Introduction to crystal elasticity and crystal plasticity
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Week-04
Lecture-08

Hello everybody! Let us start today's class regarding the continuum plasticity. So last class what we have discussed that was only first of for 1 dimension stress or strain relation specifically when we do different type of test, for example tensile test or (())(0:50). So we also discussed that the effect of the strain hardening effect, yielding, all this phenomena in terms of 1 dimensional stress state or in case of mostly called tensile stress-strain.

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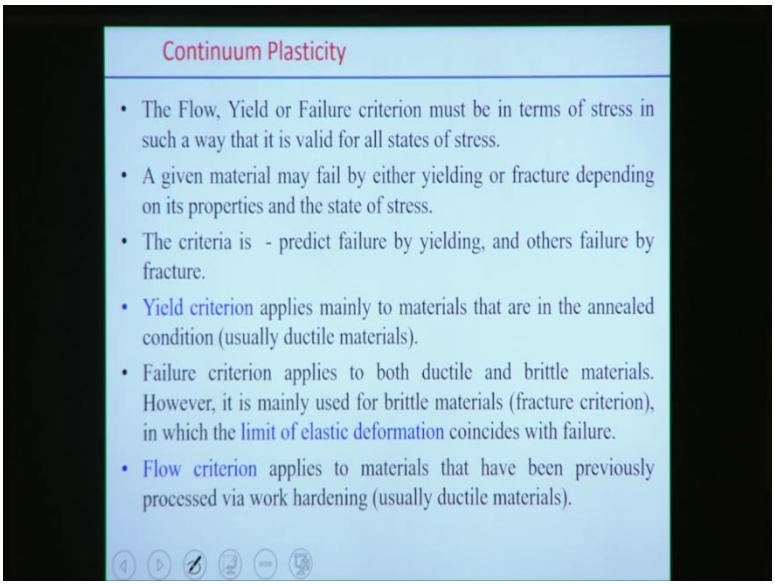

Now today we try to discuss the continuum plastic behavior of the materials in actual case, for example, if practically when the state of stress in 3 dimension then in this case how we can evaluate different plasticity model to represent the actual cases. We discussed that flow, yield and fracture criteria actually represented in terms of the stress in such a way that it is valid for all states of stress.

So of course here the uni–directional state of the stress also equally important here and that we will try to correlate in some equivalent form with respect to the state of the stress of strain. Second point is that a material can be consider failure either by yielding or fracture depending on its properties and the state of the stress applied to the specific material. So the criteria is to set that predict the failure by yielding and other failure mechanism but here focus is only on the plastic deformation of the material not the different failure criteria.

So yield criteria specifically applies to a material that is in annealed condition for the ductile materials. And the failure criteria applied actually both ductile and brittle materials however, failure criteria is mainly focused on the brittle materials in which the elastic limit of the deformation actually decides the failure criteria, specifically in case of brittle material.

But flow criteria applies to the materials that have been previously processes via work hardening mechanism and that is specifically ductile material. So along with the yielding criteria and the flow of the criteria basically the main focus of continuum plasticity and specifically in this case when it is subjected to 3 dimensional state of the stress.

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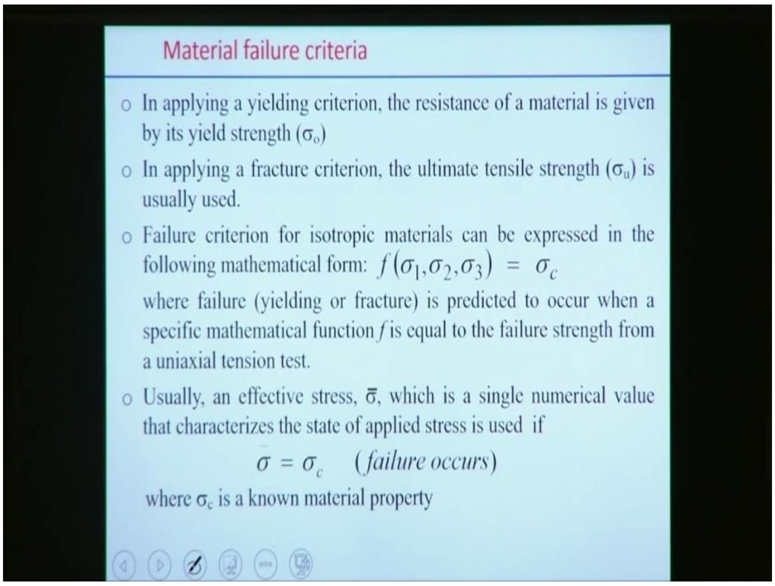

So first in applied any yield criteria the resistance of the material is actually limits by the yield strength and that yield strength is actually measures from the uniaxial tensile test. But in case of fracture criteria the ultimate tensile state is generally considered and that data is also extracted from the uniaxial tensile test, from specific material.

But failure criteria for isotropic materials can be represented in terms of the three principal stresses and when it is equal to the yielding and fracture in case of uniaxial tensile test data, then we can say the criteria in that way, or specifically to decide this culture we generally try to link between the 3 dimensional stress state to 1 dimensional cases.

But here the functional form, when it is the function of 3 different principal stresses that functional form need to decide here that better represents the plastic behavior of the material in case of 3 dimensional stress state. Now usually an effective stress which is the single characteristic numerical value that represents the 3 dimensional stress state and if we consider that equivalent stress called effective stress is equal to the 1 dimensional stress state.

For example here is the sigma C that is the whether it may be yielding or whether it may be the ultimate tensile strength that is decided to the set the failure criteria. And this 1 dimensional stress state whether yield stress or ultimate stress state is actually considered as the material property.

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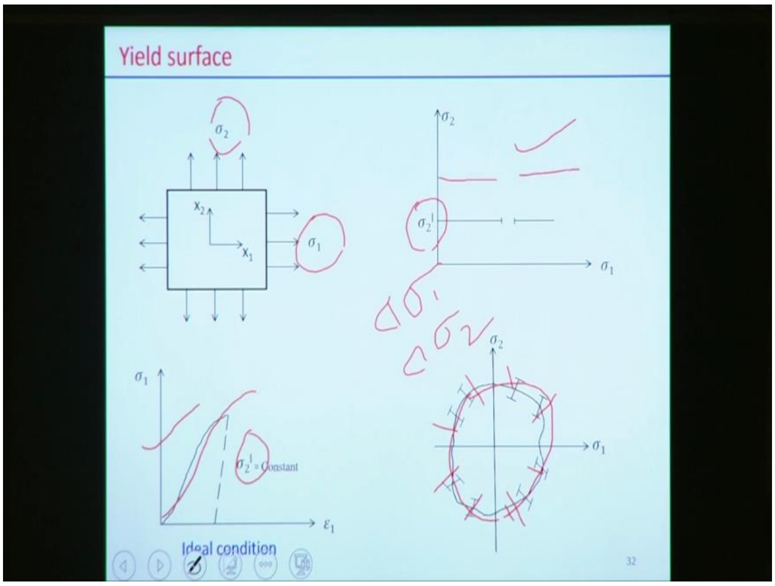

Now we can look into the 3 dimensional stress state, we can evaluate the yielding or yield surface specifically for different types of materials. So if you see the figure actually this figure, here the expression of the stress versus strain typical diagram, but when we do actual stress versus strain diagram of a specific material in a uniaxial tensile testing machine, in this case we keep the constant other variable for example, we keep the rate of deformation of the material as a constant, we keep temperature as a constant.

Then we evaluate the relation between stress and strain and for the specific other function parameters we can decide the yield stress value, we can decide the ultimate stress tensile value, we can decide the fracture strength for a specific material and that is very true in terms of uniaxial loading. But actually when we try to do, represent the biaxial stress state for example one element is subjected to the different state sigma 1 and sigma 2.

In this case how we can evaluate the stress strain diagram for this case. So 1 simple approach is that keeping other parameter for example sigma 2 as a constant and we can keep all other parameters and then we can conduct this tensile test and we can get that specific stress strain diagram and from that we can evaluate the yield form.

Similarly if we change the sigma 2 value then probably the stress strain diagram will be different. So keep on changing the other parameters keeping one parameter fix so we can evaluate the different stress strain diagram and may be the yield part will be different. So in this case it is represents on a scale that the curve represents the actually the different yield point on the axis of the two different stress, principal stress, here that is sigma 1 and sigma 2.

But since this change if we look into this figure also, here if you see that it is the curve actually stress strain behavior with respect to the sigma 1 and sigma 2 axis for a fix value of the sigma 2 dot that means this is the divided component of the sigma 2, so that values we are getting on constant line. Similarly if we change this value sigma 2 dot we may get other range of the stress where the yield point starts or yielding of the material starts.

Now keeping on the variation of this, in terms of like delta sigma 1, it is simply changing the value of successive different tests, sigma 1 changing the value successive for different test, we can (())(10:07) the several yield data point for the specific material. So finally when you try to represent these things it can become curve that actually represent the yield curve or if there is a 3 dimensional stress state we can say the yield surface can be generated in this way when we are considering the several stress by changing the stress level this different types of surface can be generated and we can say this is the start of the yielding.

But we can, it is better to represent the start of yielding point not as single point probably it can vary for range because here the variation of the stress, two different stresses we can conduct the test but at the same time if we change the rate so probably there is a small variation of the yield point happen so it is better to represent the surface not this thing, it may be scattered within the small range, that range is represent so we can represent the small range.

And if we try to fit one specific curve within that range probably we can get that yield curve or yield surface when it is subjected to not 1 dimensional case, when it is subjected to like 2 dimensional stress state or it is 3 dimensional stress state. But remember all these cases we are representing that state of the stress or maybe we can say that state of the yielding with respect to the principal stress axis.

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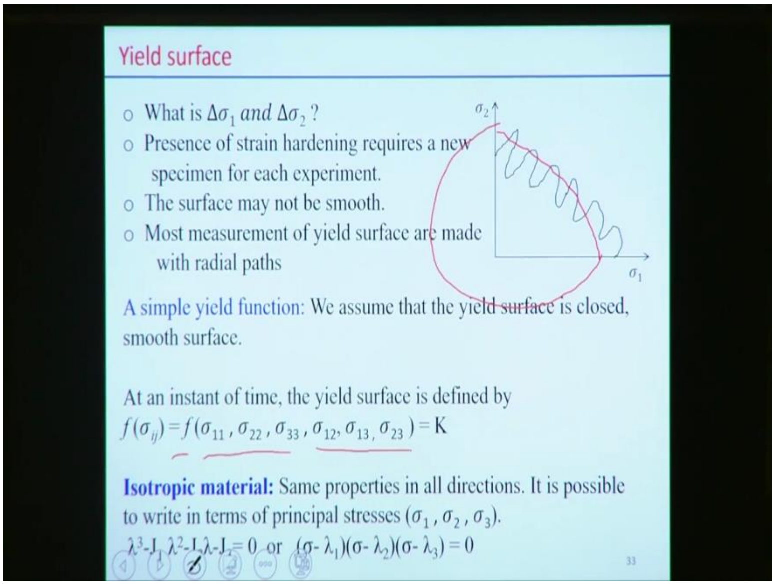

Now it is a point here that when you conduct the different tests by increasing either sigma 1 and sigma 2 then what maybe the implement of the consecutive test, so that should be the sensitive to the experiments, so probably if we do this variation delta sigma 1 and delta sigma 2 and over the stress axis it cannot be a very smooth curve that it can be a very zigzag kind of pattern but the variation depends on the what are the increment value delta sigma 1 or delta sigma 2 for the successive test we have consider.

At the same time presence of the strain hardening also requires a new specimen for each (())(12:49) during the experiment. So every time if we want to consider the strain hardening effect of the metal we need to consider every time, so another new specimen. So on this way it may be require a huge number of experiment to decide the yield surface of a specific material.

So the yield surface may not be smooth at the same time most of the measurement of the yield surface are made with the radial path, so that zigzag path, but this is the actual pure experimental data and it is obvious that if we want to predict the yield surface for a specific material we need lot of experiment and lot of data and cost also involved to conduct all this experiments.

So if we think in the other way, if we try to represent the yield surface in a mathematical form so we can define some yield function but in this case we can assume that yield surface is closed surface and it is a smooth surface then we can define the different functional form of the yield surface and accordingly we can predict the different theory, following the different theory the shape of the size of the yield function for a specific material.

Now the approach maybe like that, if we assume that the yield surface may not be in this zigzag, maybe it is smooth curve or it can be a close curve in this case we can define the functional form of the yield surface like that, f is the functional form, that f is the function of stress state and we can consider the normal stress as well as shear stress component, when we consider symmetric stress tension, so there is 6 component and that is equal to some constant value K.

This is the very general form, form to define the yield function for a specific material, now if it is isotropic material so that means the same properties in all direction, so it is possible to write in terms of the principal stresses so we can further reduce the function form of the yield surface not in terms of the exact, the 6 components of the stress rather we can represent in terms of the principal stress components.

It will be more easier to represent this thing and that principal stresses will know can't form that path, that cubic equation of the stress connected to the stress invariance so here j1, j2, j3 are the stress invariance and we can represent the equation is like that, lambda 1, lambda 2 and lambda 3 are the root of the this equation and that actually represents the principal stress. So it is more approachable to represents the yield function in terms of principal stresses as well as the individual 6 components of the stress.

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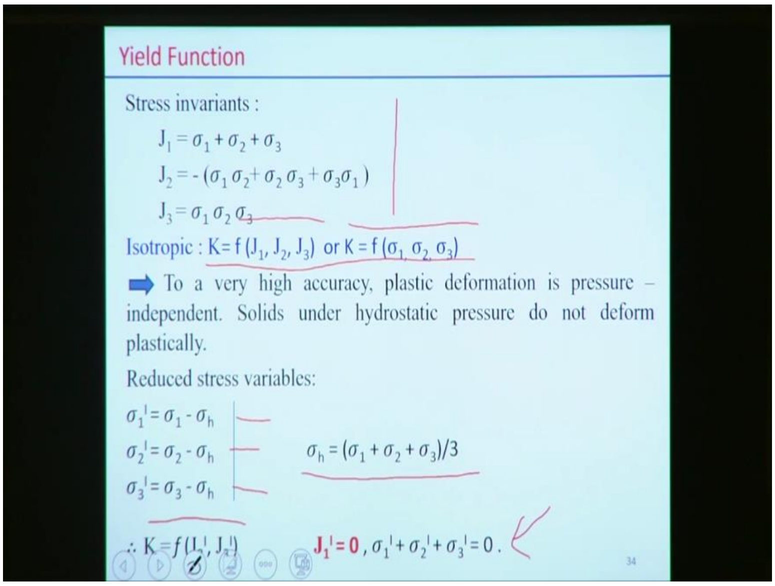

Now that we have already derive this is the stress invariance in terms of the principal stresses, sigma 1, sigma 2 and sigma 3. Now for isotropic materials since the properties are equivalent or properties are same irrespective of the directions then we can represent instead of the stress state, actual stress state we can represent either in terms of the principal stress or either in terms of the stress invariance, because indirectly the stress invariance having all the components of the stress state.

For example, the j1, j2, j3 can already represented in terms of the principal stress components. So both way we can represent the yield surface in case of isotropic material. Now we can further reduce the number of dependencies of the stress state or this functional form of sigma 1, sigma 2, all the stress invariance in that way, but if we consider the very high accuracy plastic deformation is pressure independent.

So that is generally observed in case of solids under hydrostatic pressure actually do not deform plastically. So hydrostatic do not contribute to the plastic deformation of the material so in this case we can further reduce the stress component in terms of the deviatoric stress component. So sigma 1 dot, sigma 2 dot and sigma 3 dot you see that this represents in terms of the deviatoric component and that sigma h is actually the hydrostatic stress component and that is the simply one third of the summation of sigma 1, sigma 2 and sigma 3. There are 3 principal stresses.

Now to do that, not let us say if the plastic deformation is independent over the hydrostatic stress state then we can represent the functional form is that way, that K equal to f of J2 dot and J3 dot, so these are the two stress, deviatoric stress invariant but it is not a function of the first stress invariant j1 because the hydrostatic stress component if we consider here then this j1 dot actually becomes 0 since the principal, this hydrostatic stress component of three principal stresses summation is equal to 0, that means the first stress invariant equal to 0.

So we can further reduce the function form dependency, now it only depends only on the two stress invariance j2 dot and j3 dot and that is also for the deviatoric components.

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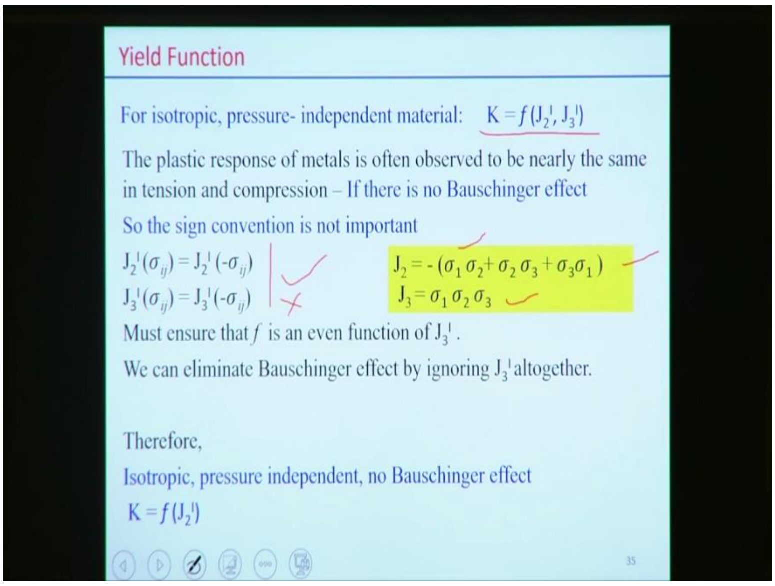

Now we can say that for isotropic pressure independent material that K is represented like this, function of j2 dot and j3 dot, this two stress invariance. Now the plastic response of the materials is often observed to be nearly the same in tension and compression that means if we consider there is no Bauschinger effect then mathematically we can say that if there is no Bauschinger effect that means yielding with respect to tensile and compression is same then in that case the sign convention is not important here that means it should follow like that j2 dot sigma ij equal to j2 dot minus sigma ij and j 3 dot sigma I j equal to j3 dot minus sigma ij so this mathematical expression actually significant or holds true when we consider that the yield point with respect to tension and compressive load same, that means there is no Bauschinger effect.

But if we look into the expression of the this stress invariant j2 dot j2 and j3 if we see that if you change the sign sigma 1, sigma 2 and sigma 3 so expression does not change for j2. But if we change the sign for sigma 1, sigma 2, sigma 3 this expression j3, in this case if you see that actually j3 changes the sign. So we can say that this convention actually not satisfied for the j3 dot that means what we can say that j3 dot actually not independent of the sign, it depends on the sign.

If there is a change of the sign from tension to compression, for example if we consider tension as a positive and compression as a negative sign, so if you change it but it is not satisfying in this case. So we can say that f should be must with the event function of j3 dot. So we can eliminate the Bauschinger effect by ignoring j3 dot. So but here actually j2 dot is satisfied by changing the sign but j3 dot is not satisfying, so we can neglect this if we consider the metal is not having no Bauschinger effect so therefore we can further reduce the dependency of the functional form, here it is the function of only the second stress invariant so K is a function of only j2 dot if metal, if metal is isotropic pressure independent and there is no Bauschinger effect.

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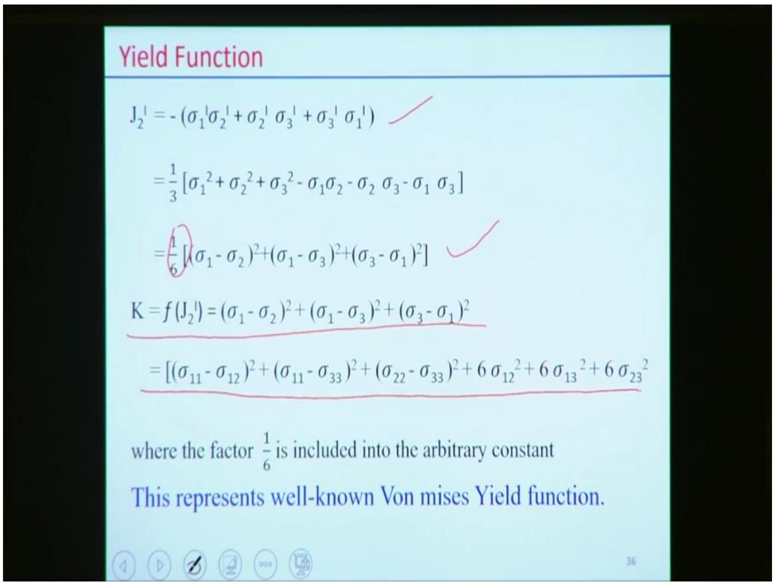

Now further we can derive the j2 dot stress invariant, it is like that and we can find out in terms of the principal stresses 1 sixth of this that sigma 1 minus sigma 2 square plus sigma 1 minus sigma 3 square plus sigma 3 minus sigma 1 square, so this is the stress invariant, maybe if we consider the one by 6 this is the factor, included in the arbitrary constant term.

So this is the actual functional form in terms of principal stresses or in terms of the actual stress state that is in terms of sigma 11, sigma 12, or actual stress tension. Now this expression, actually shape of the function is well known Von Mises Yield function so that is widely used in case of plasticity analysis or plastic behavior of the materials.

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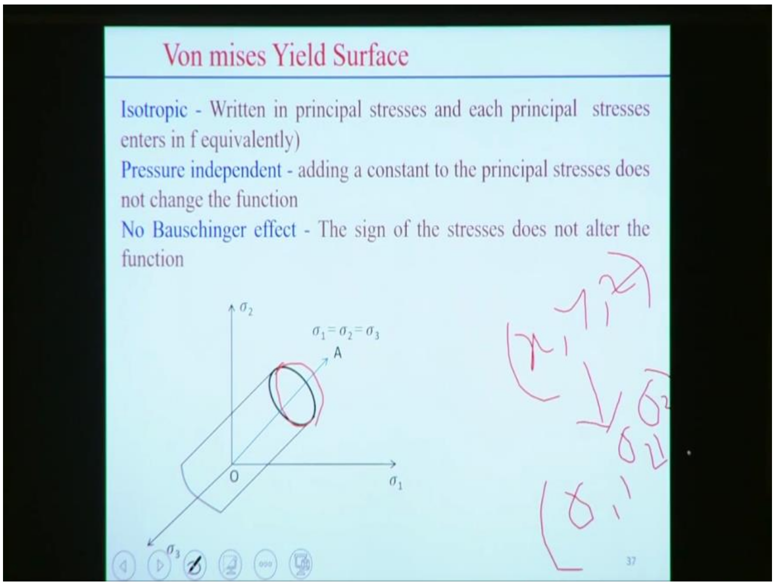

Now we can cross check this thing that by looking into the expression of Von Mises Yield function that if we look into the principal stresses and in each principal stresses if we look into that expression that each principal stresses actually enters in the functional form equivalently that means the, it actually assumption over the isotropic metal behavior is actually following by this confusion that all the term, all the principal stresses actually enters here equivalently.

Second point is that pressure independent, adding a constant term to the principal stresses actually does not change the function form. If we add the constant term with the principal stresses this expression it actually does not change the functional form that means this expression, this functional form is independent of the pressure. No Bauschinger effect – The sign of the stresses does not alter the functional form.

If you change the sign sigma 1 to sigma 2, sigma 3 then it does not alter the functional form, also it is the same functional form and we can retain it that means no Bauschinger effect is considered here. So this is the, so this equation actually is actually considered here. So this equation actually, this is a very important equation to represent the yield surface of a material, this is known as Von Mises Yield functional form.

Now if you do the further analysis you grpahically represent the yield function here, we can draw the one axis OA. On this axis basically making the equivalent angle with respect to sigma 1, sigma 2, sigma 3 and on this axis actually represents the sigma 1 equal to sigma 2 equal to simga 3. So with respect to that if we draw the surface that actually represents the yield surface in case of 3 dimensional stress state, but 3 dimensional stress state here is x, y, z is the actually (())(26:10) system when the initial stress state is define but here we are converting the stress state in sums of the principal stress sigma 1, sigma 2 and sigma 3. So on this stress state or on this stress axis sigma 1, sigma 2 and sigma 3 actually we are representing the yield surface here.

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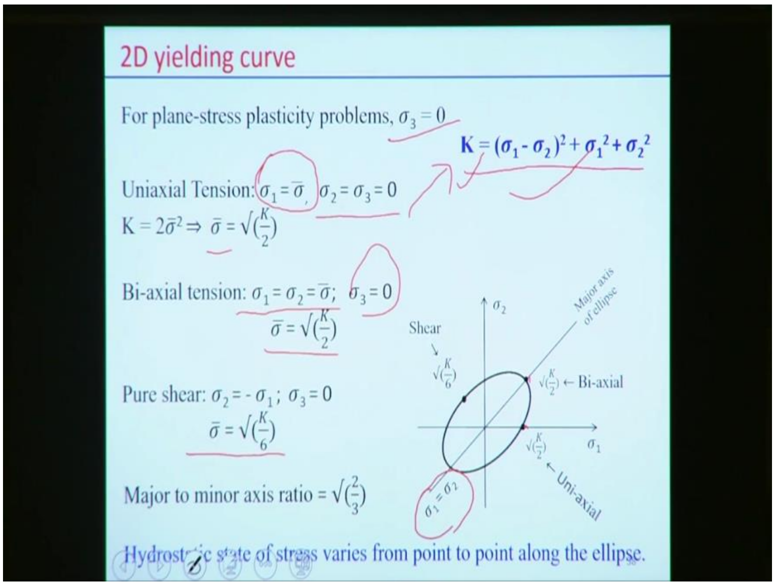

Now from this 3 dimensional stress state we can represent further in case of 2 dimensional yielding. For plane stress plasticity problem sigma 3 equal to 0, so that K can be represents like that, sigma 1 minus sigma 2 square plus sigma 1 square plus sigma 3 square. Now if this is the, actually K equal to this expression represents the functional form of the yield curve here. Now if we do the Uniaxial tension testing and if we try to put the condition here so Uniaxial tension testing for example, sigma 1 equal to sigma dot.

So one principal stress equal to sigma dot and sigma 2 sigma 3 equal to 0 from the Uniaxial tension test and putting this condition here we can find out sigma dot equal to root K by 2. So that means Uniaxial tension testing the yield point, if it is yield point is sigma (())(27:41) that is equal to root the power of K by 2. So we can represent replace the K in terms of the yield value in case of Uniaxial tension test data and then we get the functional form accordingly.

Similarly we have the tension, we have the data for the Bi-axial stress state and for example sigma 1 equal to sigma 2 equal to sigma bat and sigma 3 equal to 0, in this case but here you can find out sigma bar is actually root the power of K by 2. Similarly if we consider the Pure shear test, from Pure shear test in terms of principal stresses the condition is sigma 1 equal to minus sigma 1 and sigma 3 equal to 0, if we put these values this expression then we can find out that sigma bar equal to root the power of K by 6.

So depending upon the different test data, for example whether is Uniaxial tension test data, whether it is Bi-axial tension or whether it is Pure shear test, all this, the K value actually deviates. So in different cases different K values exist. So according to the availability of the tension test data or shear stress data we can (())(29:13) the yield functional form by putting this thing because this sigma bar actually we can directly evaluate from the test data during the actual test.

Now how this varies actually, all this data if you plot it on the axis sigma 1 and sigma 2 the two principal stress axis then see that this point actually represents the for Bi-axial root the power of K by 2, here also it is Uniaxial root the power of K by 2 because Uniaxial is along one axis, sigma 2 was 0 here, so that point exist at this point along the sigma 1 axis. Similarly Pure shear it was the sigma 1 equal to minus sigma 1, so at this point we can put the root the power of K by 6 so all these constant values actually represent the, can be plotted on a ellipse.

So the major axis of the ellipse is basically sigma 1 equal to sigma 2 and we can plot this thing, and major to the minor axis ratio is basically root to the power of 2 by 3. But in this case the hydrostatic state of the stress varies from point to point along the ellipse, so this curve actually represent the different constant value for the different test condition.

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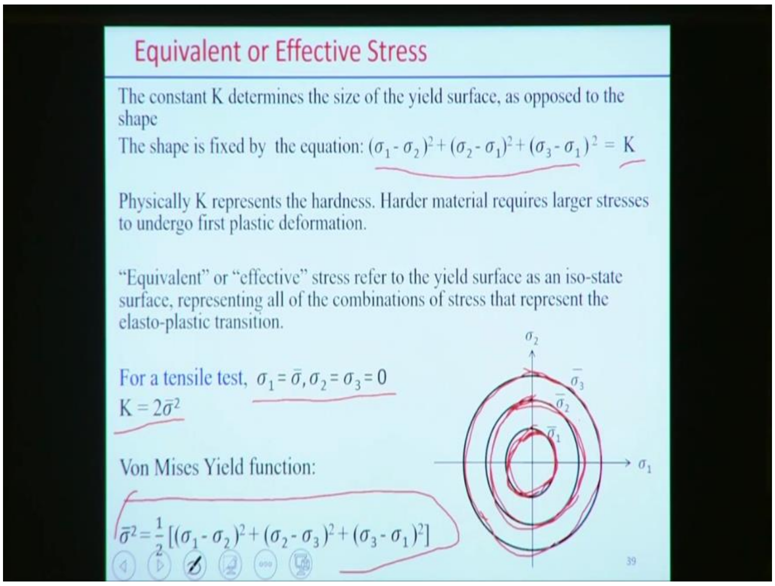

Now if we represents the actually the constant K determines the size of the yield surface as oppose to the shape. The shape of the curve is fixed but K actually decides the size. Now K can vary depending upon the load condition. So the shape is fixed by the equation that is true and which is equal to K constant. Now K actually physically represents the hardening, so harder material requires larger stresses to undergo first plastic deformation or if it is softer metal probably the K value should be the less. So depending upon the K value the size of the yield surface or yield curve can also be decided.

Now the point is that so equivalent or effective stress is basically the characteristic the value, basically we represent the one single value which is entirely calculated from the 3 dimensional stress state, it maybe in terms of the principal stresses. But here the effective stress refer to the yield surface of an iso-state representing all the combinations of the stress that represent the elasto-plastic transition. So let us look into for a tensile test that we have already discussed these conditions.

So K becomes twice sigma bar square, so Von Mises Yield function becomes likes this. So sigma bar square is equal to half of this, so this is the functional form in case of Von Mises Yield function and the sigma bar actually represents the data for the Uniaxial tension test. So according to the value of the sigma bar we can represent the different shape of the size of the curve. So suppose this is the shape but size can be increasing if sigma, actually sigma bar increases for the different material.

For example one case the yielding starts at the values of the sigma 1 bar so this further equation of the yield surface, for the specific material, another material maybe the sigma bar is different, so this is the representation of the curve and sigma 3, if it is sigma 3 bar this is the representation of the curve. So basically here sigma 3 is greater than sigma 2 greater than sigma 1. So accordingly depending upon the values of the yield point we can represent the yield surface by simply changing the values of the K.

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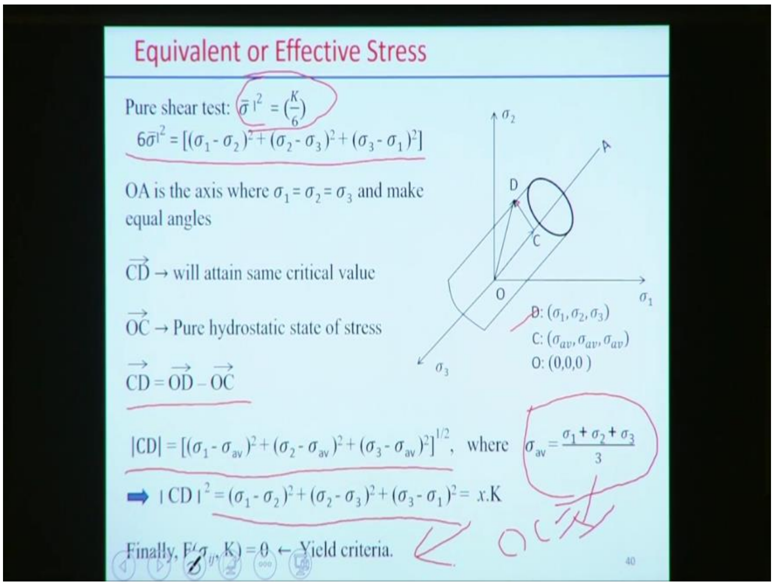

Now if it is Pure shear test for example we have the value or we can conduct the test in case of Pure shear condition so for Pure shear condition this are the relation between the K and the shear stress value then the representation of the yield surface in case of Pure shear is this is the function form which is different for the actual tension stress data. So that is the variation of the in shape and size of the yield function according to the type of taste or according to the data available whether it is shear stress data or what it is having the tension stress data.

Now graphically how we can represent this stress state, let us look into that what we have discussed that O is the axis that represent the sigma 1 equal to sigma 2 equal to sigma 3 and it is on the principal stress axis system sigma 1 sigma 2 and sigma 3. So D is actually state of the stress that is sigma 1, sigma 2 and sigma 3 and C point actually represent the average stress value so that average stress value, or maybe we can say the hydrostatic stress component here. There is a one third of sigma 1 plus sigma 2 plus sigma 3.

So basically OC represent the pure hydrostatic state of the stress in this case, and CD will attain some critical value because D point is actually represent, the actual principal stress value sigma 1 sigma 2 and sigma 3. So that actually represent the actual state of the stress in terms of the principal stresses sigma 1 sigma 2 and sigma 3. Now CD can be represented OD minus OC in the vector form, then CD can be calculated like that because according to the (())(35:55) so OC actually represents that this sigma average value sigma 1 plus sigma 2 plus sigma 3 by 3 and from there we know the coordinate C and D, so CD can be represented this value sigma 1 minus sigma average square plus sigma 2 minus sigma average square, so like this.

Now CD can be square can be represented this thing, sigma 1 sigma 2 equal to and maybe it can be represent some, x is the some numerical value into K. So here CD is the actually some critical value so, okay OD is actually represent the stress state that consist of the OC plus CD. Now looking into this thing we can find out, we can predict the yield criteria like that F is the function of sigma ij and K, that is the yield criteria, but K not only decides the yield point F but at the same time we can take care of the strain hardening effect in case of plastic deformation of the material. Now we see how we can look into that part.

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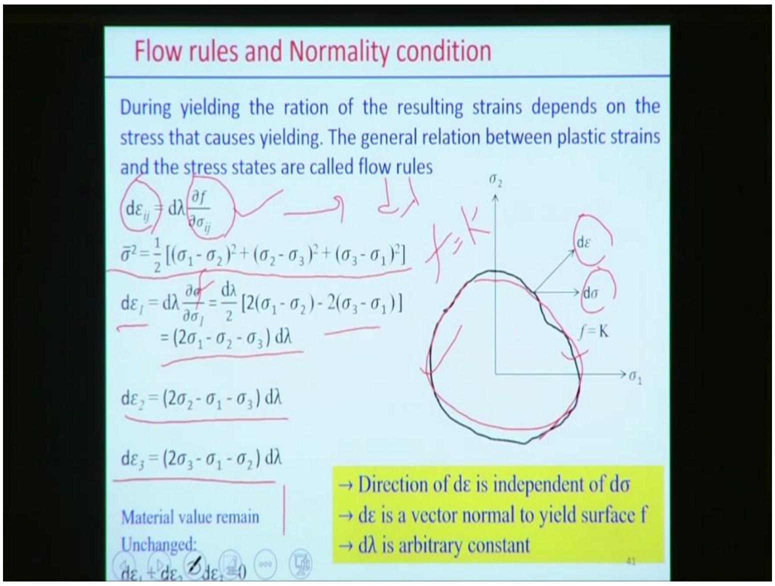

Now if we try to after identification of the yield point, of the specific material that means when it just cross the elastic limit then with the further deformation of the material it goes to the plastic stage and there is a increment of the stress value possibly due to the strain hardening mechanism. So in this case we can predict further the stress state, the plastic (())(38:02) also if we consider the effect of the strain hardening. But normally we consider the increment of the stress state with respect to the increment of the stress strain path.

So actually this analysis, this whole analysis is based on the how we can update the stress state considering the effect of the increment of the strain value in the plastic deformation (())(38:34). For example because of that we need to predict the flow rules and the normality conditions. So flow rules in the sense that during the plastic deformation (())(38:46), how we can sustain the plastic deformation and that can be better represented in terms of the flow of the material during the plastic deformation.

So during yielding the ration of the resulting strain depends on the stresses that causes yielding. The general relation between the plastic strain and the stress state are called flow rules or decided as a flow rule. For example suppose this is the yield curve on the stress axis sigma 1 and sigma 2, and on this yield curve with the further straining, with the further deformation of the material what maybe that increment of the plastic strain.

Now increment of the plastic strain is actually proportional to the, if we define some functional form of the yield curve or if we define the functional form of the yield surface, so that percentage of the yield function with respect to the stress state and proportional and that proportionality constant we can consider the d lambda, so basically d lambda is the constant value here, constant term here and that d lambda actually the sometime the deciding factor, what is the values of the increment of the plastic strain amount.

So it is a combines effect of the increment of strain actually depends on the what are the yield function we define, so what is the nature of the yield function or what is the mathematical form of the yield function and what is the stress state and some constant term that constitute the amount of the increment of the plastic deformation or plastic strain during the yielding of a material. Now this law states that the flow rule actually decided by the few factors yet the direction of the strain increment actually d Epsilon is independent of the d Epsilon.

So the increment of the strain and increment of the stress actually, they are independent with each other direction and this curve of course it represents one yield function, suppose f equal to, f is the functional form and equal to K, this is the shape of the that f decides the shape and K decides the size of the yield surface. Now d Epsilon is a vector, consider it as normal yield surface so we define the direction of the increment of the d Epsilon is basically normal to the yield surface and d lambda is arbitary constant.

So this is sometimes called the normality constant in case of plastic deformation of a specific material. So when you define this incremental mode of the plastic deformation as a function of this constant and the function form of the yield surface then we can further reduce or we can further deduce assuming the Von Mises yield function, then we can further manipulate the different information here that increment of the strain ratio here.

For example this we consider the yield function so f equal to K and that sigma bar is define basically in case of the Uniaxial tension. Now if we do these things and if we try to evaluate the d Epsilon 1 so either we can evaluate d Epsilon ij or in terms of the principal strain component also d Epsilon 1, 2, and 3, all these three components in terms of. So d Epsilon 1 equal to this formula here and del sigma by del sigma 1 principal stress and so I think it would be the f, function form here and equal to derivation we can find out this expression and finally we will be getting this expression.

So increment of the strain in terms of the principal stress value and the constant term d lambda, similarly d Epsilon 2 and d Epsilon 3. So this we can find out the ratio of the increment of the strain component d Epsilon 1, d Epsilon 2 and d Epsilon 3 from this normality condition, but at the same time it to be noted that all this plastic deformation we need to assume that the material value remains unchanged that means d Epsilon 1 plus d Epsilon 2 plus d Epsilon 3 equal to 0.

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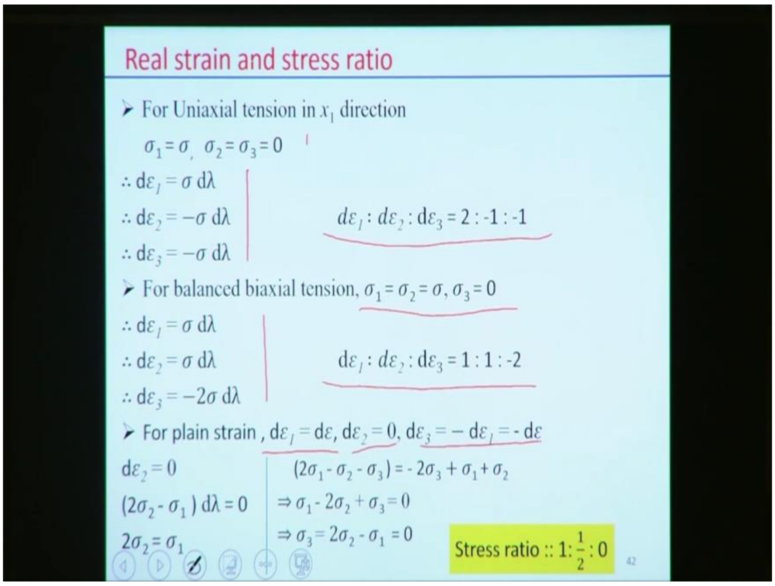

Now we can do further analysis on this thing, for example for Uniaxial tension stress in x1 direction that sigma 1 equal to sigma, sigma 2 sigma 3 equal to 0 so we can find out this increment of the strain Epsilon 1, Epsilon 2 and Epsilon 3 and further values can be evaluated like this. Similarly for Bi-axial tension we can do put this condition and we can find out this ration and from that we can find out all this ratio. Even for plain strain also d Epsilon 1 equal to d Epsilon and d Epsilon 2 equal to 0, we put it and we can find out the stress ration here, 1 is to 1 by 2 is to 0 or 2 is to 1 is to 0 like that.

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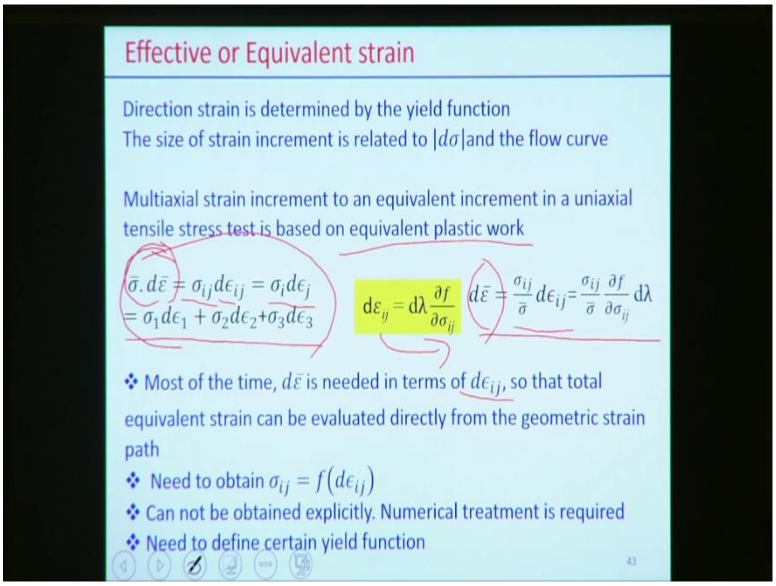

Now similarly like equivalent stress of we can find out the effective for equivalent strain component also so direction of the stress is determine by the yield functional form, that increment of the strain part is always normal to the yield function, the size of the strain increment is related to the increment of the stress value and the flow path, it depends on these two things, but Multiaxial strain increment can be represented to an equivalent increment in Uniaxial tension stress based on the equivalent plastic point.

So like the equivalent stress which represents actually the stress strain in terms all the principal stresses and that equivalent stresses one single value consist of the stress strain here. Similarly we can represent the increment of equivalent strain which consist of the increment of the strain in case of Multiaxial direction. So in that case maybe that amount of the equivalent strain can be evaluated from the Uniaxial tension stress data and from the amount of the equivalent plastic work.

So if we see sigma bar, if we say this is equivalent stress and d this is the increment of the stress, so that is stress and the increment of the strain that actually equal to the amount of the work done due to the increment of this strain value. So amount of the work done pet unit volume is represented like this, so this two cases the single value one is the stress and other is the incremental strain that represents in the actually stress strain and incremental of the stress strain, so sigma I by t Epsilon j that means in terms of the principal components.

So we can represent that sigma 1 d Epsilon 1 plus sigma 2 d Epsilon 2 plus sigma 3 and d Epsilon 3, this is the equivalent plastic work metal and from here we can find out the correlation like that so this d Epsilon bar equal to sigma ij by sigma bar into d Epsilon ij. So that we can further use the normality condition here also to bring the constant terms so this is the typical representation of this thing.

Now most of the time increment of the equivalent increment of the strain is needed in terms of the d Epsilon ij, so actual strain component of course in the incremental form so that total equivalent strain can be evaluated directly from the geometric strain path. Second point is that we need to obtain the sigma ij as a function of incremental strain path that cannot be obtained explicitly so numerical treatment is also required so in this case we need to define the yield function.

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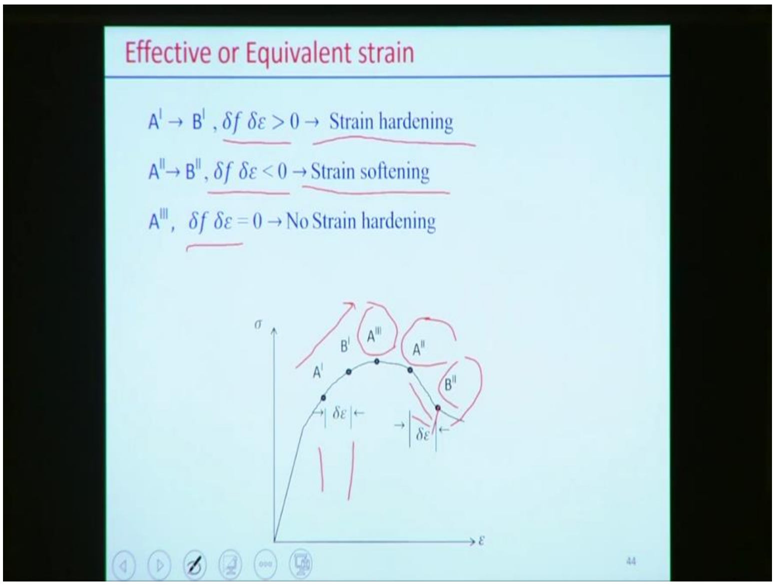

So let us see how we can represent this strain hardening effect here, so if we look into that from A dot to B dot with the increment of the strain amount delta Epsilon so in this case it is obvious from the graphical representation of the stress strain that there is an increment of the stress, so mathematical condition is that delta f functional form so delta f basically represents the functional form of the yield surface and delta Epsilon.

So multiplication of this thing should be greater than 0 so that represents actually strain hardening of the material. So strain hardening simply we understand that increment of the stress from A dot to B dot with the deformation, with the further straining of the material. Similarly if we consider from A triple dot to B triple dot here, sorry A double dot to B double dot so here that strength level actually decreases. So that is called actually strain softening here.

So mathematical representation of this thing delta f and delta Epsilon should be less than 0. Similarly at A triple at this point actually there is no increment of the strain so no strain hardening is happening at that point, so basically the increment is equal to 0, probably we can say this is the optimum point where there is no strain hardening. So after that A dot there is a increment of the strain due to the strain hardening and after that there is a decrement of the strain due to the strain softening.

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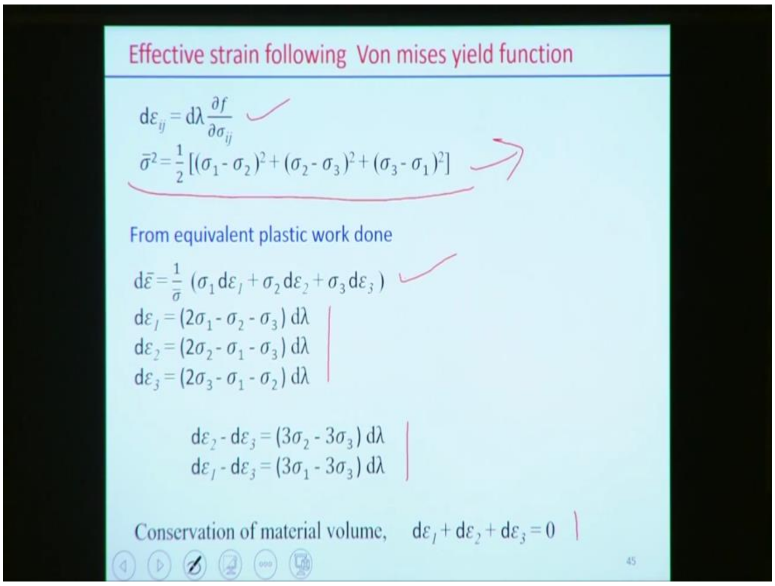

Now effective strain or better to say that incremental strain that can be derived further assuming the Von Mises Yield functional form, for example this is the plastic normality condition so that represents the increment of the plastic strain but we need to define the different parameters, the arbitrary constant d lambda and we need to define the functional form of f, yield functional form and suppose here we using the Von Mises yield function in tensile. This is the functional form.

Now we can find out that from the equivalent plastic work done that we have already derived the d Epsilon in terms of the sigma 1 d Epsilon 1 here. Now this is also we have already discussed the d Epsilon 1 equal to in terms of principal stresses and plastic (())(50:59) probably d lambda the constant term. Now with the manipulation we can find out this and also use this relation the material volume point of view d Epsilon 1 plus d Epsilon 2 plus d Epsilon 3 equal to 0 when the conservation of material volume is maintained here.

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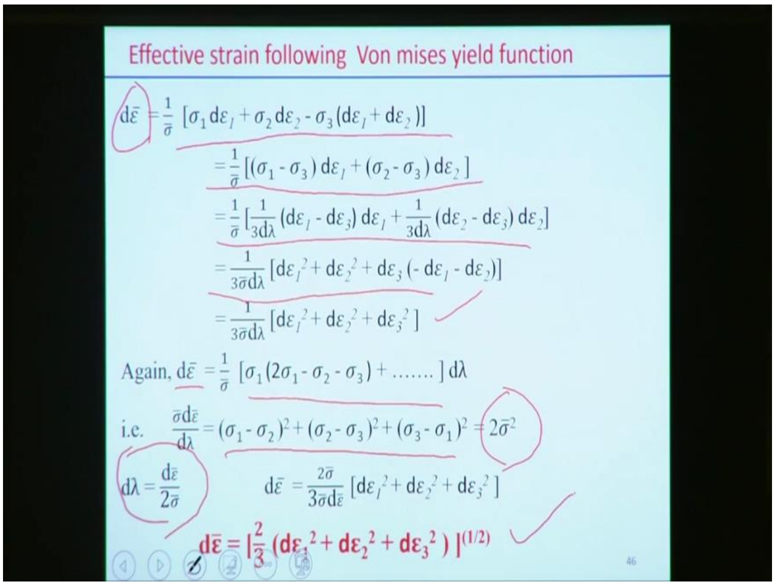

Now equivalent strain here in terms of the principal stresses and strain component we can do the manipulation here putting all these values, we put it finally we can get this value. Now incremental strain in terms of stress component it is there. Now if we do these things we are getting this expression that is equal to the sigma bar square and that comes from the Von Mises yield functional form for the tension stress. And then from this relation we can find out d lambda equal to in terms of this and finally we will be able to find out d Epsilon bar equal to 2 third of d Epsilon 1 square plus d Epsilon 2 square plus d Epsilon 3 square root of that.

So this is the amount of the equivalent strain in the incremental form, in terms of the individual component d Epsilon 1, d Epsilon 2 and d Epsilon 3. Of course this equation is valid if we assume the Von Mises yield functional form in case of Uniaxial tension itesting.

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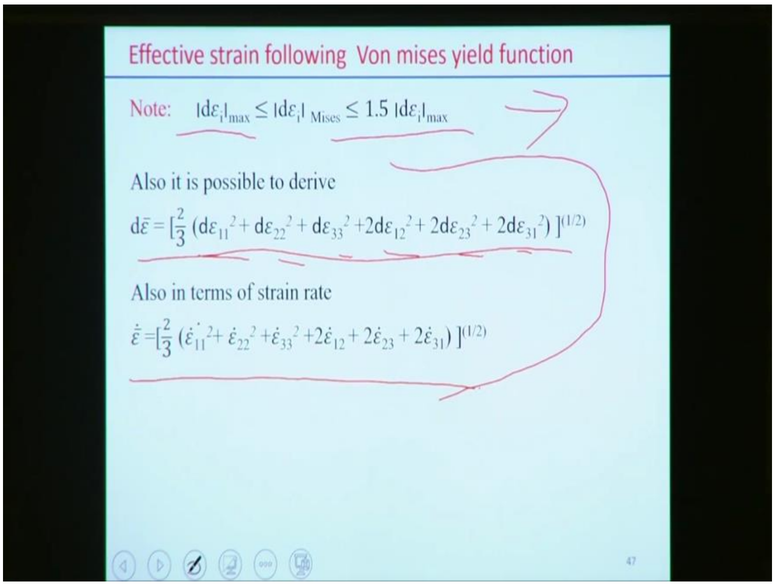

Now this expression is more practically representation of the equivalent form is more practically valid if the individual component max less than equal to d Epsilon I max or 1.5 times of the Epsilon into max, so this is just practical guidelines, the accuracy of this form of the equivalent strain in this case. Of course with the further manipulation it is possible to express this equivalent form in terms of the actual component 11 22 33, 12, 23, 33 if we know the geometric strain path and all the shear component and the normal strain component then we can further represent this way.

Similarly in terms of the strain rate we can represent the effect strain rate during the deformation in terms of all the strain component and of course all this expression definitely we are following the Von Mises yield function of form.

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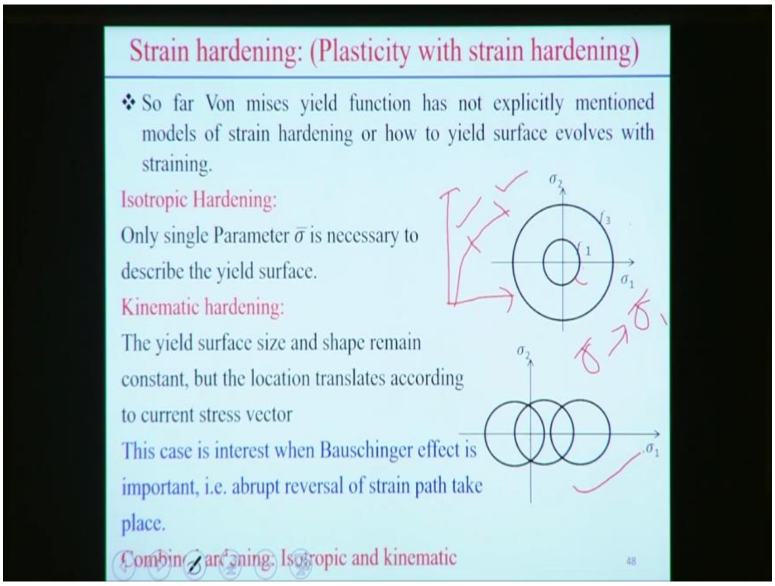

Now we come to that point strain hardening or basically we can say that plasticity with the strain hardening if we see that Von Mises yielf function has not explicitly mention of strain hardening or how the yield surface is evolved during the application of the load or during the plastic deformation stress that explicitly not (())(54:12) Von Miese yield functional form. The yield functional form just represents the shape of the curve, we can modulate it, we can decide it according to the different stress condition.

Now if we consider the effect of the strain hardening maybe we can see how this function of form actually evolve, first case is the isotropic hardening, more general case we use often use this thing, the only single parameter sigma y is necessary to describe the yield surface. Maybe this is the, if we look into graphically this is the one function of form f1 but the constant term K1 or sigma bar in cases, now if the sigma bar actually changes to sigma bar 1 for example, the shape remain the functional form f1 to f3.

Actually functional forms remain the same, only there is a change of the constant term, so that evaluation from one stage to another stage can be represent like that Uniaxial tension stress from here to here, so that mean from this point to that point is basically incremetn of the strain value due to the effect of the strain hardening. Similarly in case of Kinematic hardening the yield surface size and shape actually remains the same but the location translates according to the current stress vector.

If you see graphically the shape and size remains the same but the center point actually translates according to the new stress state, so this interesting in case of the Bauschinger effect is important to consider or that is the abrupt reversal of the strain path if we consider, in this case maybe Kinematic hardening model is the more appropriate, but certain material behavior in such a way that the material behavior can be represent both the models of the combined isotropic and Kinematic hardening.

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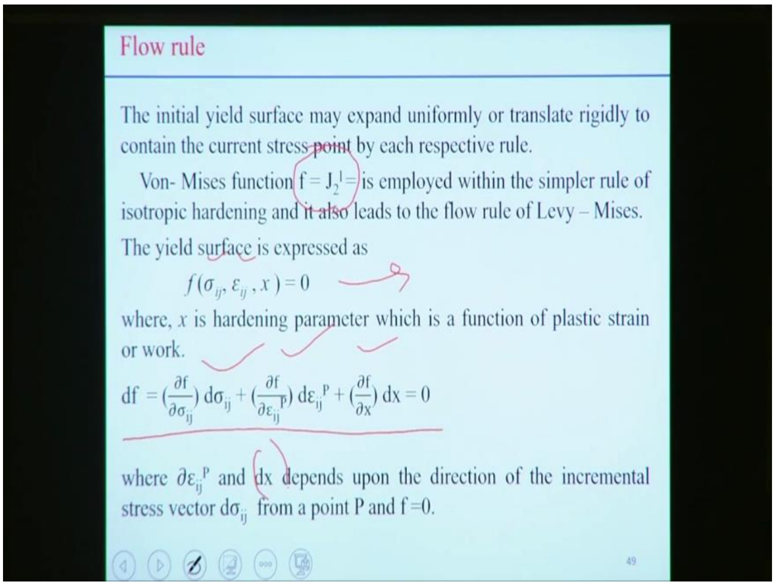

Now if we look into the flow rule, the initial yield surface may expand uniformly or translate rigidly to the current stress strain point by each respective rule. So once discuss that flow rule or else we try to represent in such a way that deformation of the material or flow of the material happens from the during the plasticity from one point to another point. So there are several flow rules exist, Von Mises function we represent simply that f equal to j2 dot is basically the second invariant, the deviatoric invariant, it represents the Von Mises yield function of form.

It is often used within the simply rule or isotropic hardening and it also leads to the flow rule of Levy – Mises. In this case the functional form yield, or maybe we can say that yield surface is represent in terms of this stress component and strain component and another is the hardening parameter. That hardening parameter we can find out (())(57:42) is the equivalent form of the plastic work done.

So this is the that simple function of the increment of the df in terms of the variable here. Where dow Epsilon ij plastic and dx actually depends on the direction of the incremental stress work done d sigma ij from a point p and f equal to 0. So that means f equal to 0 on that point and from specific point what is the direction of the stress vector incremental stress vector, the x or basically hardening parameter depends on that.

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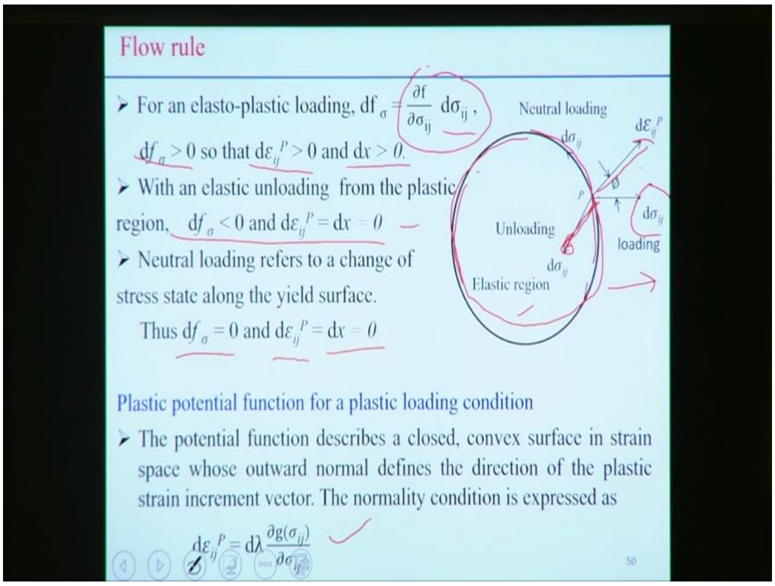

Here if you see that this is the expression of the Von yield surface for a specific condition and at a point t if you see that for a elasto-plastic loading that function of df keeping sigma as a constant in this case, this is the expression of this thing, del by dow sigma ij, d sigma ij, so in this case df sigma is greater than 0 one condition so that increment of the plastic strain positive greater than 0 and hardening also. So there is a hardening parameter so dx, change of the hardening parameter basically equal to greater than 0.

So this condition represents like that so this is the one specific functional form, so this side increment of the strain and this along this direction this is the d sigma j and making some angle Phi, so this represents the loading condition and that is the elasto-plastic loading condition. Now when plastic unloading happens so that means plastic recovery happens from the plastic region then that is the representation is that dx function of form is basically the (())(59:47) direction so that is less than 0 and increment of the plastic strain and increment of the hardening parameter is consider as a 0, so this is the physical representation of the…

Neutral loading is basically refers to a change of the stress state along the yield surface but increment of the df equal to 0, d Epsilon also 0 and hardening parameter also 0 so that path actually represents the neutral loading, there is no hardening effect or loading refers to a change of the stress state along the yield surface. So these are the typical representation of the typical loading and unloading condition when you try to consider this things in a plasticity model to analyze the plastic deformation of the material.

Now sometimes we define the plastic potential function for a plastic loading condition, the potential function describes actually closed convex surface in a strain space whose outward normal defines the direction of the plastic strain increment vector. In this case the normality condition is define like this which is same as we discussed earlier the normality condition during the plastic deformation but the functional form maybe different here.

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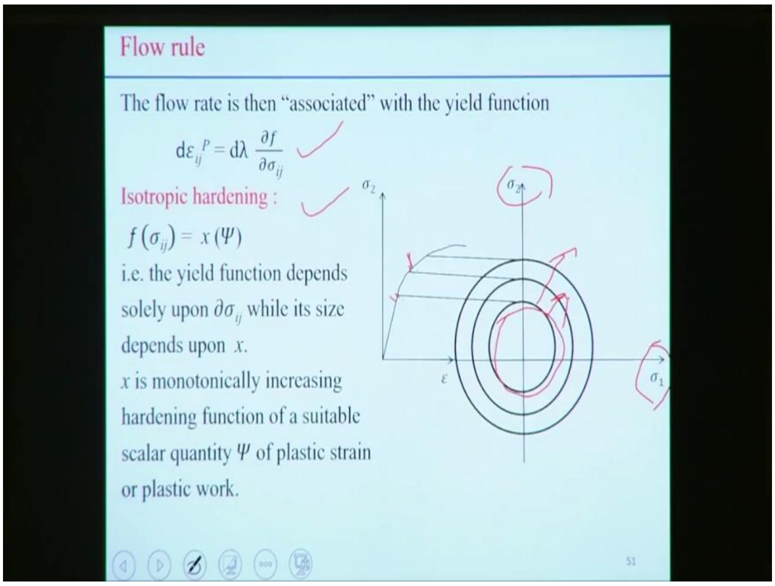

Now sometimes we use the flow rule is the associated with the yield function, it is better to describe or else decide the flow rule during the plastic deformation of the material that is associated with a yield functional form so that is the normality condition that defining and here we see the isotropic hardening, how we can represent this thing. So isotropic hardening means on this strain principal strain axis the represents the maybe first circle or maybe first functional form represents the corresponding to the yield point of the just for the elastic limit then the strain level actually increasing with the further deformation that is because of the strain hardening.

So if it correspondent to this yield point for the material this point and when we consider strain hardening of the strain so yield surface actually evolves this way in case of isotropic hardening, this is the graphical representation, this is the second point then when the yield point or strain level actually increases from first yield point to the second so that actually evolves the yield surface or change of the shape, sorry change of the size actually depending upon the hardening effect for this specific material. So this is the representation of this thing.

Now yield surface depends only upon of the increment of the stress whereas size actually depends on the hardening functional form of the hardening parameter. So x axis is monotonically increasing hardening function of a suitable scalar quantity Psi of the plastic strain or plastic work that if that can evaluated from the amount of the plastic work done in this case.

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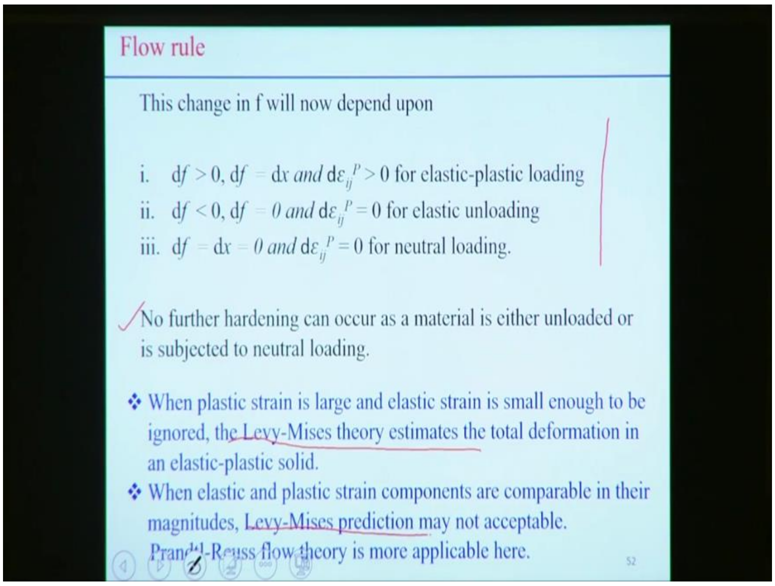

So change in f will now depend upon the del f, df greater than 0 or df equal to dx on these three different conditions elasto-plastic loading, elastic unloading and for neutral loading, that we have already discussed. Now no further hardening can occur as the material is either unloaded or is subjected to the neutral loading. So basically here we are not considering any hardening effect when there is unloading, unloading condition or whether there is a neutral loading, there is a importance.

Now when plastic strain is large and elastic part is very less, or small enough to be ignore then we generally consider the Levy-Mises theory to estimate the total deformation in an elastic-plastic solid. When we (())(64:13) elastic and plastic strain components are comparable in their magnitudes then Levy – Mises equation may not be acceptable, in this Prandtl –Reuss flow theory is more appilicable.

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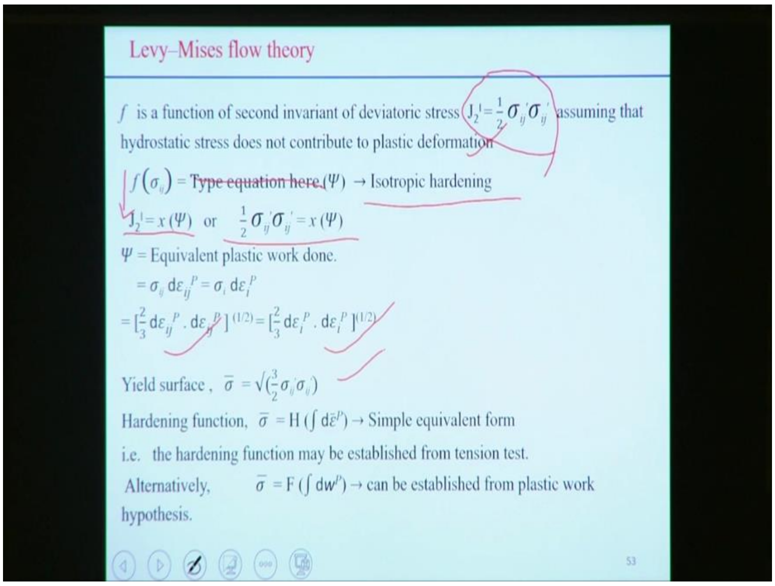

Now we try to get the overview of the Levy –Mises flow theory or Prandtl – Reuss theory, first we consider that if f is the function of the second invariant that we have used and the deviatoric stress component to derive the Von Mises yield functional form and assuming that hydrostatic stress does not contribute to the plastic deformation that is and no Bauschinger effect. Now this f sigma ij, this must be constant here, is a function of the Psi which is the isotropic hardening.

So in that sense the this function is basically equivalent to the second invariant of the stress invariant so that we can write this way or this is the expression. Now while the equivalent plastic work done we can find from the equivalent plastic work done and yield surface can be evaluated in terms of the deviatoric stress component here. So there is a several way to define the hardening function also so sigma bar, is a function of the hardening but this is the simple equivalent form so basically we can find out this hardening parameter for hardening function looking into the equal amount of the plastic work done. That hardening function can also be established from the tension test or also can be established from the plastic work done hypothesis also.

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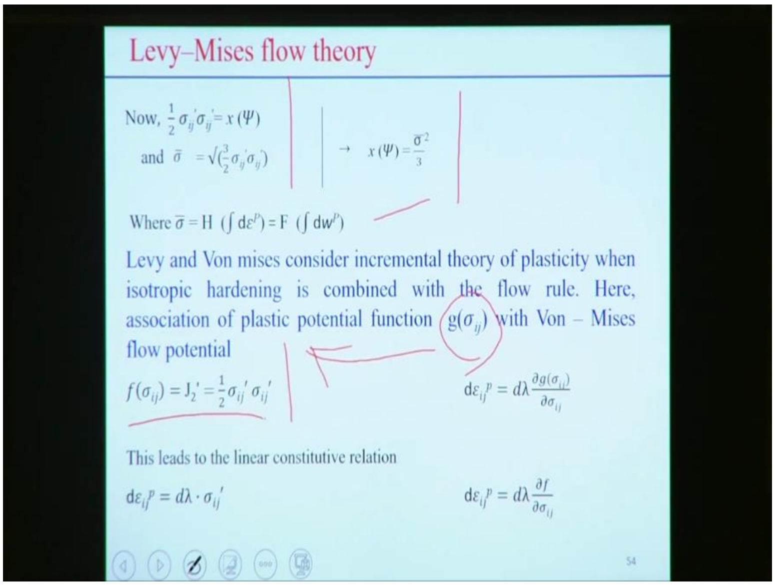

Now Levy-Mises theory we can look into this further expression whether the hardening parameter function of the form can be represent the sigma bar square by 3 but you can find out the sigma bar is the function of h or f like that. So Levy and Von Mises consider the incremental theory of plasticity when isotropic hardening is combined with the flow rule. But here the association of the plastic potential function with the Von Mises potential function. So basically the plastic potential function is here considered as the Von Mises yeild function in this case.

Now this Von Mises yield function the functional form in terms of the deviatoric stress component and of course second stress invariant and see that actually the increment of the plastic strain component is proportional to the deviatoric stress, we can use this relation.

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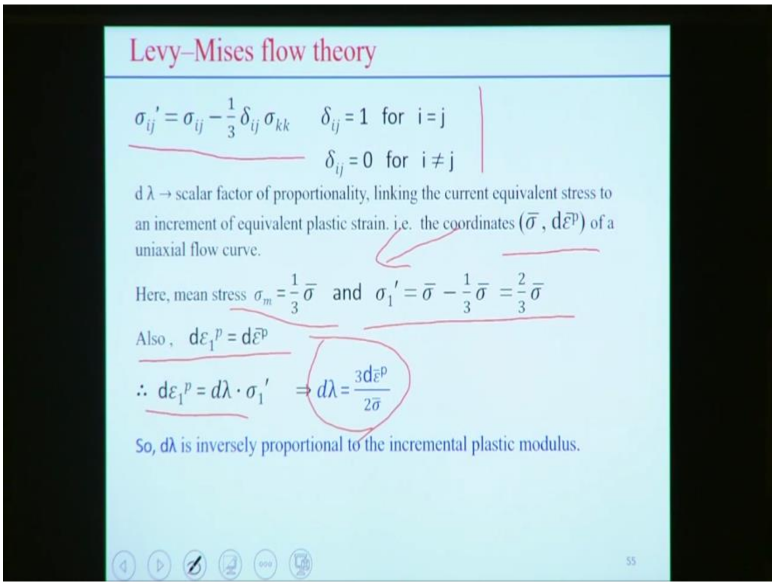

So we can finally find out this is the deviatoric stress component is the index form and we can write it in this way, d lambda is the scalar factor proportionality and that linking to the current equivalent stress to an increment of equivalent of the plastic strain where the coordinates from the curve is basically sigma by and d Epsilon bar of a Uniaxial flow curve. So that is link can be Uniaxial curve, we can use in the as a coordinate sigma bar and d Epsilon bar.

Now the mean stress here is the one third of sigma bar and this is the deviatoric component for the 1, for Uniaxial case and we can find out d lambda equal to this. So here we can see that d lambda is inversely proportional to the incremental plastic modulus. So d lambda can be evaluated actually if we evaluate the equivalent or effective stress value or equivalent or effective incremental of the strain and accordingly you can decide the this d lambda value in this case.

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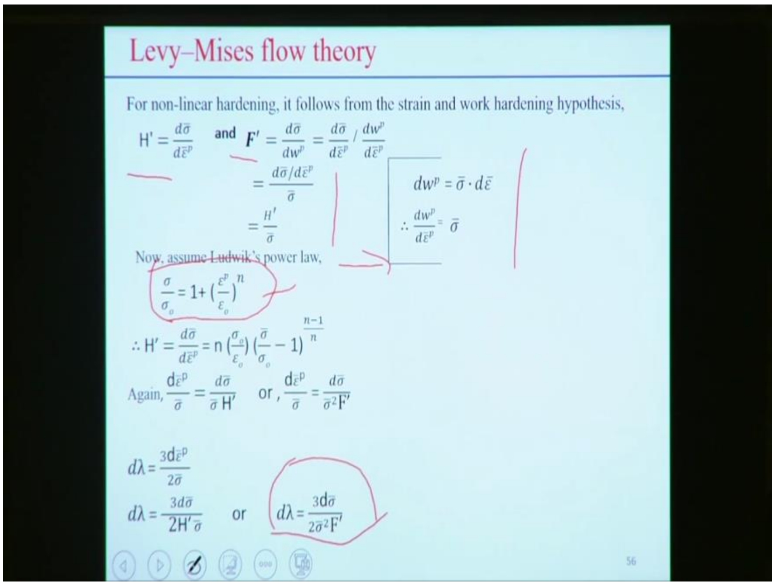

Now for non-linear hardening from the strain and work hardening hypothesis we can find out the functional from H and F, in terms of equivalent stress and equivalent work done, but all this can be derive if we consider the Ludwik's power law and this is the expression, we can find out, from this expression we can find out the d lambda in this case or we can find out simply, okay in this case we are trying to find out the d lambda here for this problem.

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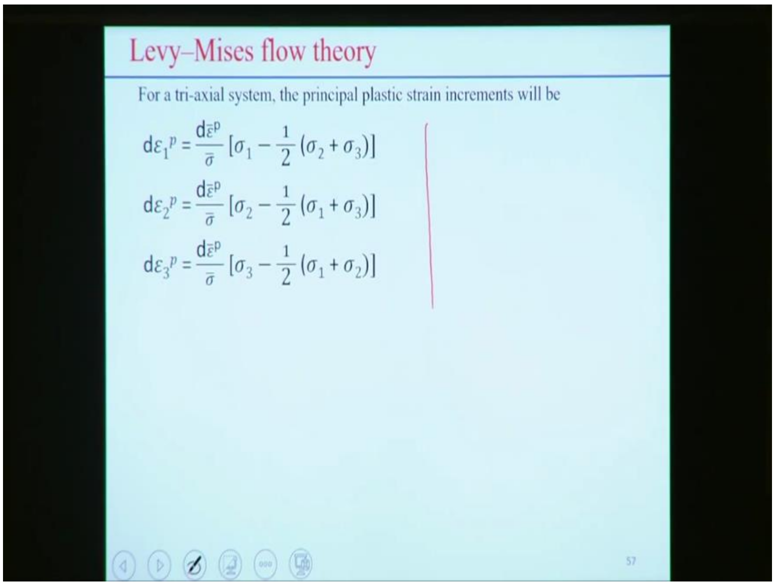

So for the triaxial strain system the principal plastic strain increments in terms of principal plastic strain increments can be represented in terms of the principal stress of equivalent amount of the incremental plastic strain and that corresponds to the fall in the Levy – Mises flow theory.

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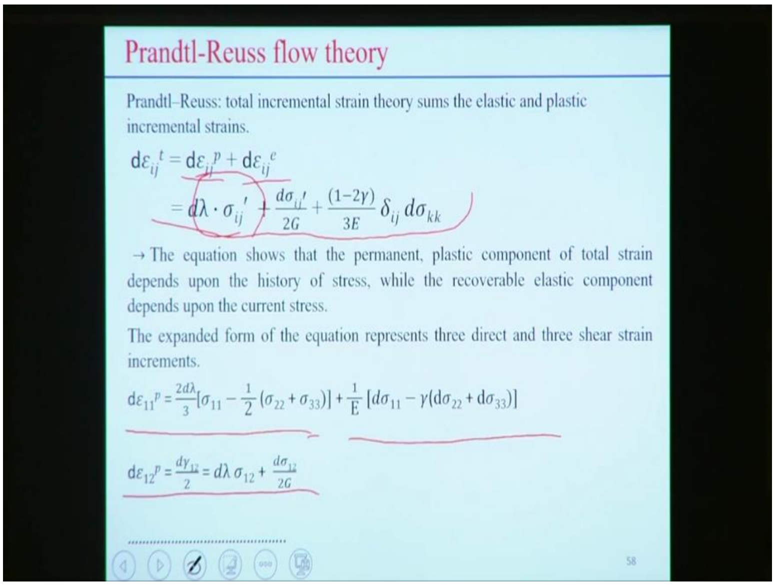

Now Prandtl theory telling that total equivalent strain theory, some both the parties there, elastic and plastic incremental part is there, so total incremental of the strain consist of the not only of the plastic path if there is elastic path and there is a plastic path as well. So in this case we can represent this is the, with the manipulation we can represent this thing, so first part represent the plastic path and second represent the elastic path. Of course this theory is basically applicable when you try to capture the elastic recovery during the plastic deformation that means elastic and plastic component here consider.

Now the expanded form or the equation can be represent like this similar, this is the plastic path and the elastic path and this is the shear component, so shear component consist only on the in terms of the single component is there.

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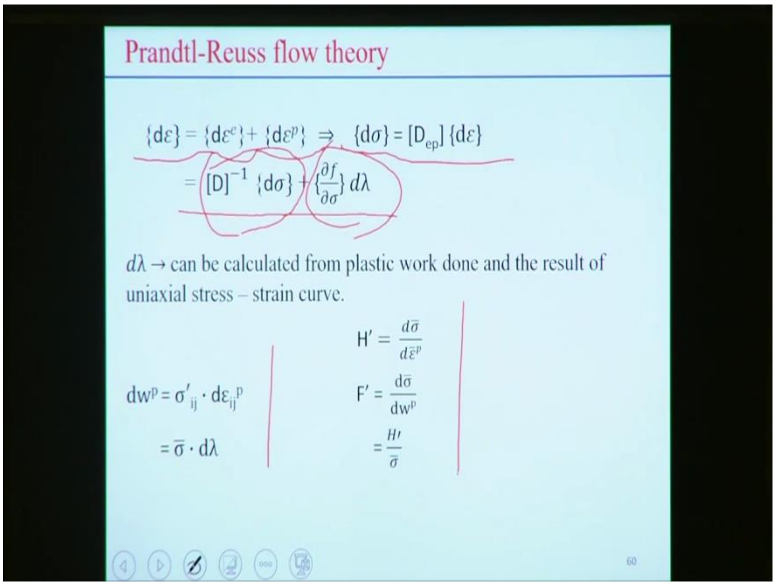

Now if we consider the elastic and this is the simple expression for the stress strain relation with the elastic limit and all the normal stress component, normal strain and shear stress and shear strain component, so we represent this component in terms of this expression stress, I think in terms of the column vector, d matrix is call Eveready elasticity or stress strain matrix and Epsilon is the strain component shear or the strain column vector.

Now according to the Prandtl Reuss plastic increment of the strain is actually proportional to the deviatoric component and we can define this d Epsilon and that deviatoric component can be represent as the (())(71:17) dow f by d sigma and that actually comes from the normality condition. So when f is definr actually yield function.

So basically this is the total strain of the consist of the incremental elastic strain and incremental plastic strain and then we can represent the stress in terms of Dep matrix with the incremental strain so finally we can say that d inverse d Epsilon that is corresponding to the elastic path and this is the plastic path. Now d lambda can be calculated from the plastic work done and the result from the Uniaxial stress strain curve so we can use this relation here to find out the that d lambda, from the d lambda actually actually here you can find out by looking into the Uniaxial stress strain diagram and putting that value we can do the further analysis.

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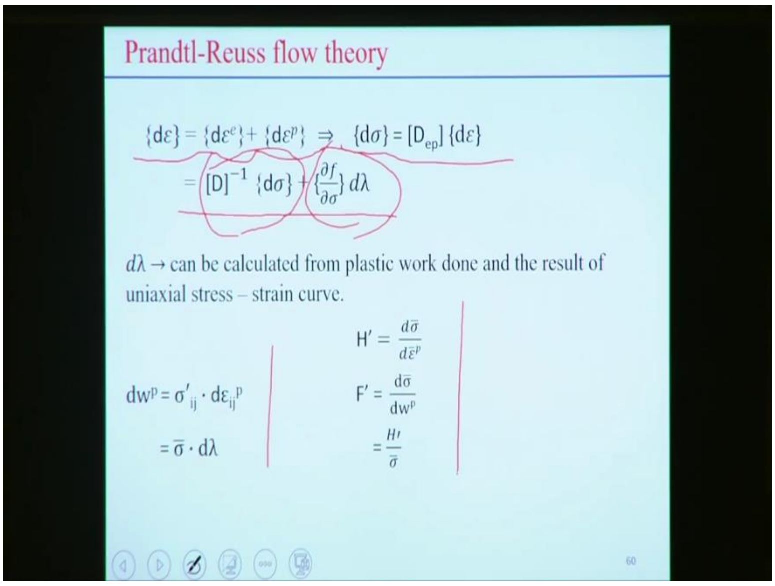

So that was the way we can do the plasticity model when we have considering during the deformation when both the elastic component as well as the plastic component earlier by following the Prandtl Reuss flow theory. So according to the practical application of the mateiral we can use the Levy Mises flow theory during the plasticity analysis or we can use the Prandtl Reuss flow theory. So this was just an overview how we can use the flow theory in the plasticity model.

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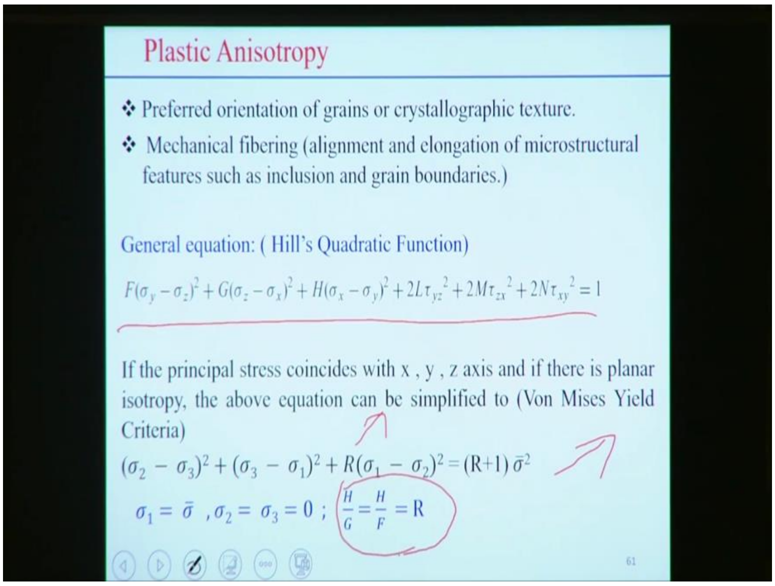

Now we come to that point plastic anisotropy cases, here preferred orientation of the grain or crystallographic texture actually introduce the anisotropy plasticity within the material itself, sometime mechanical fibering that means alignment and elongation of microstructural features such as inclusion and grain boundaries actually introduces the anisotropy conditions in terms of plastic deformation, so general equation that is the Hill's Quadratic equation is represented this way so there are, here the in terms of the actual stress state, stress component in case of (())(73:41) system x, y, and z.

So all the normal stress and the shear component by introducing the different constant of f, g and h, l, m, n, it is possible to bring the anisotropy behavior by simply using the quadratic equation. Now with further simplification that if the principal stresses coincide with the x, y, z axis and if there is no planar, if there is planar isotropy then above equation can be simplified to this equation is basically following the Von Mises yield functional form, here this Von Mises in functional form introducing the one constant path, we can bring the planar anisotropy condition in during the deformation of the material.

So here if you see H by G, H by F is basically we have defined as a constant term R and as a Uniaxial tension test condition sigma 1 equal to sigma bar, sigma 2 and sigma 3 equal to 0.

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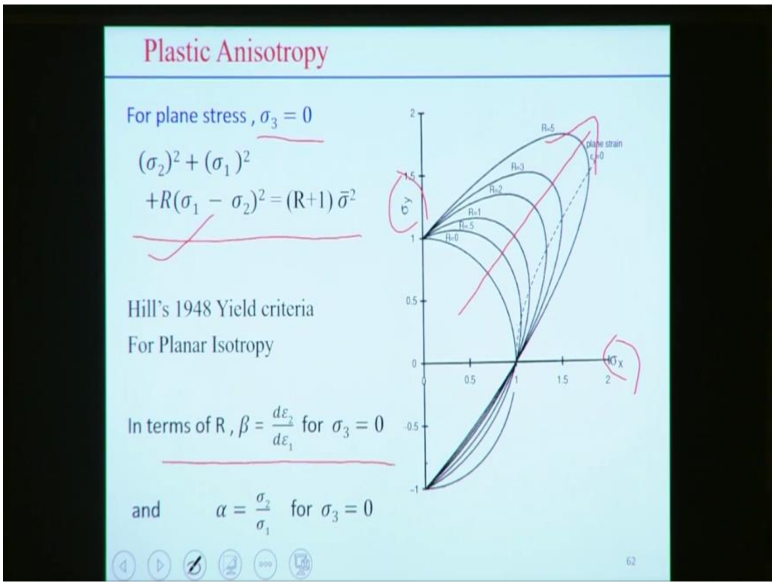

We can find out and of course for plane stress condition sigma 3 equal to 0, we can further modify this equation, for example in this case, for plane stress condition this is the equation of the anisotropic condition or anisotropic yielding. Now here the anisotropy actually introduced in the sigma x and sigma y, or I can say sigma 1 and sigma 2, by introducing the constant term R, and if you see there is the change of the R value simply if R equal to 1 is the simple represent the 2 dimensional Von Mises yield functional form in case of tension test.

But if R equal to if we vary the different R value, if R equal to 0 then it becomes a circle equation of a circle, and accordingly if we vary R equal to 1, 2, 3, 4 and R equal to 5 then we can see there is a deformation actually happens in one specific direction. So that direction is the represents the anisotropic behavior of the material and this represents the curve, this curve actually represents on the 2 dimensional, 2 dimensional case, it represents the effect of the anisotropy simply by changing the values of the R and represents the yield function of form at different values of the R.

Now this is called actually (())(76:54) 1948 yield criteria for planar isotropic, now we can further analyze in terms of the R we can do these things, we can define the beta is the ratio of the d Epsilon 2 by d Epsilon 1 for sigma 3 equal to 0 that means plane stress condition and alpha equal to ratio of the sigma 2 by sigma 1 for sigma 3 equal to 0.

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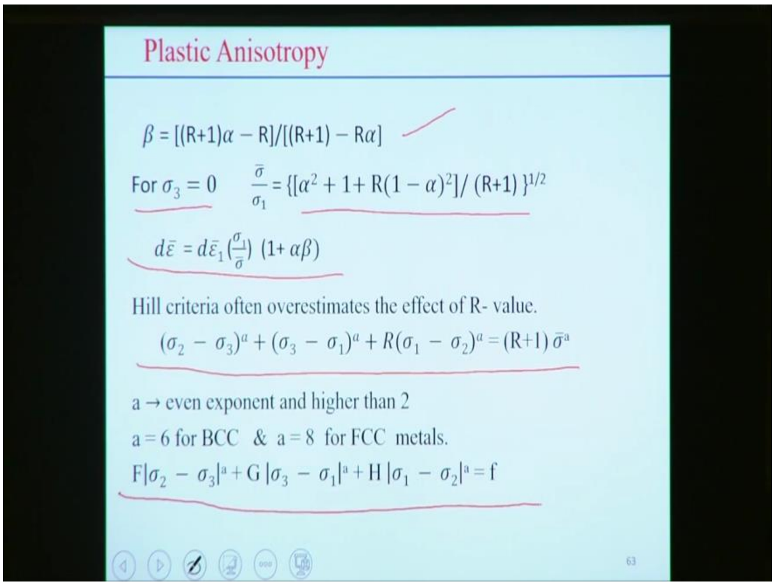

And then we can derive this formula beta equal to in terms of R or in terms of alpha, so if sigma 3 equal to 0 then we can find out sigma bar by sigma 1 in terms of R alpha and R, and finally we can represent the d Epsilon bar equal to 1 by sigma 1 by sigma bar 1 plus alpha beta.

So but in this case sometimes Hill criteria often overestimated the effect of the R value. Now the if by changing the x point we can represent the general form of the equation consider the isotropic and anisotropic behavior in terms of the changing the exponent A. A is generally considered the even exponent and which is higher than 2. For example A equal to 6 for BCC material and for A equal to A for the FCC material.

So that function of form can be further modified in this general way by introducing the F, G and H is three different constant to bring the anisotropic behavior of the 3 dimensional stress strain and the A the exponent is the different in all the cases. Now this has all about the theory of the plasticity I have to cover the very basic idea that we generally use during the analysis of the plasticity not much going into the in depth analysis of the different theory and their mathematical derivation, but looking into the knowledge of the theory we will try to focus on some numerical problem which can be covered here.

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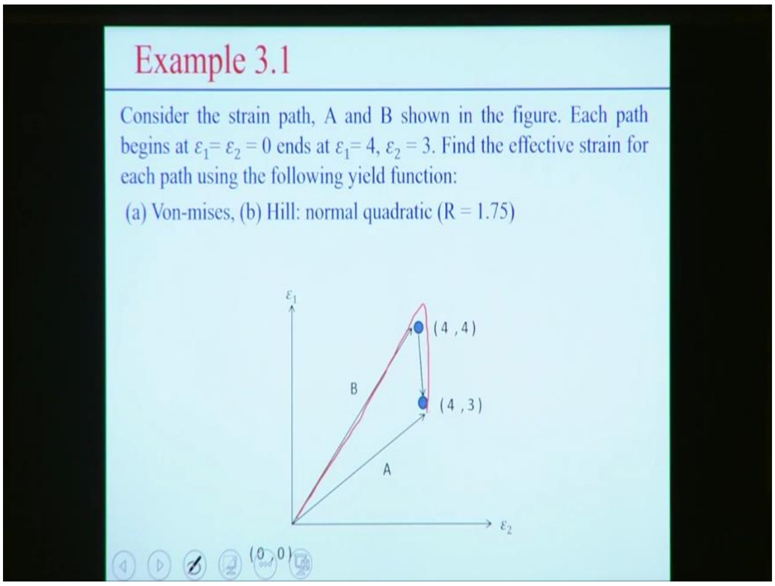

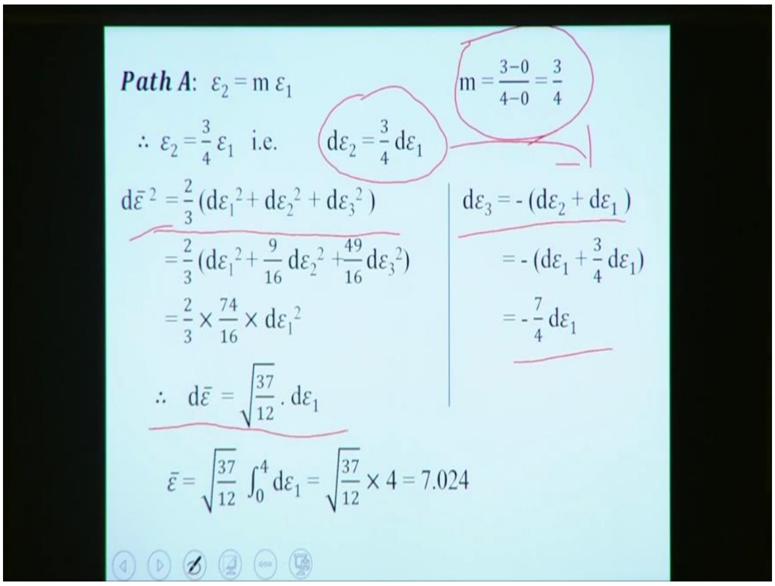

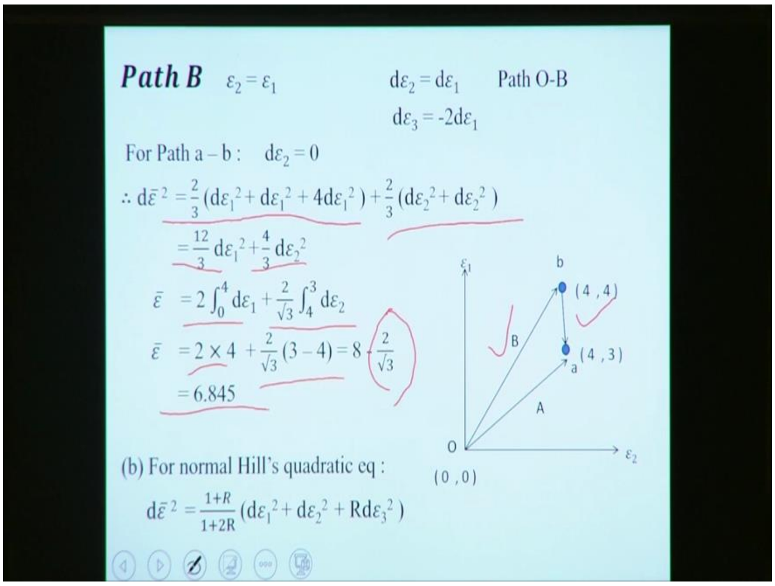

For example, first example if we consider the strain path A B shown in the figure each path begins at Epsilon 1 equal to Epsilon 2 equal to 0 and ends at Epsilon 1 equal to 4, Epsilon 2 equal to 3. Find the effective strain for each path using the following yield function. So this problem is like that this is the 0, 0 point and we define two path, A path actually it comes directly and another path is B path it comes from here to this point.

So in this two path we need to find out the effective strain assuming the there is a Von Mises yield function or assuming the normal quadratic there is a R. So to do that first we need to find out the correlation between Epsilon 1 and Epsilon 2. So if we look into the path A, Epsilon 2 is represent the straight line, Epsilon 2 equal to m Epsilon 1. So m the gradient basically can be obtained from the geometric point because this is 0 0 coordinate and this is 44 and 43 coordinate and this is 44 and 43 coordinate, so accordingly to the geometric coordinate we can find out the slope and from here we can find out the relation between the Epsilon 1 and Epsilon 2 and we can use this during the derivative and we can find this relation.

Now from here when another point is there material volume remains constant, so we can use the this relation and d Epsilon 3 in terms of the d Epsilon 1, now the effective strain in the incremental form, we have already derived and this, this is again it is valid if we consider the yield function as a Von Mises functional form, then to find out then we are getting this relation between the effective strain and this d Epsilon 1. Now if you look into that d Epsilon 1 actually varies from the 0 to 4. So if we put the (())(81:04) limit from 0 to 4 we can find out the absolute value of the incremental effective strain for the path A.

Now similarly for the path B we can similar analysis we can contact here, first path B consists of the two paths, 2 linear path this, and another one is this. So basically 10 to B, another is the B to A, so that path, so we can follow the similar analogy here and this is the path O to B and second path is the B to A, so we can get two components here and to the individual (())(81:46) here if we find out this thing, so remember maybe noted here that from B to A, that (())(81:57) limit from 4 to 3 so there is a decrement, so second component is actually negative component here. So finally effective increment of the strain rate is show. So by following the normal Hill's quadratic equation we can derive the effective strain rate in the similar way.

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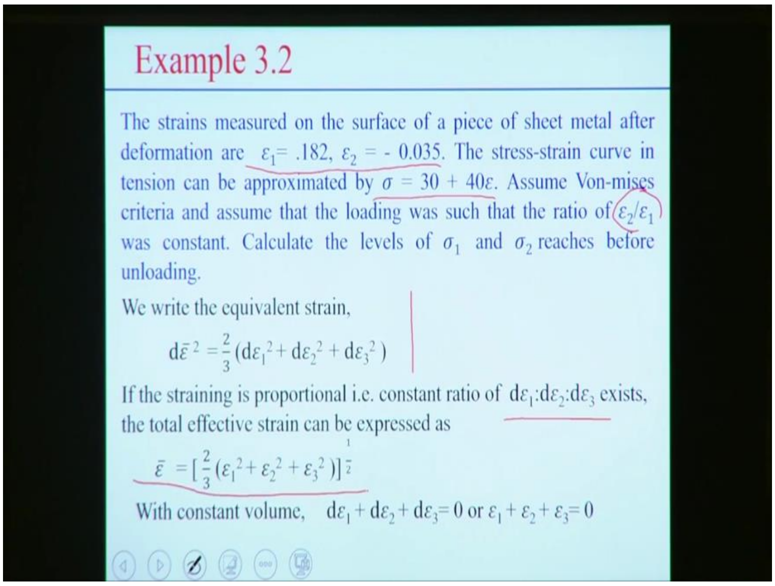

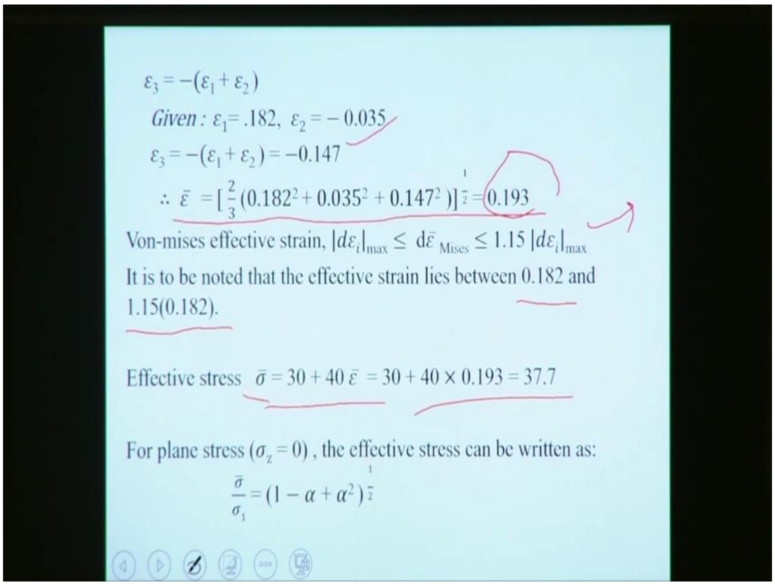

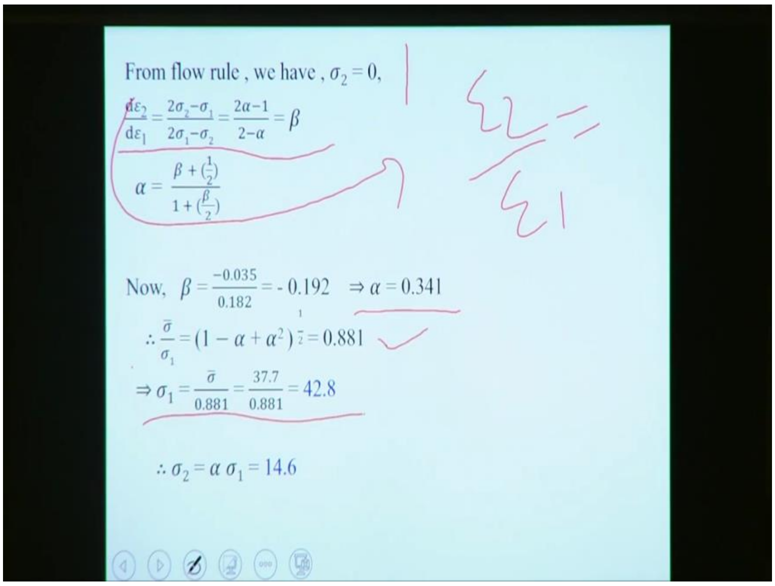

Now if we look into the another example, strain measures on the surface of a piece of sheet metal after deformation, so this types are measure after deformation, the stress strain curve is represented approximated by this one, assuming the Von Mises criteria and assume that the loading was such that the Epsilon 2 by Epsilon 1 actually was constant calculate the levels of sigma 1, sigma 2 reached before the unloading, so from here you can find out the equivalent strain here.

Now if the straining is proportional so Epsilon 2 and Epsilon throughout the deformation then we can say that ratio exists so that so that total effective strain can be represents at a single form other than incremental form here. Now from the constant metal value we can use this one or this one, then we can find out the relation, the Epsilon 1 and Epsilon 2 are given here so we can find out, Epsilon 3, from here we can find out what is the effective stress value in this case.

Now we check if these effective values in between the guidelines or not, so if you see that maximum is point 1 2 and 15 percent of the maximum is the values of the effective strain, so it lies in between that, basically you are following this guidelines so we can consider the calculation so the effective stress here. So effective stress sigma bar equal to the stress strain relation here is given but that when the stress strain relation is given here so we need to consider what is the effective stress value and what is the strain here then we can find out the effective stress value equal to this one.

Now for the plane stress the effective stress can be written this thing in terms of the alpha where sigma 2 by sigma 1 ratio is define by alpha here. Then we can find out the ratio of according to the flow rule. We have sigma 2 equal to 0 here, and we can find out the ratio in beta in terms of alpha or alpha in terms of beta. The alpha and beta is already define, then we can find out alpha beta in terms of alpha, beta is given because this is actually Epsilon 2 by Epsilon 1, the ratio of the Epsilon 2 and Epsilon 1 and even in the problem.

Now from here we can find out the alpha and using this relation we can find out the sigma 1 and we can find out the sigma 2. So this is arranged, sigma 1 and sigma 2 can be raised if we according to this definition of this problem.

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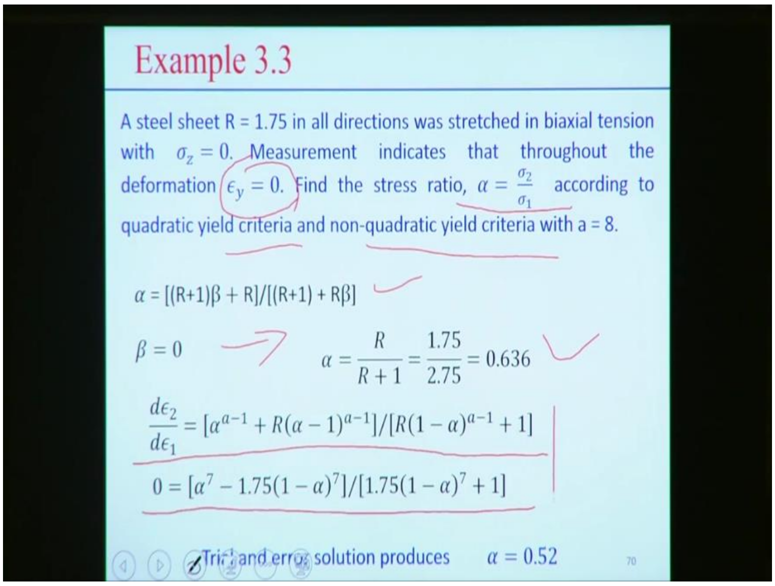

Now similarly if we given the R values is given in all direction, stress in the Bi-axial tension when sigma z equal to 0 so measurement indicates that throughout the deformation, Epsilon y equal to 0, so we need to find out the stress ratio, alpha sigma 2 by sigma 1 according to the quadratic yield criteria and non-quadratic yield criteria with a equal to 8. So this is the typical expression of the alpha, but here it is given that beta equal to 0 so we can find out the alpha here.

And similarly when there is a non-quadratic yield criteria, we can use a as 8, we use this relation for the similar analogy what we following in case of the quadratic function itself, but in this case we need to evaluate the alpha in terms of Trial and error method, so by solving this equation by Trial and error method we can find out that alpha equal to 0.52.

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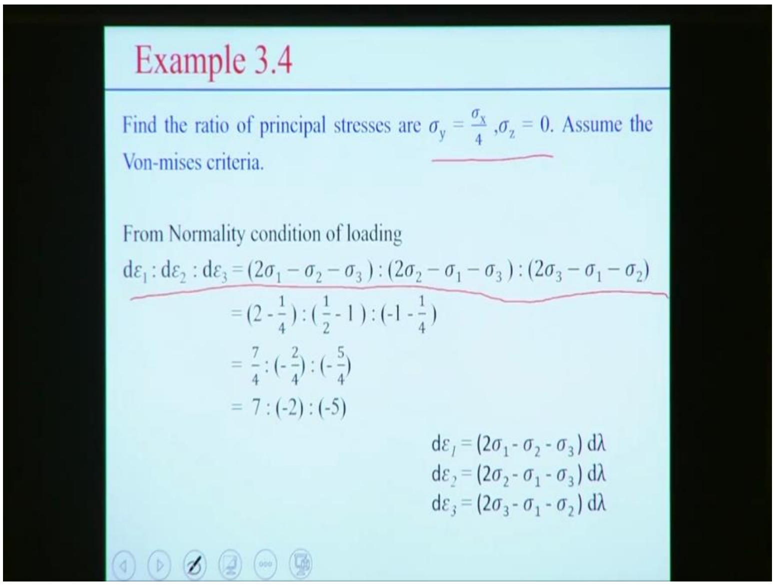

Now we look into the final problem here, find the ratio of the principal stresses sigma y equal to sigma x and sigma z equal to 0 assume the Von Mises criteria. So straight forwards assume the Von Mises criteria we can use the ratio between the incremental strain component is the in terms of the principal stresses is like this, here, if you see here if we sigma y is basically represent the principal stress sigma 1 and sigma x is principal stress, sorry sigma y is principal stress sigma 2, sigma x is the principal stress sigma 1 and sigma z is the principal stress. Sigma 3 equal to 0 here and if we put it here the ratio is like that, basically we have used this relation that we have already discussed in the theory.

So in this I can come to this section basically which is related to the 3 dimensional state of plasticity model, so in this section we have discussed how we can evaluate the strain yield surface assuming the Von Mises yield criteria and how we can put the normality condition and how we can find out the effective stress or effective strain when there is a existence of the 3 dimensional plastic stress strain and different manipulation of this thing, to find out the different constant term which is related to the plastic incremental plastic deformation and that constant terms can be evaluated amount of the plastic work done principal.

So in this case I have tried to represents the all the plasticity or plastic deformation of the material in a short period of time and in a simplify way with the certain examples, hope it will be helpful to make some understanding of the plastic deformation of the material. Thank you very much for your kind of attention.