q_E$ during the reverse cycle for ideal dielectric, whereas for a real dielectric there is a difference. This is $Q_1$, this is $Q_2$, so $Q_1 = Q_2$ in the case of ideal dielectric, which means there are no loss of charges, but in the case of real dielectric you will have loss of charges through mechanisms such as leakage or loss, ohmic loss, or non-ohmic losses.

We will see what they are. This is what we are going to analyse in this lecture, what are these losses and how can you represent them mathematically.

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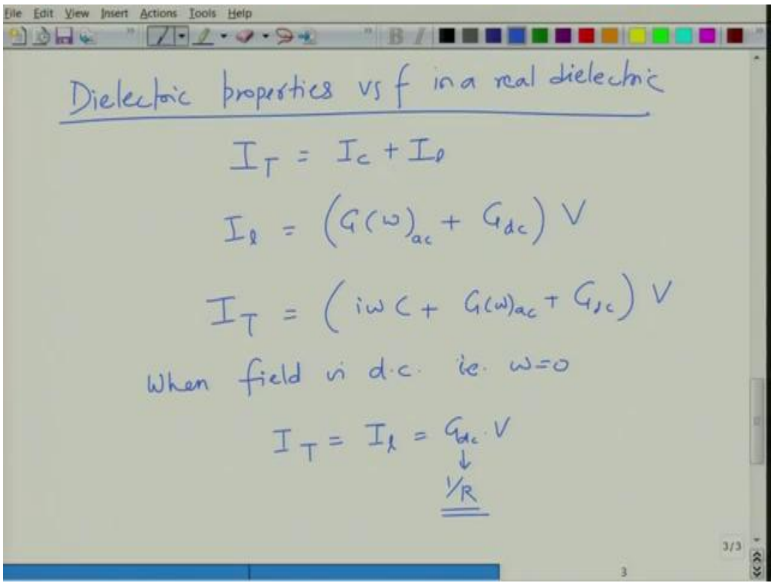


Let us now look at the case of, basically, we are saying power dissipation, we will further dwell upon power, not power, sorry, we will come to power later on, but dielectric versus frequency in a real dielectric. So, for a real dielectric we said that:

$$I_T = I_C + I_L$$

and what basically this means is that your current loss now is basically:

$$I_L = \{G(\omega)_{ac} + G_{dc}\}.V$$

So, the first is frequency dependent contribution and the second is frequency independent contribution.

So, 'G' is conductance which is inverse of resistance. So, the total current will be now:

$$I_T = \{i\omega C + G(\omega)_{ac} + G_{dc}\}.V$$

So, this is what we said last time. So, when your field applied is d.c, that is, ω = 0. In that case,

$$I_T = I_L = G_{dc}.V$$

So, what we are going to see is there is basically dc contribution. Basically the leakage current is the current that we will observe.

So, when you make dc measurements on a dielectric you are basically looking at the ohmic losses in the system which is frequency independent.

$$G_{dc} = 1/R$$

The $G_{dc}$ is nothing but inverse of R which is inverse of the ohmic resistance of the system.

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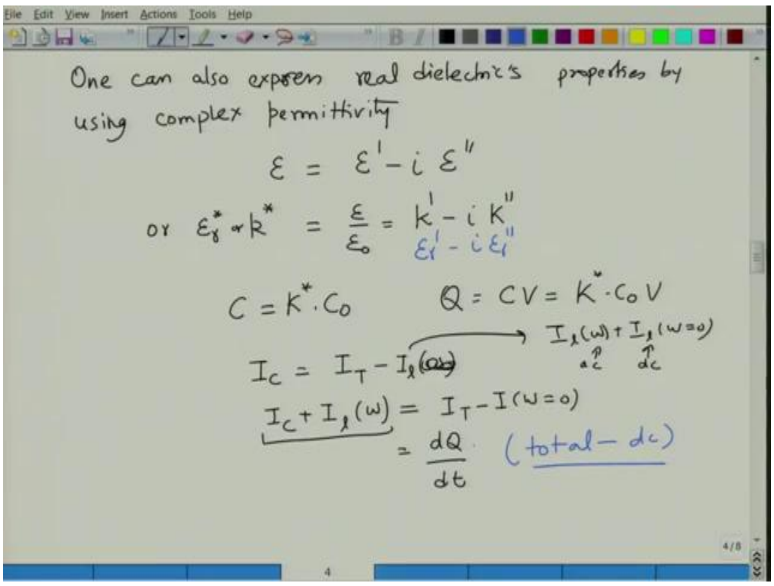

So, you can express the dielectric properties of a system in terms of current, but we will come to that later on. One can also express the real dielectric properties by using complex permittivity. So, now that the current itself is frequency dependent and independent which means the permittivity also has to be frequency dependent. So, we examine this permittivity as:

$$\varepsilon = \varepsilon' + i\varepsilon''$$

Or it can also be referred as, in terms of kappa, then:

$$\varepsilon_r^* = k^* = \frac{\varepsilon}{\varepsilon_0} = k' - ik''$$

So, basically the idea of doing that is that the current in the dielectric can be represented in terms of material property that is called as dielectric permittivity.

We will see how we do that. We can say that C, the capacitance of a dielectric, is given as:

$$C = k^*.C_0$$

$C_0$ is the capacitance of a vacuum capacitor and $k^*$ is the permittivity of a dielectric, the real dielectric that we are using. We know that:

$$Q = C.V = k^*.C_0 V$$

We have said that:

$$I_C = I_T - I_L$$

Here,

$$I_L = I_L(\omega) + I_L(\omega = 0)$$

So, the first is the ac component and the second is the dc component,
This can be further arranged as

$$I_C + I_L(\omega) = I_T - I(\omega = 0) = {dQ}/{dt}$$

Now, on the left hand side you can see that we have only the frequency dependent loss current. On the right hand side we have the total current minus the dc current.

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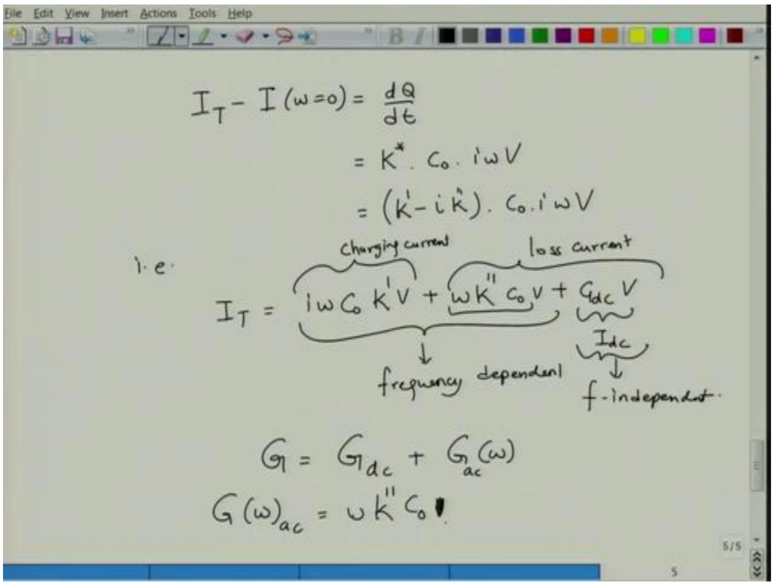


So, we can say now that:

$$I_T - I(\omega = 0) = {dQ}/{dt} = k^*.C_0.i\omega V = (k' - ik'').C_0.i\omega V$$

So, this becomes this expression which means that we can write:

$$I_T = i\omega C_0 k' V + \omega k'' C_0 V + G_{dc}.V$$

Here, in R.H.S the first term is the charging current, the sum of second and third term is the loss current. First two terms are frequency dependent. The third term is frequency independent.

So, now you have to compare the equation that we yielded. We said that overall conductance:

$$G = G_{dc} + G_{ac}(\omega)$$

So, here we say we can say that:

$$G(\omega) = \omega.k'' C_0$$

So, this basically is the conductance.

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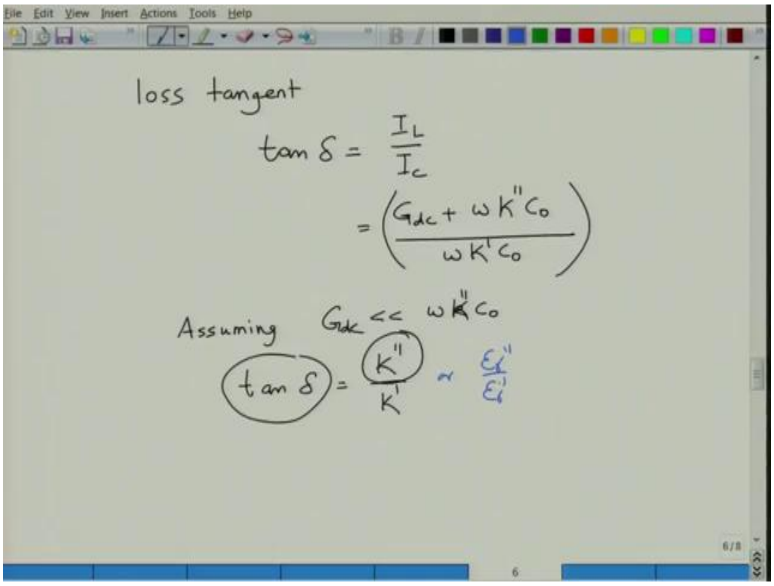


We express the tan delta, the loss, we define a new quantity called as loss tangent. It is represented as:

$$\tan\delta = \frac{I_L}{I_C} = \left(\frac{G_{dc} + \omega k'' C_0}{\omega k'' C_0}\right)$$

$I_c$ is the charging current. So, this looks like a complex expression. Assuming that:

$$G_{dc} << \omega k'' C_0$$

The loss tangent is written as:

$$\tan\delta = \frac{k''}{k'}$$

So, basically, the dissipation factor or the loss tangent which was equal to ratio of loss current to charging current considering that ohmic losses are smaller, it is nothing but the ratio of imaginary part of the dielectric permittivity to the real part of the dielectric positivity. So, if your tan delta is high, what it means is that your $k''$ is high. So, higher the loss tangent which means higher the imaginary part of your dielectric permittivity will be.

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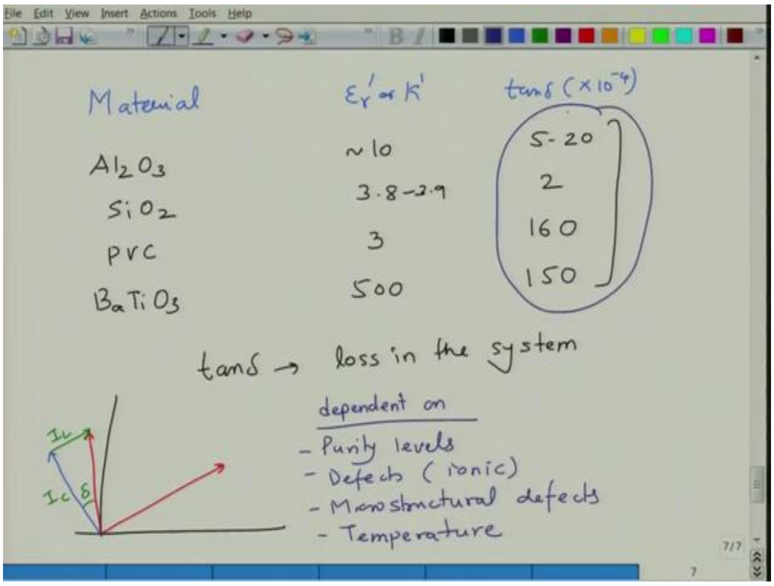


So if you look for various systems such as, let us say we take material, we take the value of $\varepsilon_r^*$ or $k'$ and then we take tan δ in $10^{-4}$. So, if you take, for example, for something like $Al2O_3$, $Al2O_3$ has a value of about 10 and the loss tangent is (5 - 20) x$10^{-4}$. For a good dielectric, if you look at $SiO_2$, its value is 3.8 - 3.9 and tan δ is about 2 x$10^{-4}$.

If you look at something like PVC, it's $\varepsilon_r^*$ is about 3 and tan δ is 160 x$10^{-4}$, more lossy system because it is amorphous, it has more defects, as a result it has more loss. And, if you look at something like barium titanate, barium titanate has very high dielectric constant and it also tends to have a little bit higher loss as compared to other systems. In general, what you will see is that the more defects in the material is, the more the tan δ will be.

So, tan δ basically is loss in the system. So, essentially in the phasor diagram as we drew, if this is your V, this is let us say your $I_T$, so this angle δ, the more this angle is, the higher the loss current will be, and appropriately your $I_c$ will reduce in number. So, basically what this represents is loss in the system. This is dependent upon things like purity level, defects, this could be ionic defects as well as micro structural defects such as porosity, grain boundaries, etc., and also dependent upon temperature.

In general, you will see that if you increase the impurities which create defects, we saw in defect chemistry that there are certain impurities which can give rise to defects, so if you have more of those impurities, you will have more defects, as a result, you will have more tan δ. Similarly, if you have ionic defects in the system because of conditions like temperature and partial pressure of oxygen, you will have more losses in the system.

Similarly, microstructural defects such as porosity, grain boundary and, you know, antiphase boundary, twins, etc., they all have potential to give rise to, may be not necessarily twins, but surely grain boundaries, small angle and low angle grain boundaries, and then temperature, in general, as you increase the temperature the loss in the system increases. And this is a very important parameter in tan δ.

So, when you measure the dielectric properties of a system, for a good dielectric application, the idea is to minimise this tan δ value. So, lower the tan δ means good dielectric.

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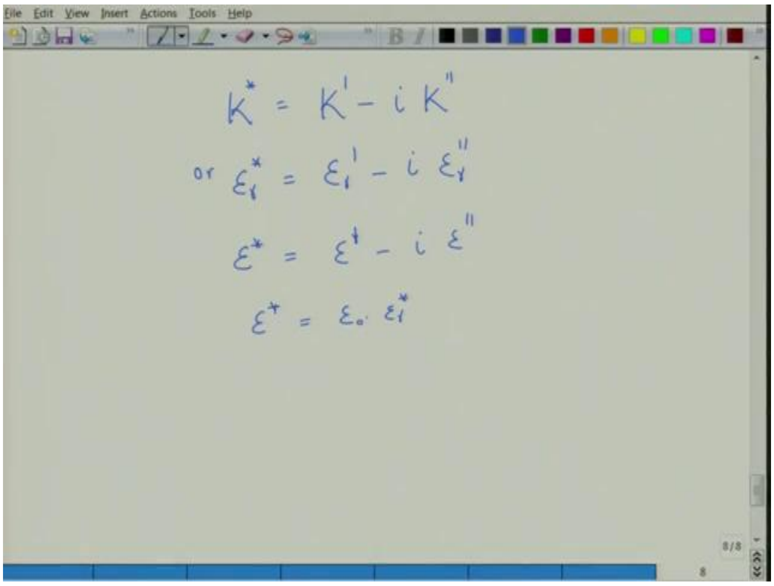

Now having known this, you can write this basically,

$$k^* = k' - ik''$$

$$\varepsilon_r^* = \varepsilon_r' - i\varepsilon_r''$$

$$\varepsilon^* = \varepsilon' - i\varepsilon''$$

$$\varepsilon^* = \varepsilon_0 \varepsilon_r^*$$

So, these are different ways of representing the dielectric properties of a given system. So, if we just briefly go through what we have done, we started with dielectric properties. We said that total current is equal to charging current plus loss current. Loss current is sum of ac and dc component. So, charging current we worked out earlier, it will be:

$$I_c = i\omega CV$$

the total current is:

$$I_T = \{i\omega C + G(\omega)_{ac} + G_{dc}\}.V$$

and correspondingly loss current will be:

$$I_L = \{G(\omega)_{ac} + G_{dc}\}.V$$

So, when the field is dc, then of course ω = 0, then the current that you measure is basically the loss current. There is no frequency dependence in the system. So, how do you relate the previous equation to the dielectric properties?

We express the dielectric properties such as dielectric constant in terms of complex properties such as **ε** can be written in the form:

$$\varepsilon = \varepsilon' + i\varepsilon''$$

Also,

$$\varepsilon = \varepsilon_0 \varepsilon_r^*$$

we can write this in terms of $\boldsymbol{\varepsilon_r^*}$ as

$$\varepsilon_r^* = k^* = k' - ik'' = \varepsilon_r' - i\varepsilon_r''$$

We also know that capacitance is:

$$C = k^*.C_0$$

and we know:

$$Q = C.V = k^*.C_0 V$$

so appropriately we can now write expression for current because we know the frequency dependent current that you measured, because now the charge is not equal to only $C_0V$, charge is equal to $k^*$ which is a complex quantity multiplied by C and V, which means this variation of charge is going to represent all the variation of frequency dependence.

So, time dependent variation of this charge is going to be the frequency dependent component of current, which is:

$$I_C + I_L(\omega) = {dQ}/{dt}$$

and this is nothing but,

$$I_C + I_L(\omega) = I_T - I(\omega = 0)$$

So, total current minus the dc component is basically the frequency variation or time dependent variation.

And from this, if you expand it further, you can see that there are two frequency dependent terms, one is $\mathbf{i\omega C_0}\,k'\,\mathbf{V}$, second is $\boldsymbol{\omega}\,k''\,\mathbf{C_0V+G_{dc}V}$. So, this first frequency dependent term is the charging current. This is the real part of imaginary constant that represents the charging current and the imaginary part represents the loss current. So, this is what we worked out in terms of tan δ. So, tan δ is the ratio of loss current to charging current.

If, let us say, the ohmic part is smaller as compared to the frequency dependent part, then we can consider tan δ to be:

$$\tan\delta = \frac{k''}{k'} = \frac{\varepsilon_r''}{\varepsilon_r'}$$

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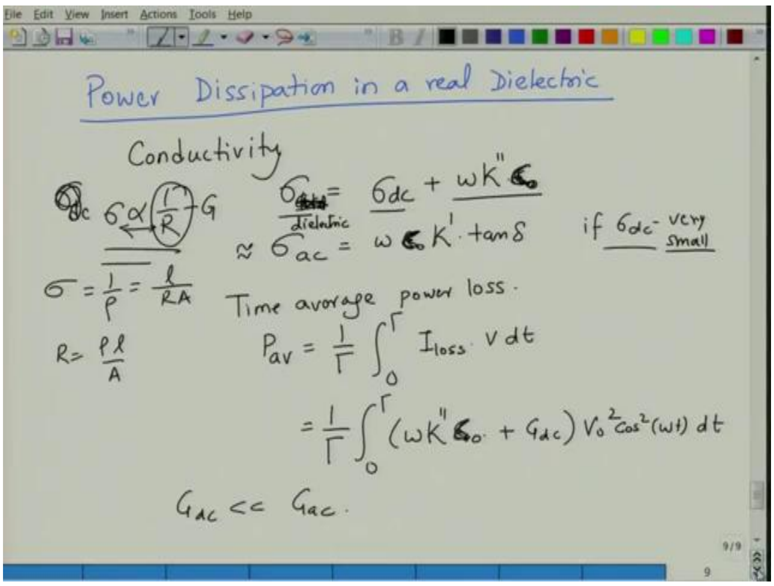

So, now what we will do is that we will look at power dissipation in a real dielectric. To do this, what we do is that we first express the conductivity of a dielectric. So, we write

$$\sigma_{dielectric} = \sigma_{dc} + \omega k'' C_0$$

basically you have a sigma dc component which is from the dc conduction plus you can say the other component that is the frequency dependent component, that will be **$\omega k'' \varepsilon_0$.**

And this is basically equal to,

$$\sigma_{dielectric} \approx \sigma_{ac} = \omega C_0 k'.\tan\delta$$

We know that,

$$\sigma = \frac{1}{\rho} = \frac{l}{R.A}$$

and,

$$R = \frac{\rho l}{A}$$

So, essentially if you leave the dimensions apart, σ is nothing but 1/R which is also 'G'. So, essentially σ scales with 'G' and so this is basically $G_{ac} = G_{dc} + G(\omega)$. So, you can say this is total connectivity essentially, $\sigma_{dielectric}$ essentially you can say. So, if $\sigma_{dc}$ is very small, then you can approximate this as:

$$\sigma_{ac} = \omega \varepsilon_0 k'.\tan\delta$$

So, the time average power loss can be expressed as:

$$P_{av} = \frac{1}{T}\int_0^T I_L.Vdt$$

And this will be:

$$P_{av} = \frac{1}{T}\int_0^T (\omega k'' C_0 + G_{dc}) V_0^2 \cos^2(\omega t).dt$$

Now, assuming

$$G_{ac} << G_{dc}$$

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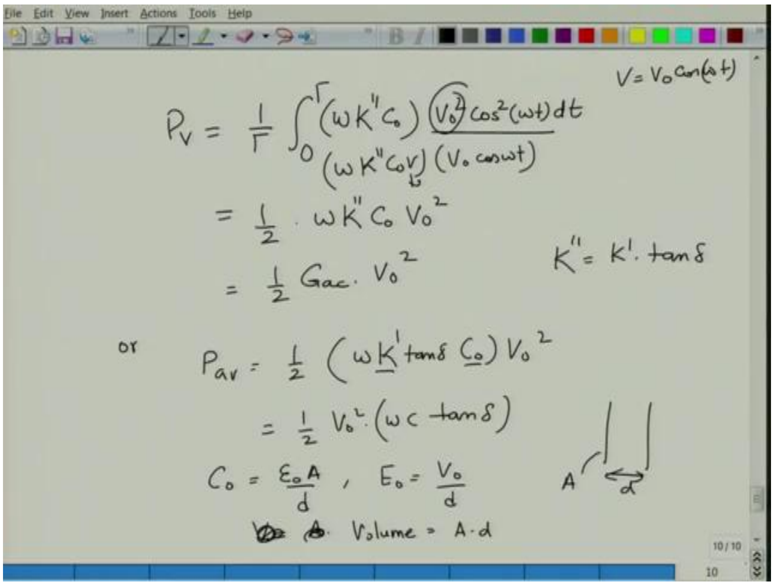


Then we can write:

$$P_{av} = \frac{1}{T}\int_0^T \omega k'' C_0 V_0^2 \cos^2(\omega t).dt$$

We know that,

$$V = V_0 \cos(\omega t)$$

So, P average in this case will be, if you now do the integration, this will come out to be:

$$P_{av} = \frac{1}{2}\omega k'' C_0 V_0^2 = \frac{1}{2} G_{ac} V_0^2$$

I can also write in terms of $k^*$ and tan δ because we know that $k'' = k'$ .tan δ. So, we can replace this as:

$$P_{av} = \frac{1}{2}(\omega k' \tan\delta . C_0) V_0^2$$

You can also write this as:

$$P_{av} = \frac{1}{2} V_0^2 (\omega C \tan\delta)$$

So, if

$$C_0 = \frac{\varepsilon_0 A}{d}$$

‘d’ which is the dimension of a system,

$$E_0 = \frac{V_0}{d}$$

so you have a capacitor with plate area A and separation as d.

So, the dissipated power density you can write it as shown, and the volume will be equal to A multiplied by d.

$$Volume = A.d$$

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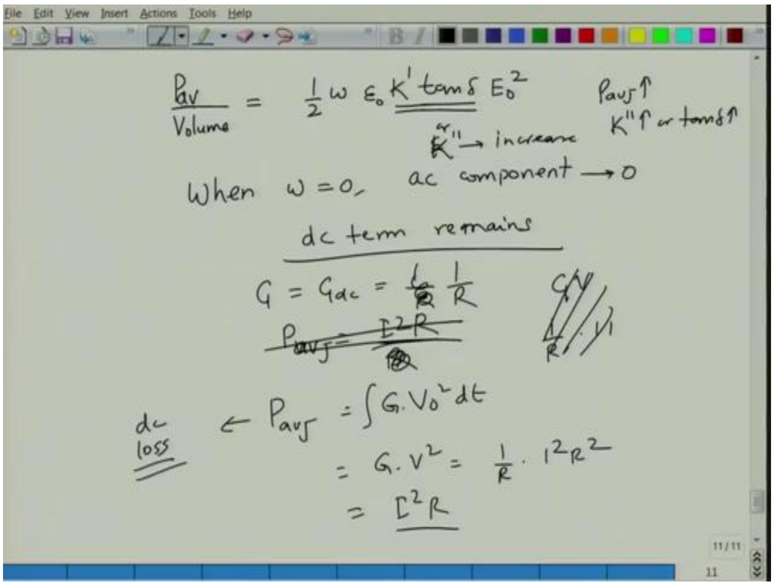


So, if you write this as P average divided by volume, this can be written as

$$\frac{P_{av}}{Volume} = \frac{1}{2}\omega\varepsilon_0 k' \tan\delta . E_0^2$$

So, if you now make the frequency dependent zero. So, the only term that you will be left with is the dc term which we have ignored. So, when frequency is zero, then ac component will vanish.

So, you are left with only dc term which we have ignored because we have considered this to be smaller than the ac component. So, if frequency is zero, then what we will have is

$$G = G_{dc} = 1/R$$

In such case your P average will be:

$$P_{av} = \int G.V_0^2 .dt = G.V^2$$

And this will be equal:

$$P_{av} = \frac{1}{R}.I^2R^2 = I^2R$$

This is what this will be, the dc loss.

This is in the absence of frequency dependent laws, that is when frequencies equal to zero. However, you can see the frequency dependent contribution is, the power loss increases as your tan δ value increases or you can say your $\boldsymbol{\varepsilon}\, k''$ increases. So, your P average will go up as your $k''$ goes up or tan δ goes up.

This also means that as you increase the frequency, then also the power loss will go up. So, this is what it is in terms of electric property representation, in terms of frequency. So, basically what we have seen is that we have a loss current and the charging current. The loss current has two parts, the frequency dependent part and frequency independent part, and the ratio of loss current to charging current is basically dissipation factor.

So, higher the dissipation factor more the power loss in the system is going to be. So, this you can work out again and go through it again and again. It is a little complex to understand, but if you go through it a few times you will understand it. In the next class we will start our discussion on, essentially looking at a little bit more detailed treatment of frequency dependence of dielectric properties from a microscopic scale.

What we have seen right now is the macroscopic scale, but because frequency dielectric constant varies in various steps as a function of frequency, we need to understand the microscopic mechanisms.