Fundamentals and Applications of Dielectric Ceramics

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Module No # 04
Lecture No # 20
Frequency Dependence of Dielectrics

So welcome again to the new lecture of the course fundamentals and applications of dielectric ceramics.

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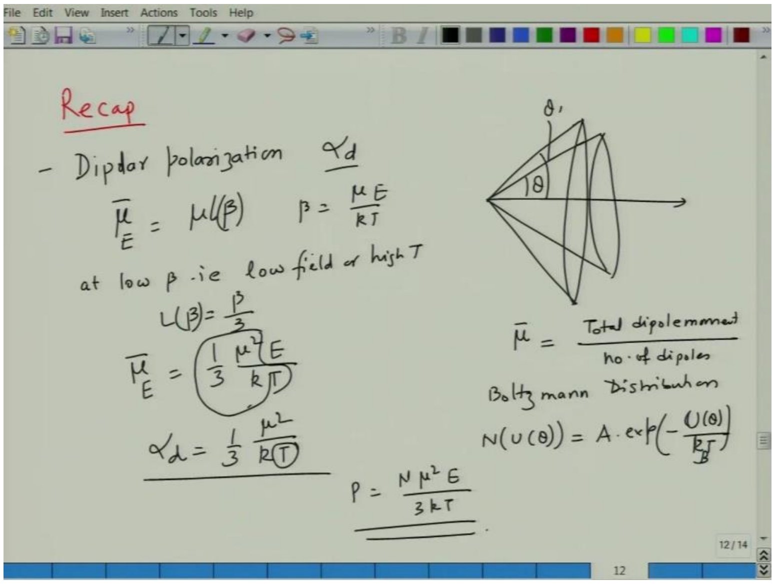

Recap

- Dipolar polarization $\underline{\alpha_d}$

$\frac{\bar{\mu}}{E} = \mu L(\beta) \qquad \beta = \frac{\mu E}{kT}$

at low β .ie low field or high T

$L(\beta) = \frac{\beta}{3}$

$\frac{\bar{\mu}}{E} = \frac{1}{3}\frac{\mu^2 E}{kT}$

$\alpha_d = \frac{1}{3}\frac{\mu^2}{kT}$

$P = \frac{N\mu^2 E}{3kT}$

$\bar{\mu} = \frac{\text{Total dipolemoment}}{\text{no. of dipoles}}$

Boltzmann Distribution

$N(U(\theta)) = A \cdot \exp\left(-\frac{U(\theta)}{k_B T}\right)$

So, in the previous lecture let us just recap the previous lecture so in the previous lecture we talked about we started our discussion on dipolar polarizability. So basically, alpha d so where we take the case that you know if you have electric field line like this then dipoles are making a cone at an angle θ with respect to applied field so this is first cone you might have another cone at another angle.

So basically, all these dipoles then need to be so this is an angle θ_1 so you need to basically calculate the average dipole moment which is basically total dipole moment of the system divided by number of dipoles. That is what we did so we took a slightly angular analysis considering the solid angle and then we also took help of Boltzmann distribution function which allows us to get the minimum free energy.

Because Boltzmann allows function says that number of dipole moments of energy $[U(\theta)]$will be

$$N[U(\theta)] = A \cdot exp\left(-\frac{U(\theta)}{k_B T}\right)$$

So, this avoids the calculation of entropy which may be a little trickier so from this we calculated what is the average dipole moment. So, the average dipole moment was calculated as $\mu \cdot \beta$ ok and this beta sorry. And this $L(\beta)$ was basically Langevin function and Langevin function at low value of β so β will be equal to here $\frac{\mu E}{k_B T}$.

So, at low beta that is low field or high temperature we approximated $L(\beta) = \beta/3$ and from this we got μE which is equal to $\frac{\mu^2 E}{3 \cdot k_B T}$ ok. So, from this we determine what is dipolar polarizability which is

$$\alpha_d = \frac{1}{3} \cdot \frac{\mu^2}{k_B T}$$

So, what it means is that basically dipolar polarizability decreases as you increase the temperature this is the analysis that we did and this is dependent on temperature unlike the other two polarizability is that at electronic and ionic in polarizabilities.

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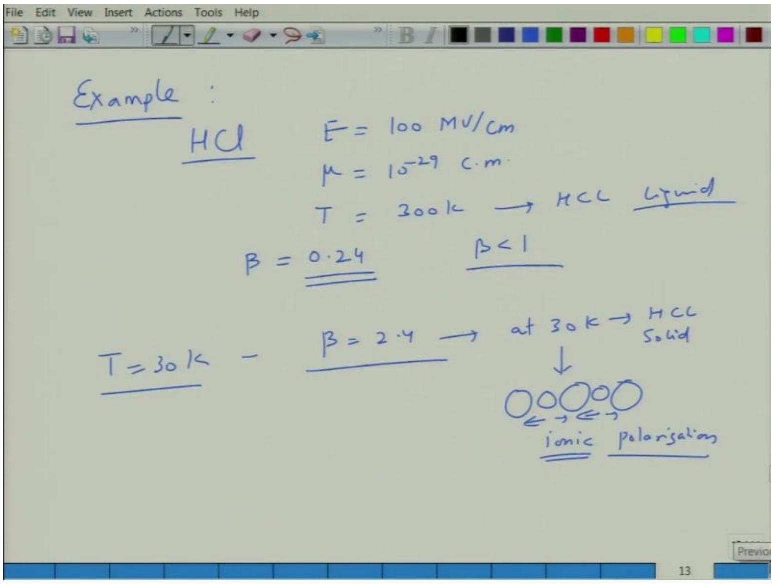

Example :

HCl

$E = 100$ MV/cm

$\mu = 10^{-29}$ C.m.

$T = 300k \longrightarrow$ HCL Liquid

$\beta = 0.24$ $\beta < 1$

$T = 30k$ - $\beta = 2.4 \longrightarrow$ at $30k \rightarrow$ HCL Solid

$\downarrow$

ionic polarisation

So for example if you do it for a system such as if you take for HCl for example so imagine that electric field is equal to about 100 MV/cm. Dipole moment is about 10^{-29} coulomb meter then at a temperature of 300 K we get beta of about 2.24 which is sort of ok because we considered beta to be small than 1 so this sort of validates analysis that approximation was not wrong.

However, if you do for example at a temperature like 30 K then this increases the β value to 2.4 so that is sort of invalidates our analysis. But the but what happens is that at 30 K HCl so at this form HCl is liquid right. So, all the dipoles are randomly oriented but here HCl becomes solid so basically in solid form if you have this kind of situation the dipole tends to cancel each other.

It becomes basically you have ionic polarization only that contributes so as a result the dipolar polarizability is not valid at lower temperature. It is only the ionic polarization that contributes for the HCl because it is in solid form so sort of NaCl structure it makes. So now let us what we have done just to summarize.

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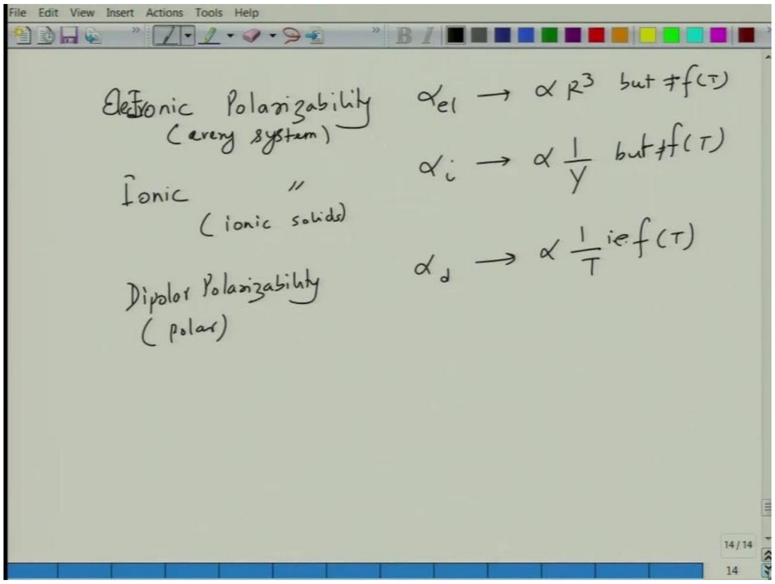

Electronic Polarizability (every system) $\alpha_{el} \rightarrow \propto R^3$ but $\neq f(T)$

Ionic 〃 (ionic solids) $\alpha_i \rightarrow \propto \frac{1}{Y}$ but $\neq f(T)$

Dipolor Polarizability (polar) $\alpha_d \rightarrow \propto \frac{1}{T}$ ie $f(T)$

Now you so you have three polarizabilities you have ionic polarizability putting everything together which is $\alpha_{(electronic)}$ which is basically proportional to R^3. But is not a function of temperature sorry electronic then we have ionic polarizability (α_i) so this is electronic polarizability is true for every system because every system will have atoms. Ionic polarizability is only for ionic solids where we have cations and anions in the solid form.

There we consider α_i and this is proportional to $1/Y$ but again it is not a function of temperature. Electronic and then we have dipolar polarizability this is for polar molecules only where molecules have dipole moment there, we defined this as α_d and this is proportional to *1/T*. So, this a function of temperature and it decreases as the temperature increases which means you need to apply larger field to polarize the system ok.

So that is why you will see in every system now polarization decreases as you increase the temperature in whether it is a ferroelectric system piezoelectric pyroelectric system or any other system the thermal randomization takes over as you increase the temperature that and as a result you have decrease in the polarization or the dipole moment of the system.

So, can we I mean so basically this analysis should be reasonably correct if you have if you know the value of *D* and if you wanted to create certain sort of some sort of dipole moment. Then you can to some extent get a feel of what should be the electric field that you require to polarize the system to create some amount of dipole moment (μ) so I said,

$$P = \frac{N}{3} \cdot \frac{\mu^2}{k_B T} \cdot E$$

So, but this is not valid for nonlinear effects where you have interaction between the dipoles.
So, here we are not we are assuming that dipoles are not interacting ok but if dipoles is start interacting then it is a different situation but to first approximation for system where you have non interacting dipoles you can do that. And in a normal solid you will also have a effect of grain boundaries as well as defects it is sort of complicate the scenario. But I mean it may give you an idea of what is that which is required to polarize the system**.** Now let us move on to so what we have done until now is that we have looked at the basic dielectric properties.

What is polarization? What is the mechanism of polarization? What is dielectric constant? And what is the frequency dependence of dielectric constant as a function of under looked at in the context of various mechanisms which mechanism is operative in which frequency range and then we looked at a parameter called as polarizability and it is an analytical treatment.

And we found that ionic and electronic polarizabilities are independent of temperature whereas dipolar polarizability for polar molecules is dependent upon temperature. So, what we are going to do now is that we are going to look at the frequency dependence of these dielectric properties in a little bit more detail. Because this is what is of interest practically speaking because most dielectrics are used in applications are requiring certain frequencies of electric field.

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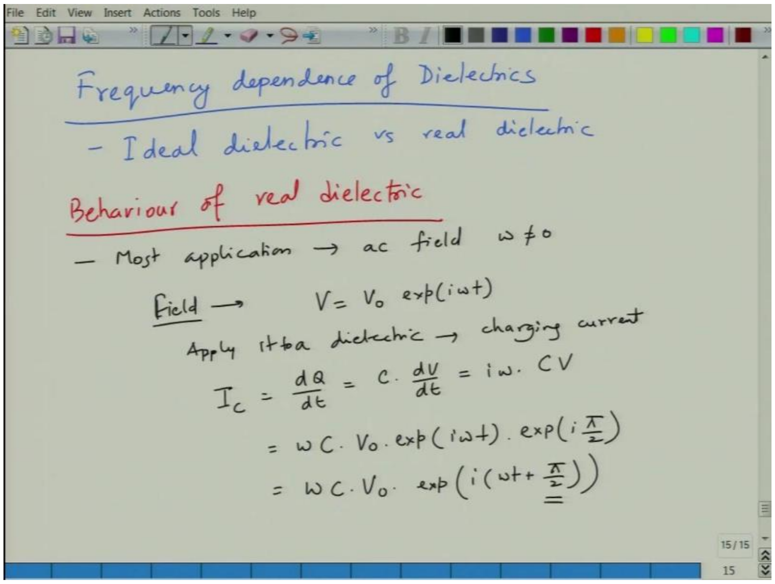

Frequency dependence of Dielectrics

- Ideal dielectric vs real dielectric

Behaviour of real dielectric

— Most application → ac field $\omega \neq 0$

Field → $V = V_0 \exp(i\omega t)$

Apply it to a dielectric → charging current

$$I_c = \frac{dQ}{dt} = C \cdot \frac{dV}{dt} = i\omega \cdot CV$$

$$= \omega C \cdot V_0 \cdot \exp(i\omega t) \cdot \exp\left(i\frac{\pi}{2}\right)$$

$$= \omega C \cdot V_0 \cdot \exp\left(i\left(\omega t + \underline{\underline{\frac{\pi}{2}}}\right)\right)$$

So now our discussion will be towards studying frequency dependence of dielectrics. So here I mean the first thing that we are going to look the some fundamentals so we will first look at the comparison between the ideal and real. So, this also so this gives you basically good information about how good you dielectric is how close are you to making a real dielectric. Basically, a real dielectric is something where dielectric by nature should be insulator so which means there should be no whatever you whatever electric field you apply to it, all the current all the electric field should be used in creating dielectric displacement that is a polarization and you should not lose any charges out from the system but systems are not perfect they have various defects and things like that. As a result, whatever current you produce should be charging current ideally but it is not charging current there is a component which is called as leakage current that is produced.

And as a result, some of the charges that you polarize leak out of the system as a result you have loss of information or loss of anything and in terms of charges. So, this is what is important this is what makes it important to understand the frequency dependence of dielectrics. Because it gives you parameters which have which can be depicted in the form of frequency dependent behavior. So, let us look at first the behavior of a real dielectric alright so basically for most applications we do not require a static or dc field for most applications require an ac field ok.

So, which means frequency is not equal to 0 it is finite and as a result so let us apply first let us so the field is a sinusoidally varying field. Let us say this field is

$$V = V_0 \cdot \exp\,(iwt)$$

and if we apply this voltage to a dielectric this leads to so apply it to a dielectric and this should give you what we call as charging current. And this charging current is basically the rate of change of charge basically *dQ/dt*. So this I_c is nothing but *dQ/dt* and we know that *Q* can be written as *C* into *dV/dt*.

So, this will be equal to

$$I_c = i\omega \cdot CV$$

and if you now do a little bit of so this is

$$I_c = \omega C \cdot V_0 \cdot \exp\,(i\omega t) \cdot \exp\,(i\frac{\pi}{2})$$

ok. So, this will become I can write this as

$$I_c = \omega C \cdot V_0 \cdot \exp\,[i\left(\omega t + \frac{\pi}{2}\right)]$$

What does this mean? This means that this term π/2 basically means that the current charging current in a dielectric leads the voltage by an angle 90 degree or π/2 in a perfect dielectric.

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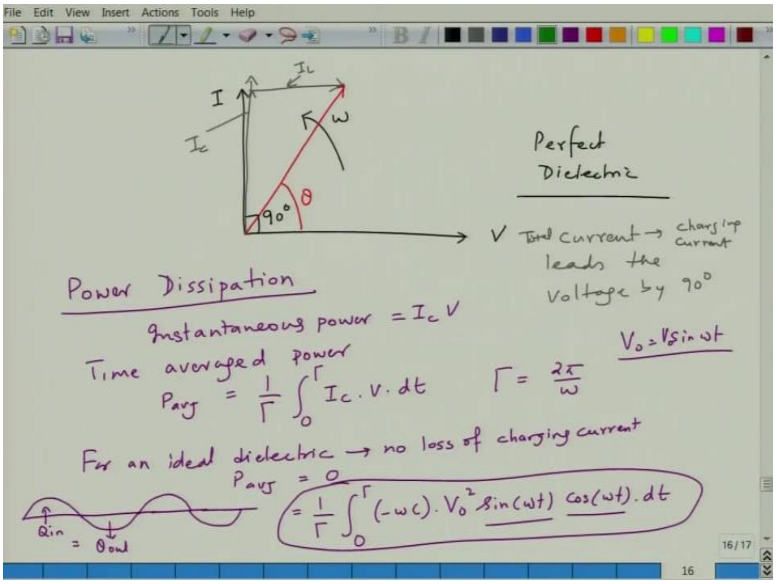

Perfect Dielectric

Total current → charging current leads the voltage by 90°

Power Dissipation

Instantaneous power $= I_c V$

Time averaged power

$P_{avg} = \frac{1}{\Gamma}\int_0^{\Gamma} I_c \cdot V \cdot dt$ $\quad \Gamma = \frac{2\pi}{\omega}$

$V_0 = V_0 \sin \omega t$

For an ideal dielectric → no loss of charging current

$P_{avg} = 0$

$= \frac{1}{\Gamma}\int_0^{\Gamma} (-\omega c) \cdot V_0^2 \underline{\sin(\omega t)}\ \underline{\cos(\omega t)} \cdot dt$

$Q_{in} = Q_{out}$

So, for a perfect dielectric if you know make the phasor diagram as we call it if this is the voltage vector then the current leads the voltage by an angle 90 degree and this is ok. This is what is the simple current voltage relationship for a perfect dielectric so this is for a perfect dielectric where the current leads the voltage by 90 degree. So as long as the current is at the certain angle at a voltage 90 degree it is perfect dielectric.

In reality what you see is that in reality the current does not lead the voltage by it leads the voltage by a angle theta. As a result, there are 2 components 1 is this component which is in the charging current direction and then you have another component sorry just I can draw it here maybe. So, this component is a charging component and this component is within the direction of voltage is the loss component.

The ohmic loss component because for a ohmic loss when the current is in the direction of voltage which means it is a ohmic current and the current is in the direction perpendicular the voltage is the charging current. So total current has now 2 components for real dielectric and that is what we will see how it comes about. How do you have so anyway so for a perfect dielectric basically the current leads the voltage by 90 degree.

And this is total current and this all of this current is basically charging current ok and what is the now in such a dielectric what is power dissipation. So, power dissipation in such a dielectric would be so instantaneous power that is drawn by the dielectric is basically I_c*V ok. So, the time averaged power now that is P_{avg}

$$P_{avg} = \frac{1}{\Gamma}\int_0^{\Gamma} I_c \cdot V \cdot dt$$

ok. So, this is the time averaged power (Γ) where time period is given as 2π/ω ok so ω is the frequency of applied field. So, for an ideal dielectric so I can probably write it here just to save some space so this is given as 2π/ω so for an ideal dielectric with no loss of charging current basically ideal dielectric there is no loss of charging current no charges are lost. So basically, the current you have is only charging current we do not lose it in some other form.

The power dissipated must be equal to and what is it this is

$$P_{avg} = \frac{1}{\Gamma}\int_0^{\Gamma} (-\omega C) \cdot V_0^2 \sin(\omega t) \cos(\omega t) \cdot dt = 0$$

that is equal to 0 ok. You can write it in exponential forms whatever you want but this is so if $V = V_0 \sin(\omega t)$ then you can calculate I_c from this, that will be $\sin(\omega t + \pi/2)$. As a result, you will have $\sin{}^{\pi}/_{2} \cos{}^{\pi}/_{2}$ factor $\sin(\omega t)$ and $\cos(\omega t)$.

But you can also write this in the form of exponential form and you will get the same answer so this power loss this power average power dissipated in an ideal dielectric should be equal to 0. For a so what it means it that during the first cycle when you charge the capacitor completely the second cycle discharges the capacitor without any loss of charge. So, during the first cycle so if this is the cycle you have this is the first cycle the charging cycle and the discharging cycle.

So, the charge in is equal to so $Q_{in} = Q_{out}$ there is no loss of charge that is what it means by a so basically it is like a perfect oscillator you know the swing or a spring whatever there is no damping in the system there is no loss. So, there is no loss in the system which means it is a perfect dielectric but life is far from being perfect there is no dielectric which is perfect. As a result, you have losses in the in every system and so when you have losses in every system how do we depict it.

Anyway, it was necessary to show how a perfect dielectric would look like so that we have something to something ref as a reference.

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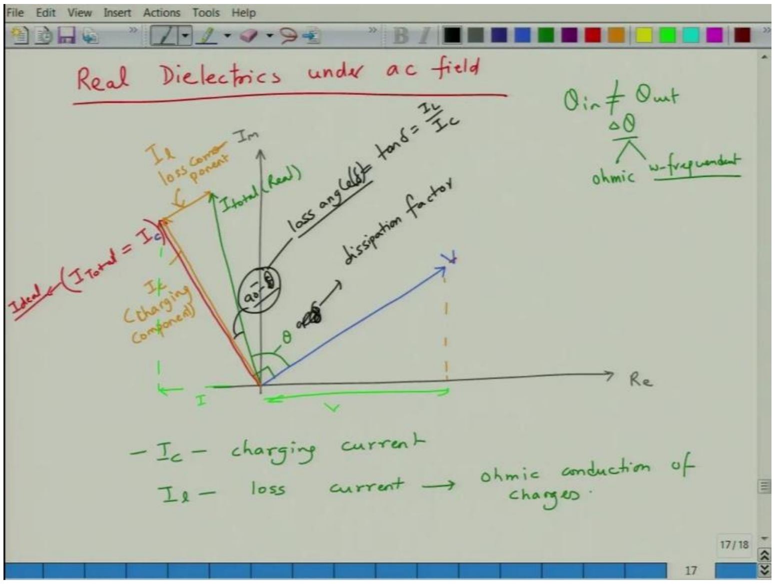

Real Dielectrics under ac field

$Q_{in} \neq Q_{out}$

ΔQ

ohmic

ω-frequendent

I_c – charging current

I_l – loss current → ohmic conduction of charges.

But let us see now how that it behavior of real dielectrics look like so real dielectrics now so real dielectrics under alternating field. So, as I said in real dielectrics generally what will happen is that as I so if you make a phasor diagram let us say and these phasor diagrams are very useful. So let us say this is real this is imaginary ok so we draw a voltage in this as we said that for a real dielectric for a ideal dielectric the I must equal to I must be perpendicular to V.

So this is I this is V, I_c ok is for a real dielectric so $I_c = I_{total}$ for an ideal dielectric. Alright now let us say for a real dielectric the situation is something like this. The current is at certain so and this angle is 90 degree by the way so for a real dielectric let us say this is your I total and this is at certain angle theta ok. So, it has 2 components the 1 first component that is parallel to so this is the I_c component and this is charging component and this would be the what we call as loss component.

So, this is I_l so it is made of 2 terms 1 is I_c + second is I_l and this is I total and basically this is what is going to give you the voltage and this is what is going to give you the current on this side. So, let us see so basically what happens in real dielectric is in addition to so I_c will be the charging current I_l will be the loss current and this loss is basically it happens because of various nature various things.

It could happen because of ohmic conduction of charges in addition you may also have frequency dependent losses ok. So ohmic conduction means there is no frequency dependence because it is in phase so generally ohmic component of the loss is depicted as frequency independent component of loss. But in addition to frequency independent loss there is a possibility of frequency dependent losses such as you know dipole rotate in a system and the dipoles rotate in the system.

They consume energy you may have traps in the system you may have defective charges in the system which may keep the dipoles bound. For example, if you have grain boundaries, grain boundaries have positive and negative charges they have certain defects. These defects may pin down the dipole the dipoles may require more energy to dominate to rotate or the amount of charges that you have stored.

As a result of when you apply voltage you create the movement of charges but these defects the extra charges are there in the system. They may trap the charges that so you said that for a real for a ideal you know Q_{in} this was equal to Q_{out} during charging discharging cycle. But if this is not = Q_{in} and Q_{out} which means this sum δQ it could either get loss because of ohmic processes or it could get loss because of frequency dependent phenomena. So, you have both the components of loss current so that is what we will see in the in our analysis.

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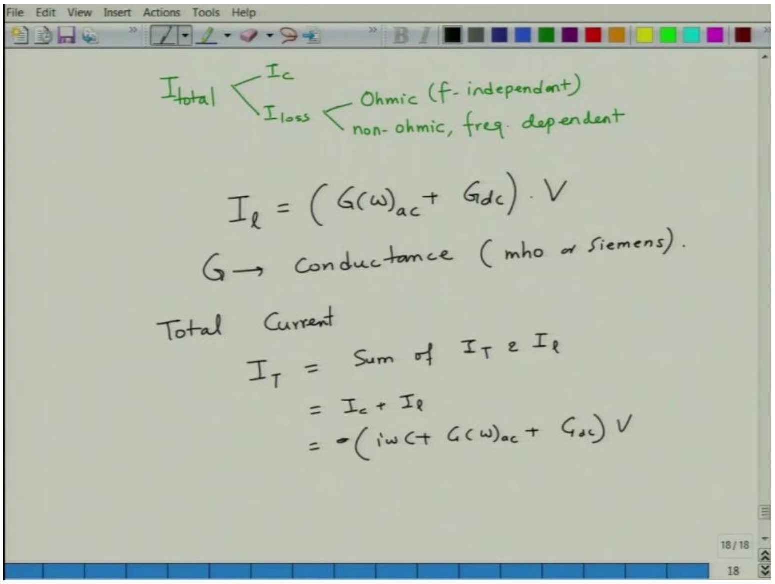

I_{total} < I_c, I_{loss}

I_{loss} < Ohmic (f- independant), non- ohmic, freq. dependent

$$I_l = \left(G(\omega)_{ac} + G_{dc} \right) . V$$

$G \rightarrow$ conductance (mho or Siemens).

Total Current

$$I_T = \text{Sum of } I_T \text{ \& } I_l$$
$$= I_c + I_l$$
$$= (i\omega C + G(\omega)_{ac} + G_{dc}) V$$

So, let us say we have both the current both the contributions so we have I_{total} this will have $I_{charging}$ I_{loss} some of this components will be frequency ohmic and some of this could be non ohmic and frequency dependent. So, this is frequency independent ok so our time dependent you can say I_{total} so let us say if so the loss current in the system is given as what we call as

$$I_l = (G(\omega)_{ac} + G_{dc})V$$

And since both of these are in phase with the applied field that this that is why it is called as loss current and this G is basically called as conductance. So, this is basically written in either mho or Siemens and this is nothing but inverse of resistance right. We know that $V = IR$ so it is nothing but inverse of resistance ok the total current as a result I_T is sum of charging and loss current and this is equal to

$$I_T = (i\omega C + G(\omega)_{ac} + G_{dc})V$$

This is what total current is and here this so this angle as we say this angle is often I have written theta you can also write as delta in some language and this is basically this delta is called as dissipation factor. So, you can see that if you look at this delta this will become $90-\delta$. So tangent of $90-\delta$ will be equal to I_l/I so basically you can say that this angle will be here you can say the loss angle.

Higher the loss angle is more the loss current will be right so let us say if this was equal to some angle let us say sorry let us say we do not write it as δ. We write this as $10 – 90-\theta$ let us say we write this as delta and then this tan delta will be equal to I_l / I_c. And more the loss in the system higher the $tan\delta$ will be for that system or the more the power dissipated or lost charges are lost ok.

So, we will do and if you have this as 0 which means there is no loss in the system. So, we will do more analysis of this in the next class we are now running out of time. So, this has this require this will give lead us to what we may expect is frequency dependent of permittivity as we will see later on ok thank you.