

Fundamentals of Materials Processing
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Lecture 38
Powder Compaction

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Example Problem

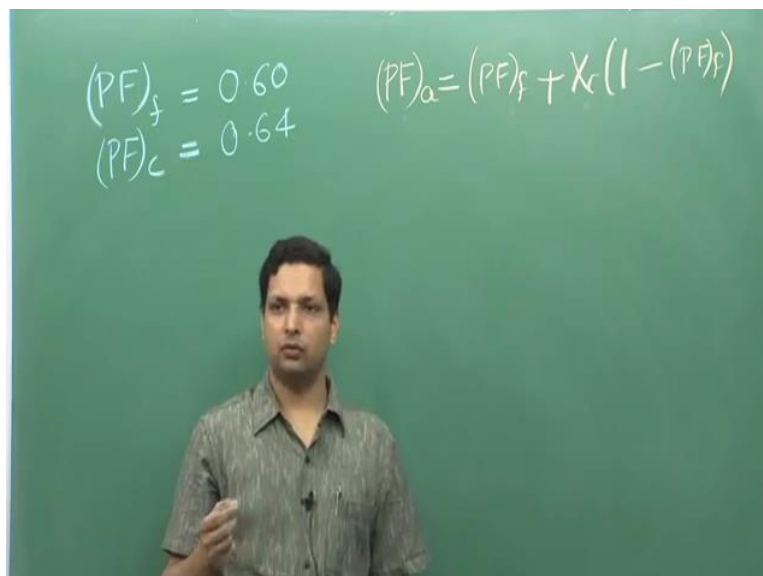
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- Calculate X_c that would yield best packing fraction, if it is known that smaller particles would pack in loose random packing and larger particles acquire dense-random packing, when required.

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So we will continue with the example problem, I hope you have given it a try. Now today we will try to solve it along with you. So here is what you have to do, let us read the problem, what does the problem say calculate X_c that would yield the best packing fraction, so that is the highest packing fraction, if it is known that smaller particles would pack in loose random packing and larger particles acquire dense random packing, when required.

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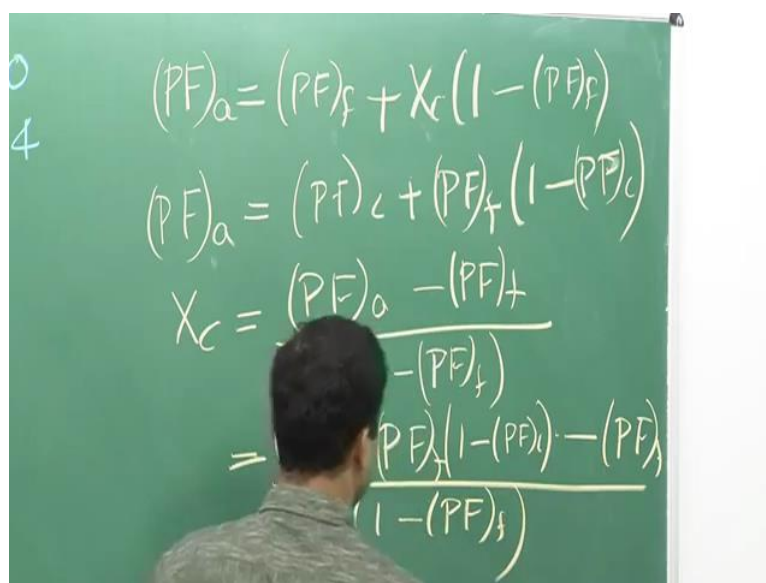


So now you know that what is the packing fraction that we had given for loose random packing and dense random packing, what were those? So let us put those numbers over here, packing fraction for loose random packing which in this case is for the smaller particles, so find you remember what it was it, it was 0.60, packing fraction for cores wherever applicable it is dense random packing whenever it acquires all the or it takes all the position available.

So in that we had said that for dense random packing PF will be 0.64, so these are the two values we know. Now we have to calculate X_c , so the equation that deals with X_c was like this, we said packing fraction apparent is equal to packing fraction fine + X_c times one - packing fraction fine. Now over here we know the packing fraction fine and we have to calculate X_c , but what we do not know is packing fraction apparent or the effective. But what does this represent? This represents when you are putting coarse particles in place of by displacing fine particles, right.

However, we get the maximum efficiency in the condition when you have all the coarse particles taken their place and fine particles are only in the pockets, in the pores and that can also be achieved by this which is the end point. So in that case, we can say that packing fraction apparent can also be given by when we have the coarse particles completely packed whatever is their packing arrangement in this case it is dense random packing and in the pockets wherever it is available, it is only the fine particles and for that we said that packing fraction is given by this.

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A person is seen from behind, writing on a green chalkboard. The board contains the following equations:

$$(PF)_a = (PF)_f + X_c (1 - (PF)_f)$$

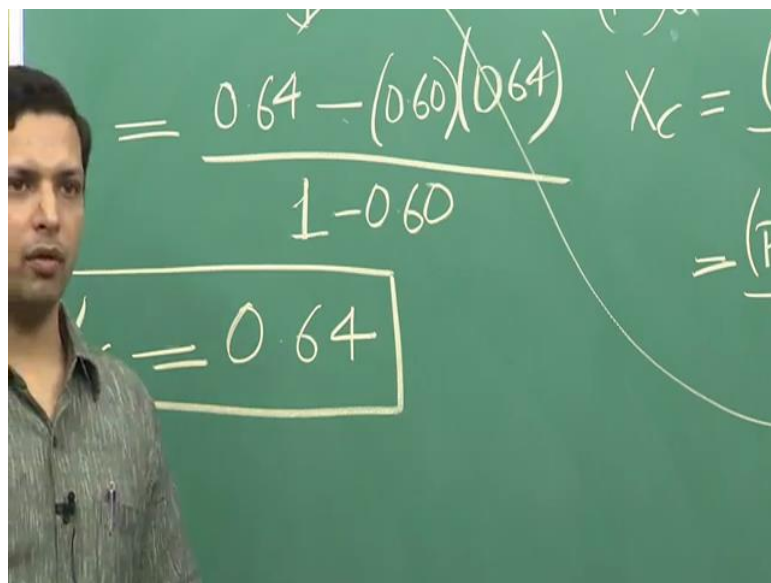
$$(PF)_a = (PF)_c + (PF)_f (1 - (PF)_c)$$

$$X_c = \frac{(PF)_a - (PF)_f}{1 - (PF)_f}$$

$$= \frac{(PF)_c (1 - (PF)_f) - (PF)_f (1 - (PF)_c)}{1 - (PF)_f}$$

So we put this value over here and then we will be in a position to calculate X_c . So this we equate $(PF)_a$ as this equal to that, and therefore let us put it over here and you would see that X_c comes out to $(PF)_a - PF_{fine}$, we are using this equation and on the bottom you have PF_{fine} . Now if you put this value over here or that equation over here, what you would get is - the $(PF)_f$. So now if you and on the denominator, we have $1 -$, so now if you open the bracket, you would see that we get, so let me bring it over here so that it remains visible. So this becomes equal to if you open the brackets or let us here, so this is one $(PF)_f$ times one - $(PF)_c$ times $(PF)_f$ and this is - PF .

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$$X_c = \frac{0.64 - (0.60)(0.64)}{1 - 0.60}$$

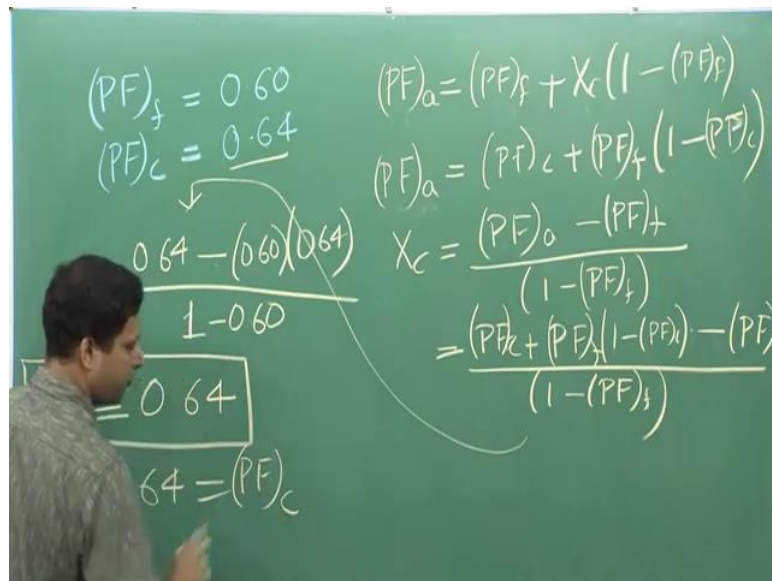
$$X_c = 0.64$$

$$X_c = (PF)$$

$$= (PF)$$

So we have, let just put the values directly and we will get 0.64 minus, so you what you get is that the fraction of cores particles should be equal to 0.64. And this you would see is similar to the number that we have over here which is the packing fractions cores.

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$(PF)_f = 0.60$
 $(PF)_c = 0.64$

$$\frac{0.64 - (0.60)(0.64)}{1 - 0.60} = 0.64$$

 $0.64 = (PF)_c$

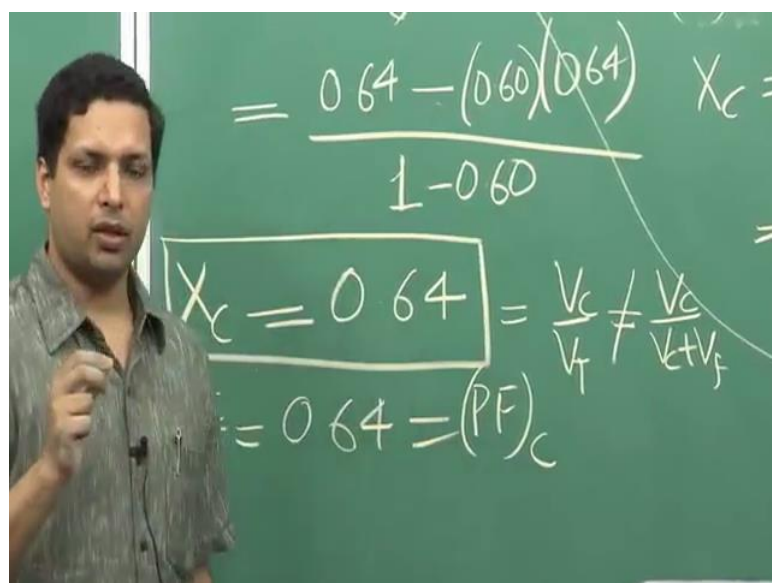
$(PF)_a = (PF)_f + X_c(1 - (PF)_f)$
 $(PF)_a = (PF)_c + (PF)_f(1 - (PF)_c)$

$$X_c = \frac{(PF)_a - (PF)_f}{(1 - (PF)_f)}$$

$$= \frac{(PF)_c + (PF)_f(1 - (PF)_c) - (PF)_f}{(1 - (PF)_f)}$$

Why is it that? It should not be a surprise actually, this is and not even a coincident that this is coming out to be 0.64. What does this 0.64 represent? This 0.64 represents the packing fraction of the cores particles when it is completely packed or it is fully filled as far as possible. And this is the condition we said is there when you want to find the maximum packing fraction. And therefore when we say $(PF)_a$ equal to this, we are actually saying that it is completely the coarse particles have taken all the major positions and the fine particles are in the pores only. And therefore, the fraction of X_c which is X_c which is equal to V_c by V_t is equal to 0.64 which is the same as the packing fraction cores.

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$$= \frac{0.64 - (0.60)(0.64)}{1 - 0.60}$$

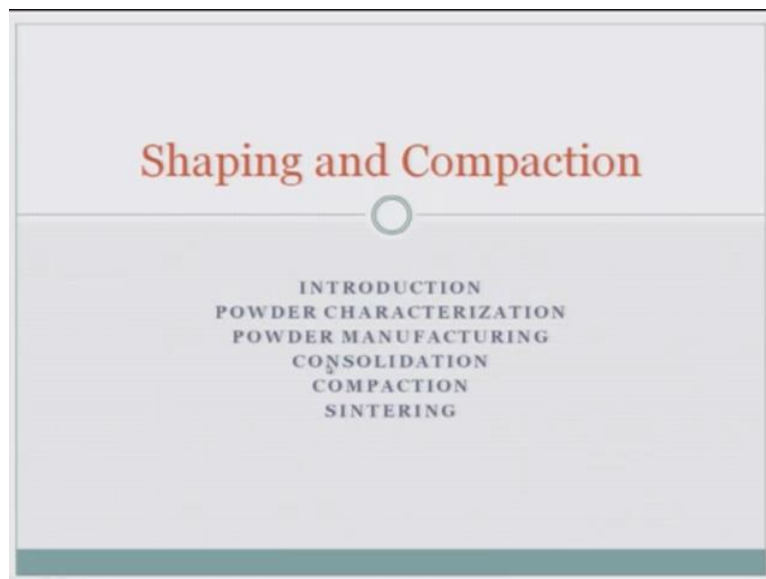
$$X_c = \frac{V_c}{V_t} \neq \frac{V_c}{V_c + V_f}$$

$$= 0.64 = (PF)_c$$

And that is why you can, you could have directly obtained that number but we wanted to show you through the steps of what is exactly going on over here. Another thing is that when we say X_c , this is not equal to remember it is equal to V_c by V_t but it is not equal to V_c by $V_c + V_f$. So if we wanted to place or put the x axis from the plot that we draw during the last lecture you will have to find this $V_c + V_f$ or in other words, you will have to find V_f . So you, that is our exercise that I will leave it for you to find what will be the value of V_c by $V_c + V_f$ at the point where you get maximum packing fraction.

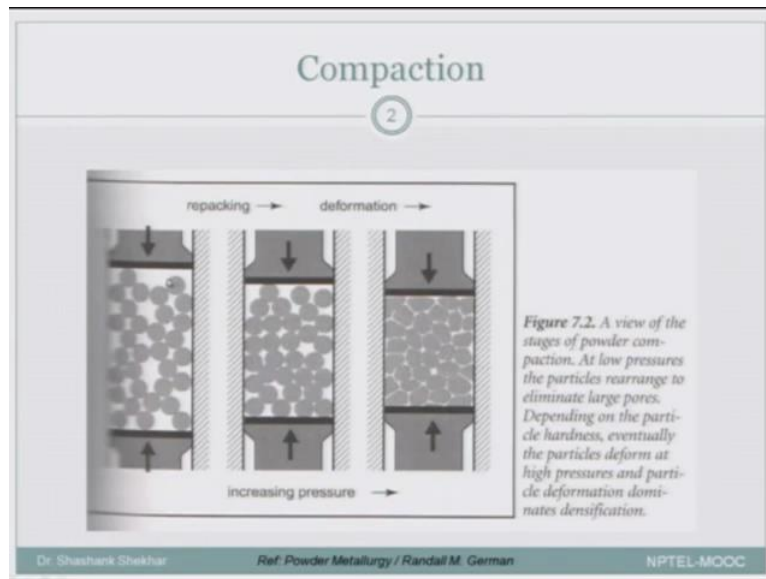
And we have not calculated the maximum fraction over here and it will not be very difficult, you can do it over here, now you know the cores, the fine and the all the parameters you would know over here, so you can put those over here and you would see that it will come to close to 0.74 something when you put those numbers. So again, you are increasing the packing fraction when you have the bi model grain size distribution.

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So with that, we come to end to this sub topic which was the consolidation and we will move on to our next topic which is compaction to have a perspective, let us see what all we have completed so far. We completed the powder introduction, then we completed powder characterization, we completed powder manufacturing, and the last topic, subtopic that we completed was powder consolidation. Now we move on to powder compaction, so powder compaction is the next step during powder processing.

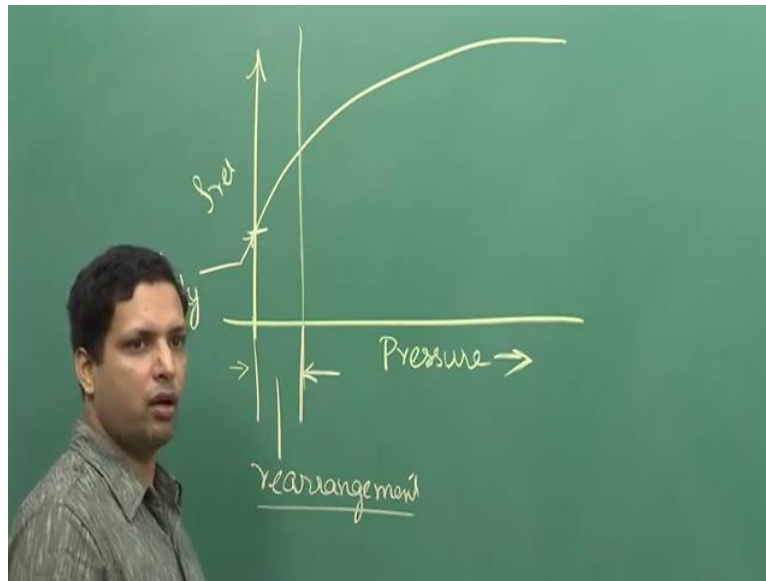
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What is actually happening over here? So in the consolidated powder what we have done is, taken away whatever remaining pores could have been just by taping. So let us say this is the taped order, meaning there are no more it cannot rearrange on its own anymore, but if you start to apply pressure on to it, so this is a piston on both the side on top and bottom and these are the powder particles and here we have for the sake of simplicity only small number of powders have been shown. And if you start to press it from the top and bottom, with just a little amount of deformation and maybe even negligible deformation, the particles will rearrange, they will rearrange in a way so that more space can be created.

And this will give a lot of a much larger improved theoretical density or improves packing fraction. So initially there will be a large increase in the relative density and if you keep on deforming or if you keep on applying the pressure, the deformation would start to increase and what happens when the deformation of the material increases? When the material deforms, it becomes strain harden and therefore it will become it will have a higher flow stress and so higher and higher amount of stress will be required to deform it or it would also mean larger amount of pressure would be require to deform it.

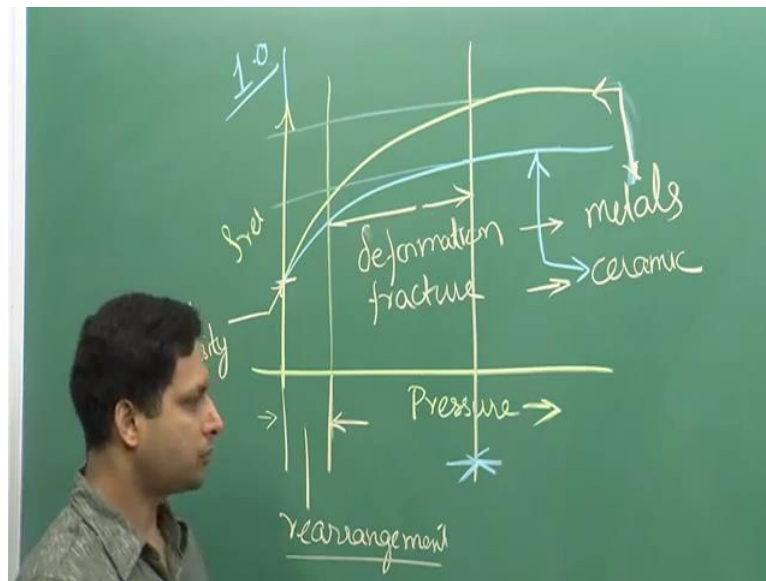
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So if you were to look at how the relative density will vary with compaction pressure and keeping in mind the picture that we have shown over here that there are two steps, at least two steps one of repacking and another of deformation, this is how our plot would look like, so let us say this is our relative density which is equivalent to our packing fraction and this is the pressure that will be applied. So this is how it will look like so μ here over here you have rearrangement going on, and what would be this value? This will be the tap density, meaning the density just after taping.

So the compaction starts where consolidation ends, so this is the point where consolidation ends and here you start to apply pressure to increase the theoretical density or the relative density even further and this, at this point only rearrangements are taking place and negligible amount of deformation is taking place. Beyond this, there is deformation which is leading to shape change in the particles so that they can accommodate the shape or they can adjust against in a better way. And as you, over here as you keep increasing the pressure, you would see that the amount of gain in the relative density becomes smaller and smaller and that is like I said because when you deform, then the material becomes work harden and therefore the flow stress increases.

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So you need even larger pressure to deform it any further. However one thing you have to keep in mind when we say deformation, this is mostly for metals, what for ceramics? For ceramics actually some amount of fracture will take place even for hard materials some amount of fracture will take place instead of deformation.

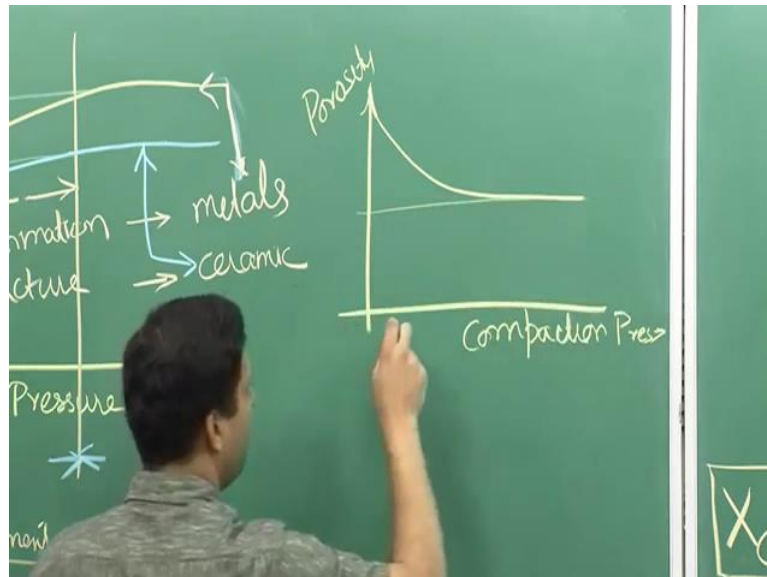
And therefore, the plot may be, say if you say we are starting from some about the same relative density, then the plot would be like this, which also means that the highest relative density that you get in the two plots or the in the two materials one in the metals and other in the ceramics will be very different. In the metals, you are able to get much higher, much higher relative density at the end while in the ceramics you will get a little bit lower density at the end of the pressure. And in another important aspect of this is that there is a limit of pressure that you can apply.

So this will be so μ here close to over here you will have the relative density equal to 1.0 that is the ultimate or the bulk density, but you will not be able attain that ultimate density just by compaction, that is the one important thing to note here, second thing to understand is that there is a upper limit of the pressure that can be applied even to get or even to get to the saturation range. If you keep applying pressure more than that, there will be negligible change in the density but you will have to apply much larger pressure for that.

So there, so you should apply pressure only up to certain limit, and that limit is decided by this plot, the point where it becomes saturated that is the point where you that is up to the point, up to which you should apply the pressure. So let us get back to, okay, there was one

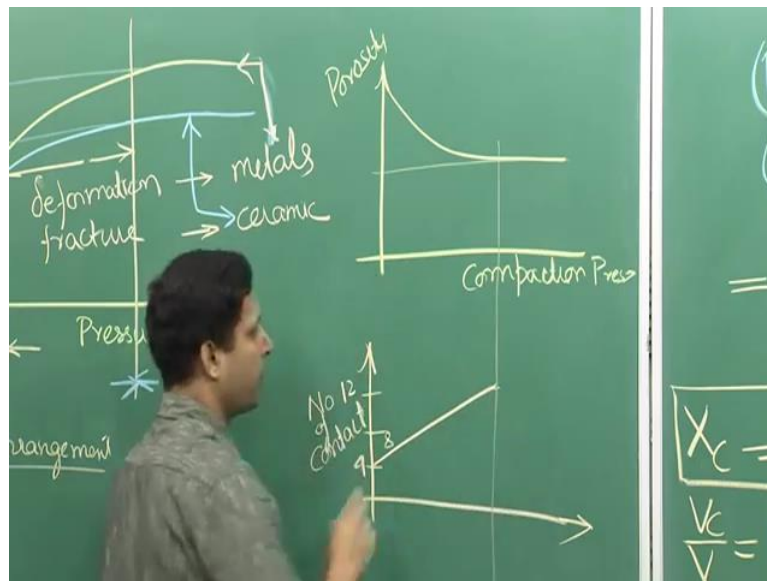
more thing before I move on. Now when you are changing the relative density, there are some other parameters that also change, for example if you look at porosity how should the plot for porosity look like? If the density is going like this then the remaining amount is what the porosity is. So the porosity plot should look like this.

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So here is our compaction pressure and porosity will not reduce to zero because we are not achieving 100 percent relative density over there it means porosity is also not reducing to zero percent over here. So porosity is one factor another is number of contacts. You remember for the highest contact point that we saw for a mono diameter particles is equal to 12 that was for FCC. So if you look at the number of contacts, the maximum you would get with increase in pressure would actually be of the order of 12.

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It will look like, it will not even it will be almost you can say linear variation with pressure up to the pressure, up to the point where you get saturation in the density. Because you are not able to get higher density, so you cannot increase contact points beyond that and this is how the plot would look like for the number of contacts with compaction pressure. So the what we see here is that pressure is changing not only the relative density but of course the other parameters which are related with relative density and if you can understand any of this, you would be able to understand the overall compaction process, so let us get back to our slide where we were looking at the compaction.

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Compaction

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- **Compaction is the process of densifying by application of pressure**
 - Initially by sliding of particles past each other (rearrangement of particles)
 - Then by particle deformation
- **Initial change is high with pressure, and later the change is smaller with increasing pressure. At higher pressure, the change in density is decreasingly smaller. Why?**

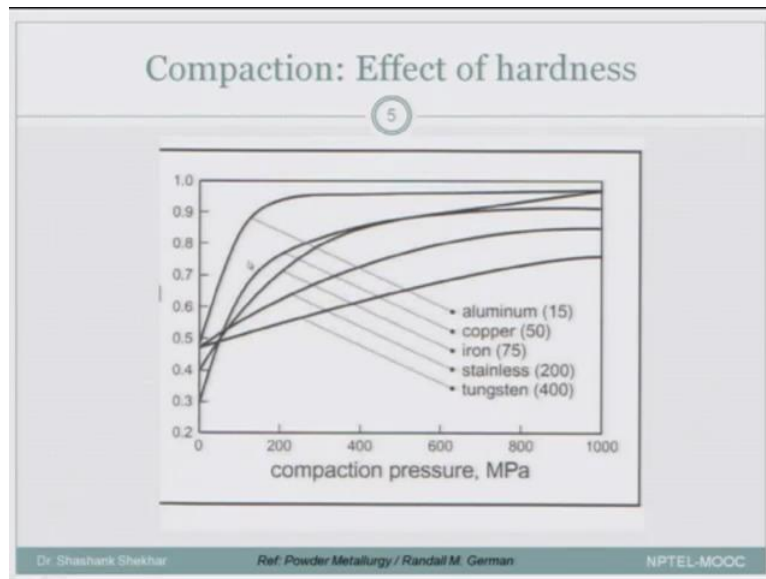
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And so this is what we said is our first region of repacking and then of the deformation. So let us move on, compaction now we are in a position to define it is the process of densifying by application of pressure. So what are we doing? We are densifying it, initially by sliding of particles past each other which we call rearrangement. So that takes place something like 10 to 20 mega pascal or up to 10 to 20 mega pascal of pressure. And in the second stage it is by particle deformation, in case of metals and in case of ceramics, it will be little bit of fracture of the particles. Initial changes high with pressure and later the change is smaller with increase in pressure.

At higher pressure, the change in density is decreasingly smaller, why is that? We already discussed that it is because of work harden particles. So compaction has two steps, rearrangement and deformation or fracture as in ceramics fracture and in metals it is deformation. And because of that, we said that there is highest density that can be achieved in metals and ceramics are different. And here we have shown, we have given some numbers what is the maximum green density that is after compaction density that you can obtain.

In metals, it is of the order of 85 percent, in ceramics it is much lower than metals because we are doing, and we are not able to get deformation only some amount of fracture. So it is less than 60 percent, not only that even materials with different hardness they behave differently, for example, softer materials attain higher relative density at much lower pressure and we will see a plot in just a moment. Harder materials on the other hand require much higher pressure to attain significant relative density, in fact there would behave more like ceramics and not attain very high relative density.

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And this is the plot that I was talking about, so here you see this is aluminum which we know is one of the soft materials, copper is also relatively soft, iron is hard, so you can see that the plot does not change much and tungsten which is one of the hardest material you can see it behaves very much like a ceramic there is just some amount of rearrangement but almost negligible or very small amount of deformation and because of that, the maximum relative density that you achieve in tungsten is also very poor compare to let us say aluminum over here you are getting just by compaction you are able to get 0.95 relative density or 95 percent relative density.

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Friction Analysis

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- Friction causes varying pressure across the pattern, along the depth, as well as along the radius
- Pressure variation across the part leads to Green Density variation
- Density gradient is not as large as Pressure gradient, but it is still detrimental to the overall shape
- Density gradient can cause varying shrinkage

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So this hardness also has a important role to play other than the class of material, the hardness also has a role to play. Next what we need to, what we want to look at is friction analysis. Now when are talking about compaction, what are we doing? We are actually compacting powder particle particulates inside some die. And inside that die it is not as simple as it looks that you just press it in desired form and you get the shape. After compaction, it needs to go to sentry, now if the pressure variation inside the compact that you have made is not uniform then what will happen is that different amount of shrinkage will take place.

And different amount of shrinkage will result in different or you can say distorted final product. So we what we need to do just take a look, quick look at understanding how this pressure can vary and one of the reason that pressure vary is because of the friction. So let us begin and let us look at the pressure variation that will occur because of friction. So we will assume a simple process where we have, we are just compacting something like tablet.

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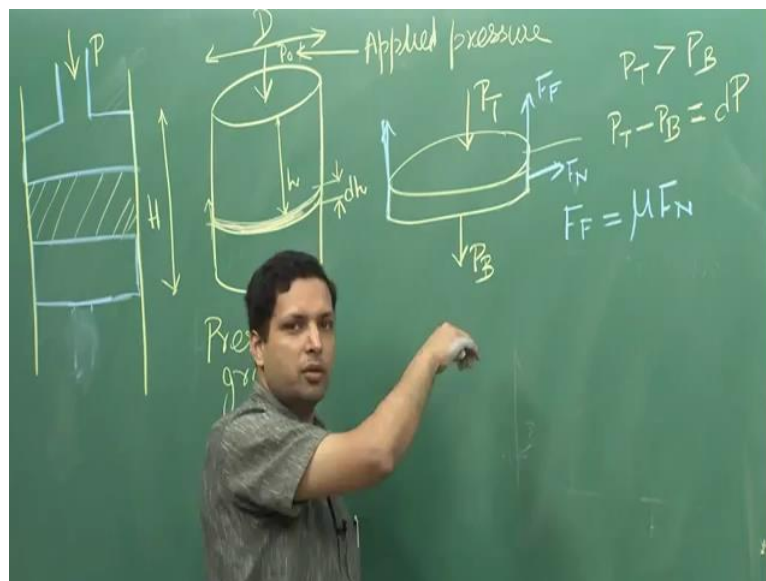
So for our simple case we are assuming we have something like this, these are our pistons and in between the pistons the pistons is our powder, so this is where your powder particles are, so this is the simple compaction process and to begin with we will even assume that we are only applying pressure from one side. So we will say that the pressure is being applied only from the top side, we will get to the more complicated state of the bidirectional pressure the next stage, first let us look at when we are applying pressure only from one side.

Now let us say we get compact like this which has height H and let say the diameter is D . Now let us do analysis at a certain cross section, thin cross section which is a distance H , h is

a variable over here small h is a variable and this is the section that we want to analyze. Now let us say this pressure that you have applied there is some pressure applied which is equal to P_{naught} , so P_{naught} is the applied pressure. Now when we are looking at this, what we want to understand is that there this whole thing is inside a cylinder or some kind of you can say piston.

So when you are applying, this things are moving and if there is some amount of friction, then this friction will oppose or act in opposite directions. So the particulates are moving in this direction and the forces will then move in the other direction and because of that the amount of pressure that is being applied here would not be same as the amount of pressure being applied here. In general what would see that the pressure would drop continuously, there will be a pressure gradient. Even qualitatively you can look and say that the pressure would be lowest at this point decreasing all the way from over here.

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And this is the case when we are doing, or when we are applying pressure from only one direction. So now let us take out these elements from over here and analyze it in more detail. So this is the thin section that we were looking at over here, so this has diameter D and the thin section we can say thickness is dh , so this is the thickness diameter image D and thickness is dh if the pressure that is being applied from the top is P_T and on the bottom it is P_B , then P_T will be greater than P_B . And you can say that $P_T - P_B$ is equal to dp or the change in the pressure Δp .

Now there is also some friction forces acting which is F_f , but this friction force would be equal to what? It will be equal to μ times n , if there is some there has to be a F_n acting in this direction. So μ times, if μ where μ is the coefficient of friction, then μ times F_n is equal to F_f . So F_f is nothing but μ times F_n . But how do we know what is F_n , F_n has to be also the normal force which is also acting in this con, in this direction has to be also proportional to the forces that are acting onto this particular thin section

So we have to take an average of the force that is acting on this point and it will there will be proportionality constant which will be able to give us F_n . So we can write F_n , so this is our stress, this is the overall area, so stress times area, this force that is the force that is acting and this proportionality constants z so this is our, this gives us our normal force F_n . Now if we know F_n , we can say what is F_f which will be this times μ . So F_f will be μ times z times P_t . Next is once we have all these forces, what we want to do is calculate or use the force balance equation.

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The chalkboard contains the following handwritten equations:

$$\sum F = A(P_B - P_T) + \underbrace{\mu F_N}_{F_f} = 0$$

$$\frac{dP}{P_T} = \frac{4\mu z}{D} dh$$

Boundary conditions:

$$h=0; P_T = P_0$$

$$h=h; P_T = P$$

$$dP = -\frac{\mu F_N}{A} \rightarrow \pi D^2/4$$

$$dP = -\frac{\mu z P_T (\pi D) dh}{\pi D^2/4}$$

$$dP = -\frac{4\mu z P_T}{D} dh$$

So next we will apply force balance on to this. We know that there is a downward force acting because there is a pressure and there is an upward force acting which is because of the μ or the friction. And the limiting condition these forces would be equal, so now let me use the force balance. So this is the force which is acting downwards and this is the force which is acting in the upward direction which is the F_f , so this is nothing but F_f which is working in the upward direction, this is the total force acting in the downward direction. And the sum of these forces would be zero and the limiting condition where it is in equilibrium or when the compact has completely reached its saturation state.

So over there we have this, now here we can write further that we will change $P_b - P_t$ to dp , so $dp = - \mu F_n$ by A , A is nothing but πD^2 by 4. So we take μF_n equal to F_f from here and therefore we have dp equal to, now over here π gets canceled, this D gets canceled and what we have is dp equal to $- 4 \mu Z P_t$ by $D dh$. So we have a dp term and a pressure term over here, so we will put this together under rest of the term or independent of dh , so we will be in the position to integrate it without any problem.

So we can write it as dp by P_t equal to $- 4 \mu z$ by D times dh . So we integrate from $h = 0$, at $h = 0$ we have $P_t = P_{naught}$ and $h = h$, P_t is equal to some variable pressure, pressure v which is P . So we will leave it over here I would, what I would like you to do is take this to the next step and come up with the equation or the final equation where P will be in terms of P_{naught} and the other parameters. And try this is on your own until we meet in the next lecture, so thank you.