Introduction to crystal elasticity and crystal plasticity
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Week-04
Lecture-07

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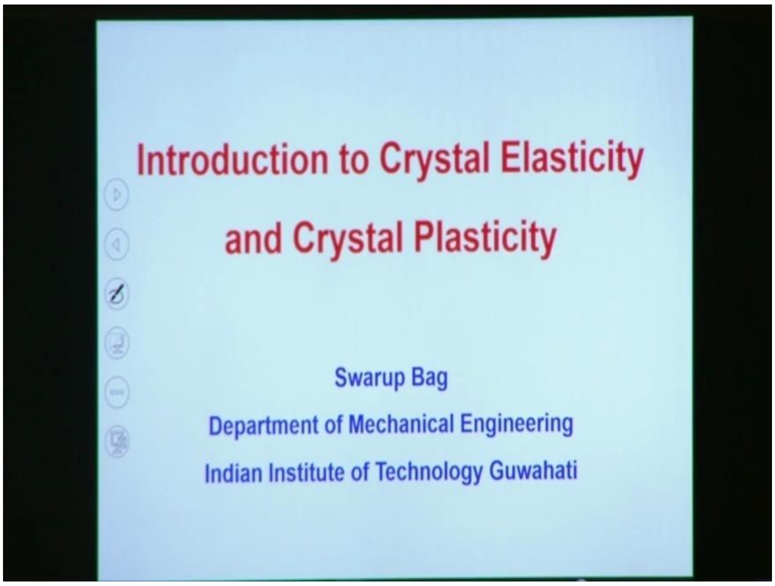

Good morning everybody. Today we will discuss about the plasticity part during the application of the mechanical load, so far we have discussed the elastic behavior and then 2 dimensional stress state, 3 dimensional stress state and how we can find out the principal stresses in case of 2D and 3D. Now we will shift to the plasticity part, so we will start from the very basic things, that in this case the continuum plasticity, specifically the theory of plasticity and when you apply the (())(1:06) material and how I can predict the in-surface when it is subjected to 1 dimensional load or if it is 2 dimensional case, or maybe in general 3 dimensional cases.

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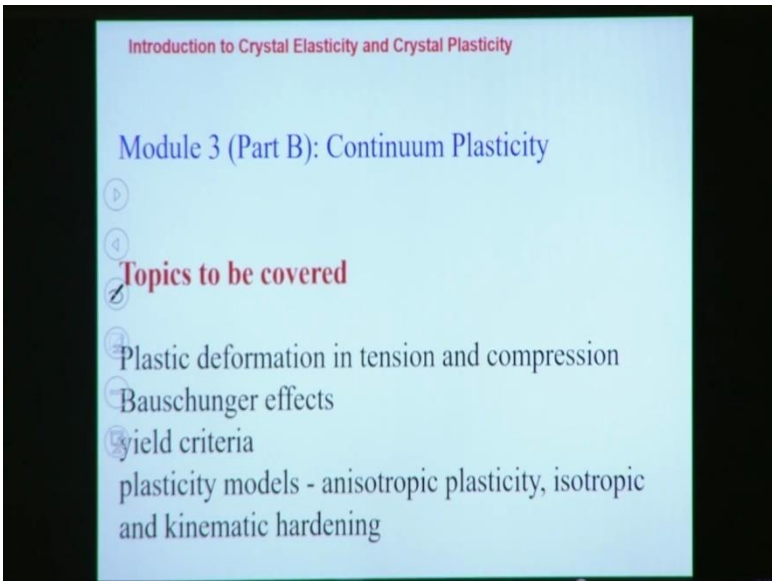

So first we will start with the 1 dimensional plasticity model and what is the effect of different (())(1:31) mechanism or here we will try to incorporate how the effect of (())(1:37) can be better explained but just by simply looking into that stress strain right now of a specific material. So first we will cover the plastic deformation intentional compression, what is the Bauschinger effect, what is the yield for the during the mechanical behavior of the material and finally we will try to discuss the different plasticity model and followed by the an isotropic model we generally use to predict the deformation of the material due to the application of the mechanical load.

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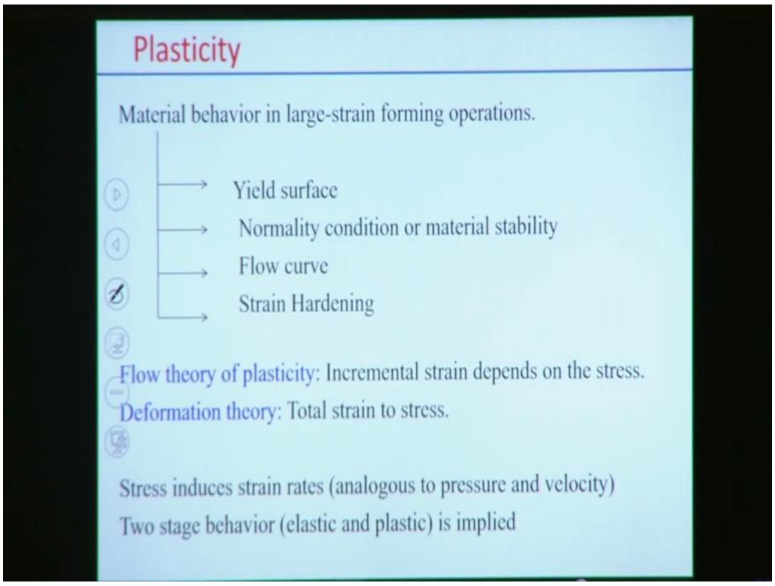

So first we will look into what do we mean by plasticity, so specifically when we use the plasticity theory, it is a mechanical behavior in the large scale strain formation operation where the elastic component is very small and mostly we can represent the deformation of the material throughout in the plastic (())(2:43). So first objective to represent the yield point when it is small dimensional load or maybe yield surface when there is a 3 dimensional load or 2 dimensional load.

Then normality condition or material stability mathematically we will try to derive from the nature of the stress state analysis of a specific material, then what different flow curve are used to model or to predict the plastic deformation of the material and finally the strain hardening mechanism, how we can incorporate mathematically and simply just looking into the stress strain behavior of our specific material when it is subjected to either one direction load or when it is subjected to in general the 3 dimensional loading condition.

So flow theory of plasticity, basically plastic deformation of the material, what is the sustainability of the deformation during the application of the load that generally is discussed because based on the incremental strain mode and that incremental strain depend on the shear state. So basically the flow theory of plasticity is generally explained in that way, but during the plastic deformation you can revive the total deformation into small part and we try to apply the theory based on the small incremental strain part and according we can predict the stress state at that point. So this is the one type of approach we can follow in the plasticity of theory.

And another way you can directly use the deformation theory that actually directly the indicate the total strain to total stress. But in this case probably the path of the plastic deformation is more important and this path generally follow in the north in very linear way, specifically it follows the non-linear path. So we generally try to follow the first approach where the non-linear path of the plastic deformation is decomposed into small small joints or small small parts and we will try to update each and every state, what is the state of the plastic strain and accordingly we can predict the stress also at that point.

So it is like that stress induces strain rate basically which is analogous to the pressure and velocity so similarly we can equivalently we can say that it is really very similar to the stress and strain like pressure and velocity. Basically the pressure difference actually induces on velocity when you try to analyze the flow through a pipe. So similarly here stress actually induces with the strain based on this approach, we will further discuss the plasticity theory applicable for one dimensional case.

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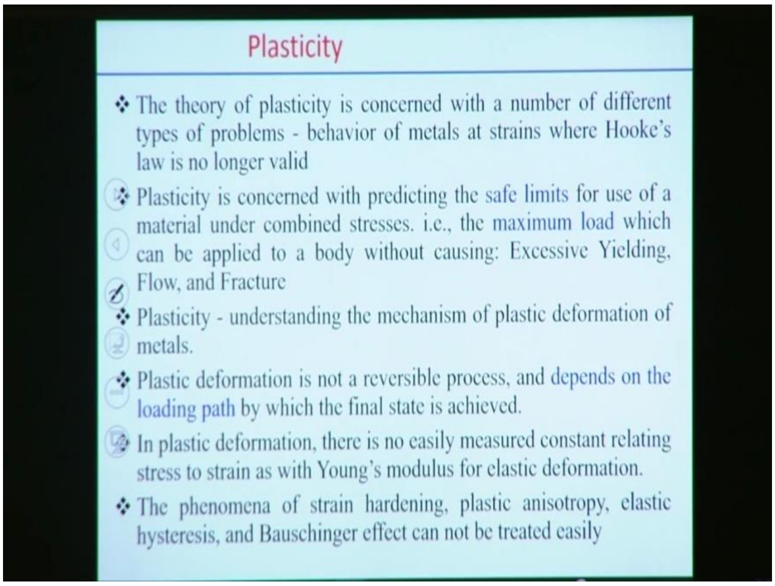

In general there are several important comments or points before starting the theoretical analysis of plasticity, I like to mention all this points, the theory of plasticity actually is concerned with the number of different types of problems, so behavior of materials at strain where Hooke's law is no longer applicable here, because we know that Hooke's law you can apply only for the plastic analysis because the stress and the strain within the plastic limit follow some linear relationship.

But in this plasticity theory or plastic deformation of the material that path is completely a non-linear. Plasticity is concerned with predicting the safe limits for use of the material under combined stresses that means the maximum load which can applied to a boy without causing excessive yielding, fracture and flow. So this way we try to uhh predict what maybe the maximum load or what may be safe limits of the materials. So plasticity actually deals with that, safe limits and maximum load.

Third plasticity is understanding the mechanism of the plastic deformation of the metals so it deals with the plastic deformation of the metals. Fourth is plastic deformation is not a reversible process and depends on the loading path. The most important thing is that plasticity actually depends on the loading path by which the final state is achieved. So it is very very different to predict at a once single step what is the final state of the plastic deformation by looking into the initial if we do not follow the path of the deformation.

Fifth is the plastic deformation there is no easily measured constant relating to the stress and strain with Young's modulus for elastic deformation. For example in case of elasticity we can easily measure the Young's modulus by simply looking into the slope, but when the that path is, the stress state path is basically a non-linear then it is not easily to determine all the constants involved in the plasticity theory.

Finally the phenomena of the strain hardening, plastic anisotropy, Bauschinger effect cannot be treated easily in the theory of plasticity, uhh but we will try to see how mathematically we can predict or we can use all these constant in the overall explanation of the plasticity theory.

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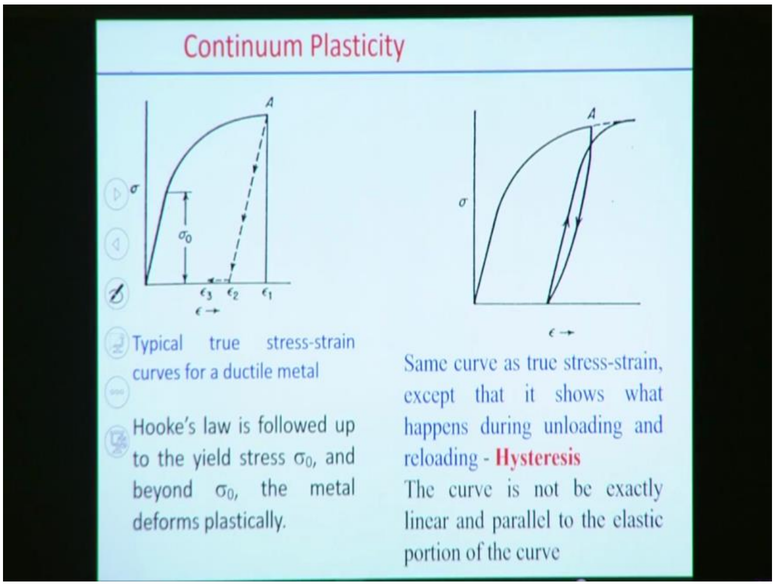

So looking into that several important points, plasticity theory, we just try to shift on the what are the loading path on specific to plastic deformation when it is subjected to universal loading condition. First we will look into this figure, here you can see that this is actually true stress-strain curves for a ductile metal. We have already discussed that true stress-strain curve and what is the difference of the true stress-strain curve from the engineering strain-strain curve during the mechanical loading of a material.

So here we see that this is a true stress-strain curve for a ductile metal and Hooke's law is followed up to the yield stress sigma 0, and beyond sigma 0 the metal deformation plastically. This is very typical deformation behavior of the material, we assume the behavior of the metal is elasto-plastic in nature. So very first part up to sigma 0 that is the elastic part and afterwards which cross the elastic limit just interest to the plastic deformation.

And at point A if you see that when you remove the load at point A it will come back to the position, at this position so that position actually with some elastic recovery part so this amount is the elastic recovery part in this case that means Epsilon 2 to Epsilon 1 that difference is the elastic recovery but from origin to up to Epsilon 2 that amount actually represents the permanent deformation of the metal.

So this is the typical elasto-plasticity behavior of a material. Now if we look into that right hand figure it represents the same stress-strain diagram but except it shows that what happens during the unloading and reloading. So up to point A we will say unloading sorry, point A is the application of the load up to point A then release of the load, basically unloading and it is following some arrow path and it come backs to the strain axis cross again we reload to the same sample, it follow the different path.

But this unloading and reloading path differences may not be very high so that gap is very small in this case, but this difference in a loading and unloading path is basically for this stresses, so this curve is not be exactly the linear and not exactly parallel to the elastic portion of the curve. This is the typical behavior we observe in the material that if that gear indicates some properties of the material, some damping properties of the material.

If that gap remains very high that indicates that the damping property of the material is very high. For example (())(12:28), I think very good damping properties, so specifically when we try to notice the stress-strain diagram from (())(12:34), that amount will be very high. But in specific strain probably that it will be more less almost equal to the reloading and unloading path, so in that case the damping properties of the material are very less, so this is the indication of the stres-strain curve. This specific phenomena is called the stresses.

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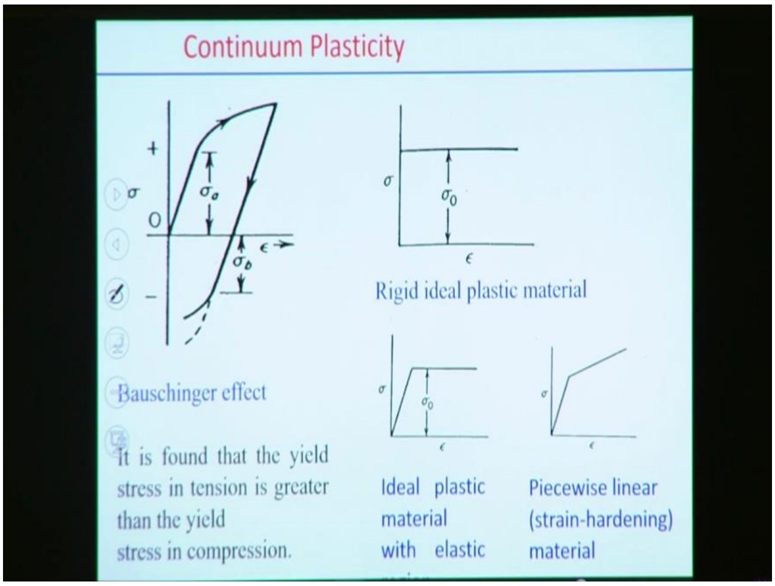

Now looking to the similar other model of the plasticity model, of the different metal, if you look into that left hand side figure as well if we see when the application of the load at point O and it is followed certain path reach up to this point and it will change the loading condition from tensile first part is showing due to the application of the tensile loading, and if you change the loading condition that means if you put the compressive load and it is follow the part below the negative side and it is come back to some other point.

So here the important point note is there, initially that yield stress in tension in sigma A here it is indication and yield stress in case of compression that is sigma B, but it is not necessary that if stress in tension should be equal to the yield stress at the compression. There is a material in this actual, there is a difference of the stress-strain tension and yield stress at compressive load so that difference in tension is greater, so the yield stress in tension is greater than that of the yield stress in compression. So this specific phenomena is called the material is having Bauschinger effect.

So that is Bauschinger effect is very much significant when material is subjected to some kind of (())(14:34) loading condition that means specific cyclic loading whether the tesnion is compression kind of loading, so in that sense the Bauschinger effect is very significant to consider further plasticity analysis of a specific material.

If we look into the right hand side there are other models also exists. First one is the rigid ideal plastic material, if we see that with respect to the strain the stress is always constant so this kind of metal behavior is called Rigid but ideal plastic material. If we look into other two models also, third and fourth model as well, then third model indicates the ideal plastic material with elastic response.

And here if you see that there is the elastic part is there up to sigma 0 is the end point here, and afterwards that yield stress is constant over the deformation, so this type of specific material model is called the ideal plastic material with elastic component. Final one is that Piecewise linear, sometimes we represents the elasto-plastic behavior of the material in terms of the two slopes, that is sometimes it is called bi-linear isotropic material.

So there is a two linear path, the first linear path actually represents the elastic limit or elastic zone and second linear path actually represents the plastic path. So that is second path is there, since it is not a constant uhh with respect to the strain like previous two models so there is a variation of the stress value specifically the there is a increment of the stress value over the strain in the plastic job, so this type of behavior is called the strain hardening material.

So that means strain level of the metal is increased with the further straining, that means with respect to the strain the stress values increases. So this type of specific model is frequently used that is called bi-linear isotropic hardening.

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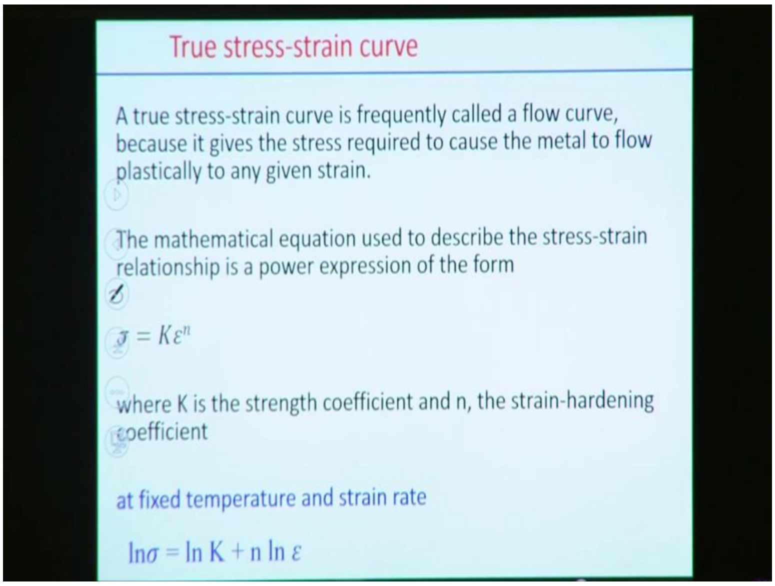

Apart from the several models, plasticity models, now we come to that point a true stress-strain curve actually frequently called also the flow curve. And we have already discussed the true stress-strain curve also mostly frequently or more significant for the plasticity analysis.

So if therefore we are focusing on the further analysis of the stress-strain that means by default we consider that is the true stress-strain curve.

But sometimes it is called flow curve because it gives the stress required the plastic deformation of the material so mathematical equation, we open or most commonly used to discuss the stress-strain relationship, is the powered expression of the form that sigma equal to K Epsilon to the power n, that K is a constant and that is called strain coefficient and n is basically called the strain hardening coefficient.

So this is very useful or most commonly used the stress-strain relation that is called the power law of relation used but the stress-strain components are represented in terms of the true stress and true strain. Now for the specific equation if we try to plot this specific equation in logarithm scale then we can find out that logarithm sigma equal to logarithm of K plus n logarithm of Epsilon.

So basically this relation represents the linear relations on the logarithm scale. Now n actually represents the slope at the logarithm scale. So lets us see how that strain hardening coefficient can be defined.

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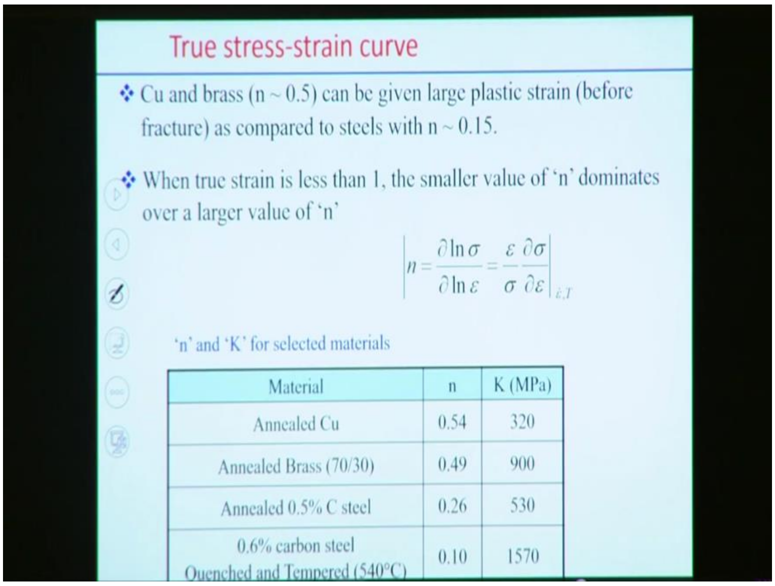

Copper and brass, the strain hardening coefficient is only 0.5 and can be given the large plastic strain before fracture as compared to the steels that means for steel the strain hardening coefficient is 0.15 as compared to copper and brass as 0.5. What physically the significant these things that means with the same deformation the n is more in the slope in the stress-strain diagram specifically I am talking about the plastic jump.

Slope is very high, so when the slope is very high with the same deformation the stress level of the material actually increases very high when there is having high strain hardening coefficient but in case of steel having low strain hardening coefficient, so in this case with the same amount of the deformation or same amount of the strain the increment of the stress level is less as compared to forward contrast.

So other way that large drastic strain can be given to the copper since it is having the strain hardening proportion is very high but low amount of the plastic strain or plastic deformation can be observed by the steel also. When the true stress is less than one, the smaller level of n dominates over the larger value of n. Since n actually represents in terms of slope from the logarithm scale.

So though logarithm sigma by logarithm n this is the stress by strain in the logarithm scale that actually represents the slope and we are representing here that n in terms of the at fix standard and fix temperature. Since the plastic deformation having the (())(21:35) or it depends on the strain rate or temperature. We will discuss the effect of the strain rate and temperature on the plastic deformation later on.

But for the time being if we see that what are the typical values of n that means strain hardening coefficient and the (())(21:53) for the different materials. The values of the different materials, the values of the different n and K indicates that for copper is having very high values of n that is 0.54 but the Quenched and Tempered steel is having low value of strain hardening coefficient that is 0.4 but if you see that strain coefficient is very high in case of Tempered steel that is 1570 but it is low as well as annealed copper. So in this case if you see that strain hardening coefficient n and K strain coefficient are, there is a that follow some vice-versa relation.

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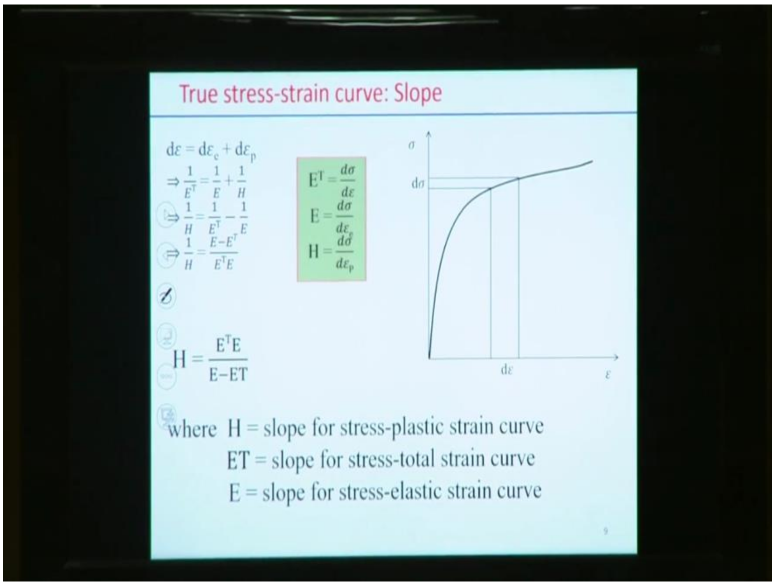

Now we will try to see two stress-strain (())(22:49) on there we will try to represent the how we can define different slope, I can say the different parameters which is analogous to the stress-strain diagram in the plastic deformation jump. First we define one typical stress-strain diagram and if we consider very small part of the diagram that is corresponding to strain is d Epsilon and stress is correspondent to d sigma.

So here we can define the several slope for example the slope for the stress strain curve, absolute value of the stress-strain curve actually represents the slope at this point, it is like that, so that slope we can say simply d sigma y, di Epsilon but since this is a material behavior is like elasto-plastic in nature so we can decompose the total amount of the strain is consist of the elastic part as well plastic part.

And elastic part separate out the elastic part is related to the elastic modulus and plastic part we can say related to the plastic modulus, but how we can correlate between all these slopes, so E here is define the, I think E is the slope for the stress-elastic strain curve so that specifically represents the d sigma y, d Epsilon e that is the difference to the elastic component of the stress.

H is define that slope for the stress and plastic strain curve so individually if we derive or define all these three different slopes then we find out the correlation right assuming the elasto-plastic behavior of the material and by decomposing plastic part, and here if you see using all these definition of the slope, if we put it here you can find out 1 by ET 1 by E plus 1 by H, so finally you can find out H that means slope for the stress and plastic strain curve.

So I can say simply, simply we can say only for the slope for the plastic component that is the ET E divided by E minus E slope, this would be E raise to power T. So that is the slope of the plastic component, so this is the simplified way, we can find out the slope of the different if we consider PO elastic, consists of the PO plastic and when it is subjected to the elasto-plastic. So ET in terms of stress and the total strain curve that we can easily measure from the diagram itself and E for the slope of the stress strain curve that also we can measure which is different as the specifically the initially slope.

So looking into that two part we can roughly estimate what is the H in this case, so this is some mathematical treatment we can find out or we are dealing the (())(26:31) to estimate the different slope of the path, then you can easily find out. So this calculation is specifically useful when you try to do some mathematical modeling using some (())(26:42) analysis.

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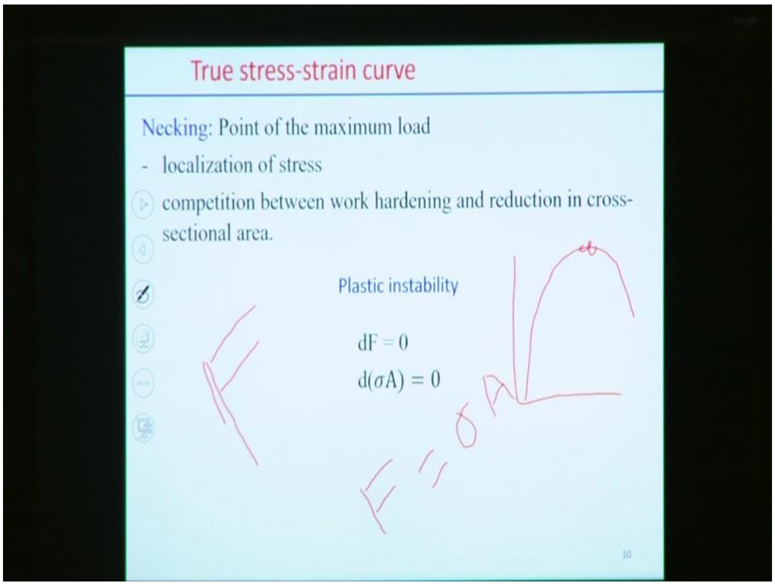

During the plastic deformation normally after the point stress there is a (())(26:55) deformation up to certain point but when it reaches some optimum point in the typical stress-strain diagram so at that point there is a competition between the work hardening and further reduction in the cross sectional area.

So the instability of the plastic deformation actually starts from that point so that typical point consider as necking point, so that is corresponds to the point of the maximum load if we consider the load deflection curve of a typical material, so specifically the necking starts with the localization of the stress so at that point of beyond that point actually stress no longer remains in the single dimension so Triaxial state of stress actually exist and (())(27:51) happen may be from the point of the defect exist within the material itself.

So it is necessary to analyze the plastic instability condition at that point specifically at the necking point beyond which there is a random reduction of the cross sectional area and finally when you reach to a certain point where the fracture of the material happen, so how we can establish this plastic instability condition at that necking point.

So necking is we consider at this point, there is a competition because with a application of the load there is a reduction of the cross sector area but at the same time there is a increment of the stress level so that is called increment of the stress level due to the work hardening or due to the strain hardening.

So since that engineering tension diagram actually represents that point as the maximum point but low deflection with low deflection curve that point represents the maximum point which is like that…

So this maximum point actually try to predict the plastic instability condition so mathematically we can say since that is the optimum point in the sense that d of applied load so f is the applied load here and so df equal to zero, this is the typical condition for the plastic instability. Now f can be, load can be represented by stress and cross sectional area, sigma into A and then d of sigma n equal to zero is the typical condition during the instability of the plastic deformation actually starts when it is subjected to Uniaxial loading condition.

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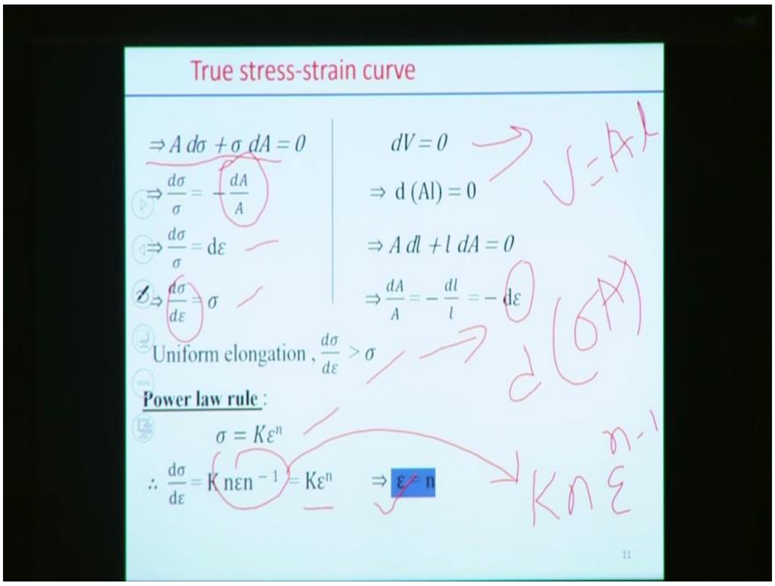

Now let me give you the further analysis or further expression of this plastic instability condition, we can find out that though D of sigma into A can be decompose into this two parts, A of d sigma plus sigma of DA, that is equal to 0. So from here you can find out d sigma y sigma equal to minus dA by A and finally what is the values of dA by A here, that also we can find out the volume in compressive of, assume there is a no change of the follow during the plastic deformation that means delta of equal to 0, DV equal to zero here.

Now again V equal to, volume equal to cross section area into length, if we put it d of A here it becomes A to download, or plus l dA equal to 0. So from this point we can find out dA by A equal to minus dl by l that actually represents d Epsilon dl by l, that is d Epsilon is two strains for the l, so we can say dA by A equal to minus d Epsilon if we put it we get this relation and finally sigma by d Epsilon equal to sigma. So this is the plastic instability condition, uhh generally used for the Uniaxial (())(31:49) conditions.

So uniform elongation will happen when this sigma y, this d Epsilon is greater than sigma. We have to explain graphically that the typical condition of the uniform deformation or (())(32:04) deformation. But with this plastic instability condition if we try to use this condition for the Power law, so the Power law means this is the typical representation of the ordinary, we can say the relation between the stress and strain for the most common materials. So what is material we will try to predict the instability condition here.

So if sigma equal to K Epsilon m then we can find out that this sigma y d Epsilon equal to K n Epsilon, Epsilon to the power n minus 1. Actually this would be like this, Kn Epsilon to the power n minus 1. So this is comes from d Epsilon by d sigma and that is equal to sigma what you use the, so that is equal to sigma, sigma is given here so that sigma comes from the power relation and from here we can find out Epsilon equal to n, this is the typical instability condtion when we consider the typical stress-strain relationship by the Power law.

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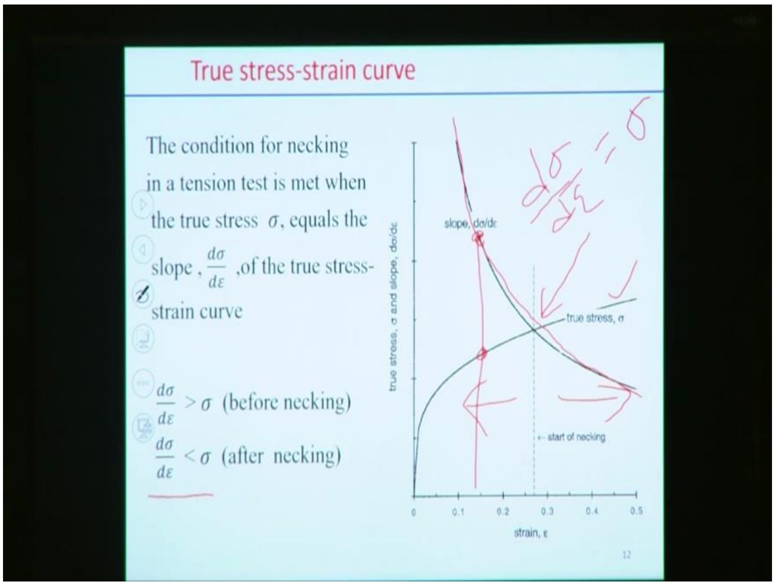

Now condition for making the graphical representation of the first graph represent the true stress-strain, this represent the typical true stress-strain diagram and this sigma by d Epsilon, this line actually indicates the d sigma that means this line slope the d sigma by d Epsilon. Now if we see the when they are crossing point that means, crossing point actually represents the d sigma by d Epsilon that is the slope, this is one curve crossing when it is equal to sigma that is correspondent to this point.

So this point actually indicates the necking, graphically this point indicates the necking but this sigma by sigma is greater than sigma before necking so this jump represents the before necking and this joint represents after necking jump. So before necking jump it is obvious that at any point we consider at any point, this is the value of the sigma and this is the value of the corresponding slope d sigma by d Epsilon.

So basically before necking the d sigma by d Epsilon is always greater than that of sigma, but after necking just reverse, this sigma by d Epsilon will be less than sigma after necking. So this is the condition of the necking is made when the true stress strain equal to the slope of the true stress strain curve, so this is the graphically representation of the plastic instability condition.

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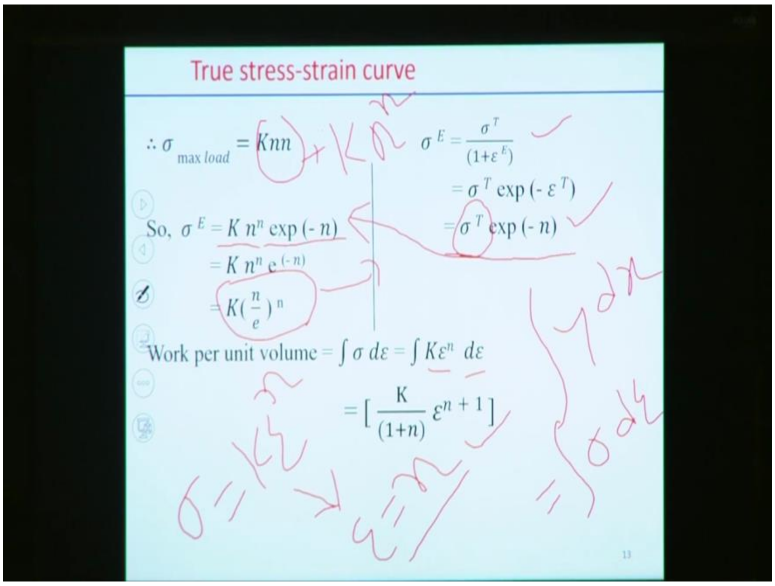

Now we can prove the further analysis of this plastic instability condition for a different kind of (())(35:39) condition. For example is that if for the Power law that makes Epsilon to the power n, this is the Power law, so what maybe the maximum load in this case, actually Epsilon equal to n, when we put it here it becomes K, since Epsilon is equal to n so n to the power n, represents the maximum load, from where the necking starts.

So before necking this is the amount of the stress, the material can sustain that when the plastic instability condition and when we are using the Power law relations between the stress and strain, so other parameters can also (())(36:51), so what maybe the engineering strain for the optimum conditions that means about to start the necking so engineering strain can also be represent the true stress by 1 minus Epsilon to power t that we have already derived.

So here we put the condition Epsilon equal to n, so we have to put it we are getting the sigma T exponential for minus 1. So engineering strain in terms of E true stress and in terms of strain hardening coefficient. Find out similarly we can find out also the uhh directly that in other way the K what will be the engineering strain here, so first sigma t equal to this is the true stress K Epsilon to the power n but Epsilon equal to n here so K n to the power n exponential minus n, this is the further calculation for that.

Now K n to the power n, n to the power minus n we can find out this is the value. So here the engineering strain in terms of the strain hardening coefficient n and the strain coefficient that can be obtained when following the plastic instability condition for a specific metal, basically at the necking point what will be the engineering strain.

Now we can also find out what maybe the work per unit volume so we know over the stress-strain diagram if we consider one element d Epsilon then generally we represent y dx the area of the part, but here physical representation on the stress-strain come that is equal to the amount of work done per unit volume. So here which is equivalent to y axis that is sigma and x axis corresponding to Epsilon so that (())(39:03) form, actually this is amount of word done per unit volume with reference to the specific stress-strain diagram.

Now if we put sigma equal to K Epsilon to the power n following the Power law and d Epsilon, we can find out this the expression of work done, so similar manipulation for similar expression can also be derived by using the plastic instability condition and when the relation between the stress-strain is actually known to us then we can use this condition then we can derive several expression from that.

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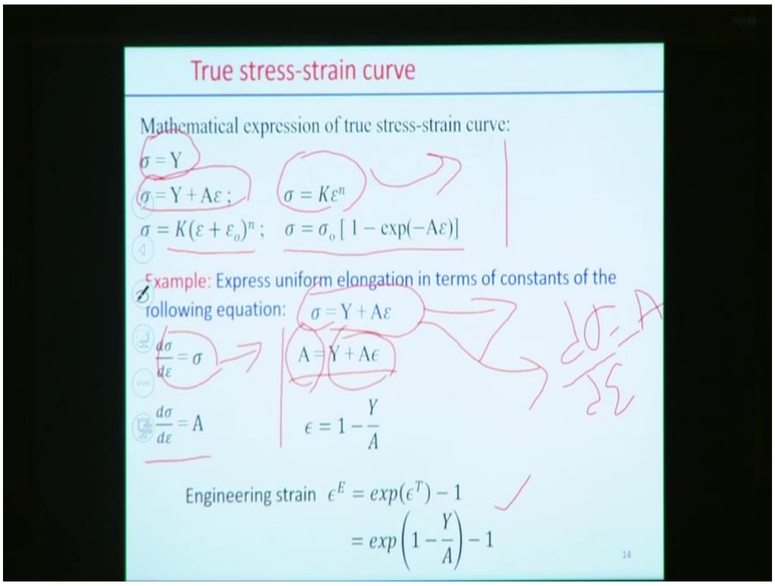

Now simply the Power law depending upon the type of material that actually follow different types of expression or different type of relations between the stress-strain. That is for example some material follow sigma equal to y that means it is constant, this is the typical stress-strain diagram. Some material follow sigma equal to this y plus Ae so basically it is the straight line, linear relation and some material follow this complex relation between stress and strain or Epsilon 0 actually indicates the initial amount of the strain and some material follow these things.

So depending upon the type of material, the stress and the strain relation can follow or can be represented by the different mathematical formulation. Now we look into one typical formulation, let us see the Uniform elongation in terms of constants of the following equation, so let us stress and strain actually follow this relation.

So here this, this is the typical relation between stress and strain and we try to find out the plastic instability condition, so this condition represents actually plastic instability condition. This sigma by d Epsilon equal to sigma, but for this specific stress-strain relation we can find out, we can find out that from here this sigma by d Epsilon equal to A so here this is equal to A that equal to sigma, sigma is again corresponding y plus A into Epsilon.

So this is d sigma by d Epsilon and this is actually sigma so from here we can find out Epsilon equal to 1 minus y by A and further calculation we can do we can find out the amount of work done per unit volume, similarly we can find out the engineering strain like this. So engineering strain can be represented in terms of the all under constant of y A in that way.

So this at the typical application so we can actually we can apply the plastic instability condition for the different type of material behavior when the mathematical relation between stress and strain is (())(42:26) who asks for different types of materials.

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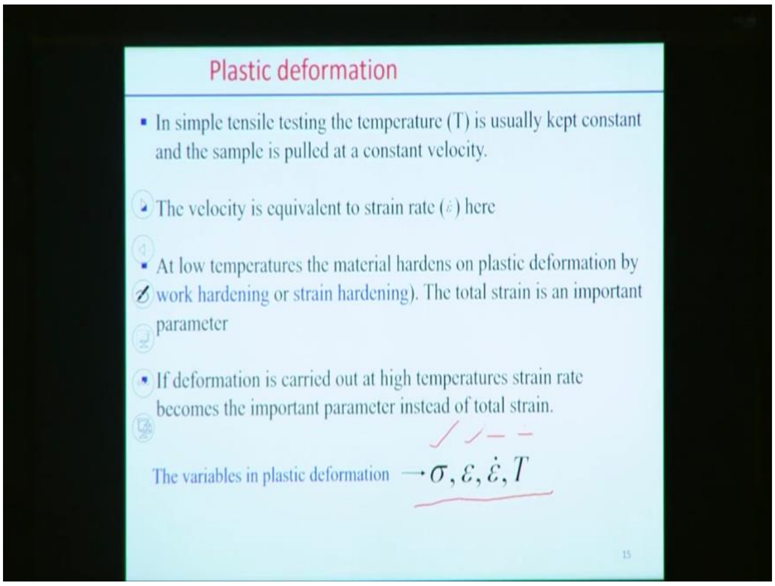

But it is mention to that point that simple tensile testing we generally we conduct we keep temperature as a constant and we use the universal tensile testing method keeping the (())(42:48) as a constant probably in a very small speed we generally speed we generally call it a tensile testing specimen. But definitely the velocity at what velocity the cross rate is moving that is also important parameter, that actually influence the nature of the stress-strain diagram.

So if it is very low, if it is very slow then probably it may not have any effect on that, but if we change the cross velocity then there may be significant change in the stress-strain diagram. So that means that cross rate velocity which is specifically equivalent to the mathematical term that is called strain rate.

So in other we can say that strain rate is also having influence on the stress strain behavior of a specific material and at the similar way if we change the temperature apart from the root temperature then also stress-strain behavior also changes for specific material. Specifically at low temperature the materials hardens or metal becomes, so that means strain level of the material actually increases due to the work hardening or strain hardening mechanism.

But at the high temperature maybe other mechanism the strain rate is more effective in this case. So definitely there uhh in the although we are showing typical stress-strain diagram, but this typical stress-strain diagram although evaluated over a constant temperature and constant strain rate, so practically this mechanical behavior of the stress-strain of a specific sample of specific material is specifically influenced by the temperature and strain rate.

So we should not neglect the effect of the other two parameter for example strain rate and temperature, so in the variables of the plastic deformation it becomes 4, one is the stress, another is the strain, strain rate and temperature.

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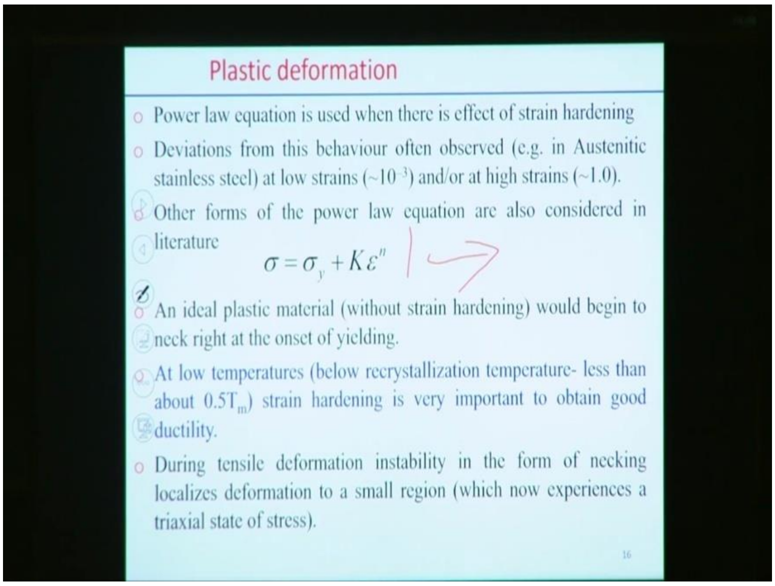

So let us see how these parameters actually effect the typical stress-strain diagram of the specific material. Now we know that Power law equation is mostly used while there is a effect of the strain hardening, even the equation, form of the equation indicates that we have the strain hardening coefficient n and the strain coefficient K.

But deviation from this behavior are often observed for example austenitic stainless steel at low temperature, sorry at low strain specifically 10 to the power minus 3 and high strain, the typical behavior of the stress-strain that actually changes, if there is change of the strain. Now other forms of the power law equation can also be considered in the literature which already we have discussed that we can predict the stress-strain with a different formula.

This is another form of the formula, deviation from the power law we can typically observe represents the stress-strain behavior of this specific material that is sigma equal to sigma y plus k Epsilon to the power n. So next point is that ideal plastic material, so ideal plastic material is that strain, stress value remains the same irrespective of the or with the further straining with the further deformation that way there is no strain value effect in this case.

So it start yielding on the neck right on the onset, right to the bigger to the neck, right to the onset of the yielding, so just start of the yielding that yield stress limit becomes constant over the strain, but if we look into the physically that low temperature specifically the below recrystallization temperature are strain hardening is very much, significant meter to obtain the ductility.

But during the tensile deformation instability in the form of necking localized deformation in a small region, now experiences the triaxial state of stress on the onset of the making of the specific sample. So these are the typical points or notes for the plastic deformation so that we can use this knowledge of the different plastic deformation which is physically justify to apply for the specific material.

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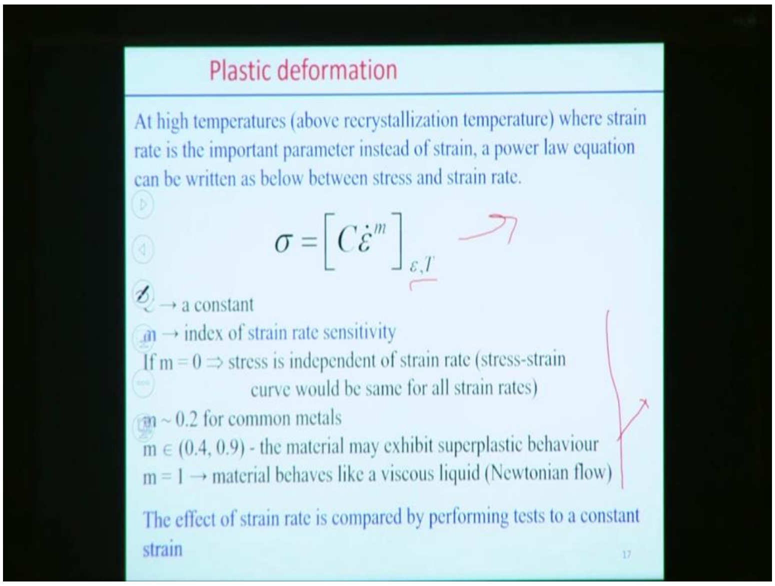

Now we come to that point, separately analyzing the effect of the strain rate similar to the power law where there is a effect of the strain hardening, at high temperature specifically above recrystallization temperature where strain rate is the important parameter in that case instead of the strain a power law equation can also be written in terms of the stress rate so here we represents the power law relation such a way that sigma equal to c Epsilon dot to the power m keeping other parameters stress.

So for (())(48:30) strain and fix temepreature. So similar kind of expression but here c is a contant similarly, but here m is the index of the strain rate sensitivity so physically when m equal to 0 the stress is independent of the strain rate that means stress strain curve would be the same for all the strain rates, but practically m equal to 0.2 for the common metals, but when m equal to very high say 0.4 to 0.9 so almost the material may exhibit superplastic behavior.

So uhh we will discuss, maybe this is not in our scope to explain the superplastic behavior. Next is the m equal to 1, in that case materials behaves like a viscous liquid, so this is a physical justification of different values of m if we use it, physically we represent all this phenomena, but in general we need to investigate the effect of the strain rate by using the similar kind of Power law equation simply by changing the coefficient here as compared to the strain hardening coefficient m.

So but the effect of the strain rate is compared by performing the tests to a constant strain that means the effect can be, strain rate effect can also be considered if we compared the test keeping the other two parameters constant, for example strain and temperatrue.

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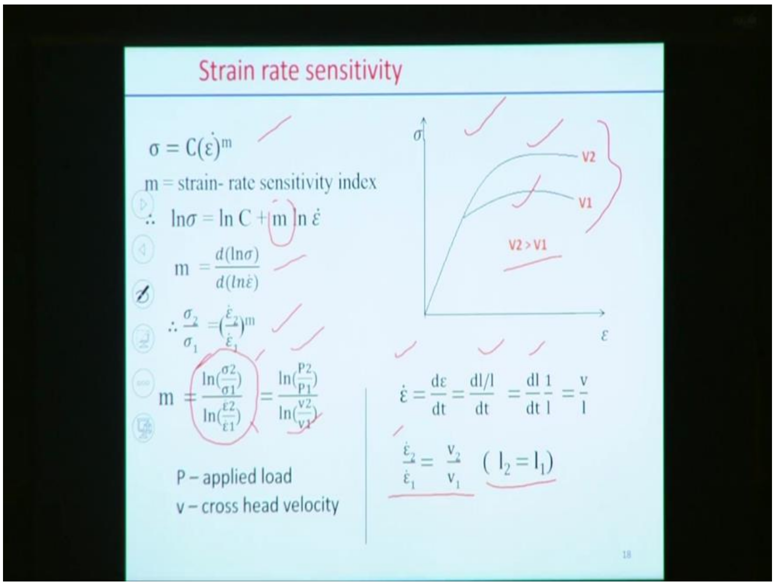

This is the typical expression for the strain rate sensitivity effect that is (())(50:21) stress and strain rate, but how can correlate with respect to the other parameters here. Here m is the strain-rate sensitivity index, similar way we can represent the logarithm scale, so in the logarithm scale m actually represents the slope so that slope actually with respect to the logarithm of stress and logarithm of the strain rate.

So if we compared the experiment at the two different strain rate for example, 2 different cross rate velocity using the simple universal tensile testing, in that case we can result that two different stress at two different strain rate and we can correlate by this relation, so sigma 2 by sigma 1 equal to Epsilon 2 dot by Epsilon 1 dot and the effect. From here we can find out that m equal to this is the expression.

So that upper side and lower side if we find out all these values (())(51:23) experiment then we can find out the strain rate sensitivity index for this specific case. Definitely in this case we are neglecting the effect of the strain hardening and we are keeping the temperature as a constant. So there is no temperature effect also here. Now this is the typical description, the graph actually represents that (())(51:49) theta stress-strain diagram typically, and two different cross rate V2 and V1.

For example V2 is greater than V1 here so when you conduct the experiment at high strain rate or maybe at the cross rate velocity then we can expect this type of graph between the stress and strain, but when it is low the strain level actually lows in case of low cross speed. So it is obvious that if there is a change of cross rate velocity in the universal tensile testing so we can get the different stress-strain diagram.

Now looking into that data, how we can find out that on the phenomena, or how can correlate the strain rate sensitivity index. There is other way also, for example strain rate can be represented by d Epsilon by dt in differential form. So again d Epsilon is the true strain that is dl by n by dt. So dl by dt, in differential form. So again d Epsilon is the true strain that is dl by l by dt. So dl by dt by 1 by l so we can say dl by dt, so length derivative that represents the velocity, so v by l.

So in this if we consider the total length of the sample or total length of the cross head movement keeping as fixed, that is l2 equal to l1, that means l2 equal to l1 the it is simply that ratio of the strain rate is equivalent V2 by V1, so V2 by V1 we can easily measure, from the universal test machines, if we put it we can find out that sigma 2 by sigma 1 simply we can correlate in terms of the load, applied load V2 by V1 and Epsilon 2, the strain rate ratio can be easily put each of the velocity. So from here we can compare how we can evaluate the values of the strain rate sensitivity through experiment.

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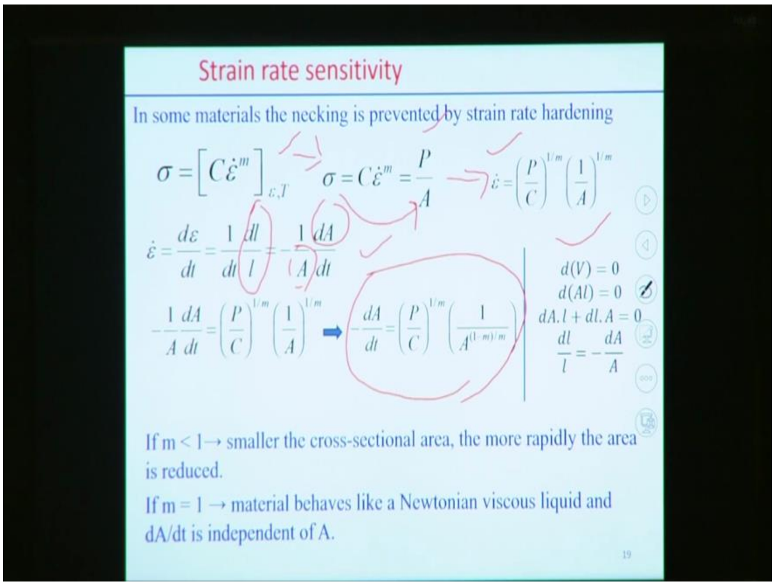

Now further analysis of the strain rate sensitivity indicates that the if some materials the necking is prevented by strain rate hardening, so when strain rate hardening mechanism will try to explain but here the explanation come from the mathematical basis which stick with an equitation this sigma equal to C Epsilon dot to the power m. From that equation we can further modify this thing, sigma equal to C Epsilon dot to the power m which is equal to load by cross section A.

Then this is direct relation sigma equal to load by cross sectional area and from here you can find out the strain rate equal to simply rearranging this equation we can find out the strain rate here in terms of load constant and cross sectional area. But strain rate again can also represented by d Epsilon by dt, so it is like that 1 by dt, dl by l, so basically dl by, sorry dl by m can also be equal to the dA by A.

So that actually comes from the keeping in mind that there is no change in the volume so from there here it is, so no change of the volume, so basically dV equal to 0, so here we can find out that dl by l equal to minus dA by A. So further convert this expression and finally we can find out dA by dt in terms of load at cross sectional area.

Now by looking into this expression if you find out that m less than 1, that means the smaller cross sectional area, the more rapidly the area is reduces but when m equal to 1 the material behaves like a Newtonian viscous fluid and of course dA by dt, so change of the process in area with respect to the time, so that is the independent of the cross sectional area. So this is the physical justification of m when you try to analyze that how there is a reduction of the cross-sectional areal with respect to time here.

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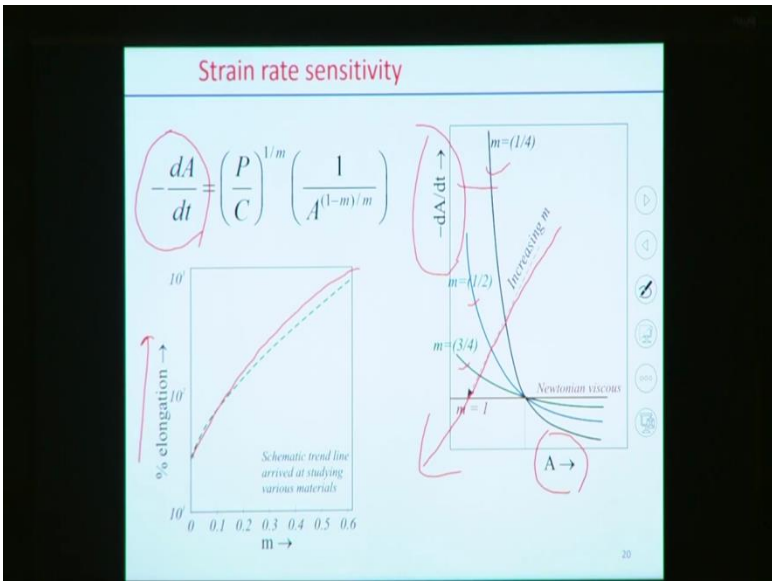

This is the physical graphical representation of this phenomena, so dA by dt, we are representing this thing, what are the rate of the interaction of the cross sectional area, the y axis represents that and x axis represents the simply cross section area. Now assuming that P by C 1 by m, sorry P by C as a constant of, to find out that for different values of the M we can draw different graphs, so for example this one M equal to 1 by 4, in this case m equal to half, in this case m equal to 3 by 4 and gradually when m equal to 1 it becomes a horizontal line.

So what does it mean, so increasing of m is basically towards this direction, so if m is tensed to 1 then it becomes the Newtonian viscous fluid, but if m tends to lower value then we can see that there is a reduction of the cross sectional area is very high, m here is very low value and quickly there is reduction of the cross sectional area at the low value of the m as compared to the high value of the m.

But if you accumulate the different various materials, if we consider the in this thing the percentage of the elongation which is measure of the ductility also, the percentage of elongation if you see, and the typical m value is basically related to this diagram. So m is very high, so the percentage of elongation also very high, so m is very low the percentage of elongation also low. So this is the physical representation of the change of cross sectional area, when there is a strain rate sensitivity is there.

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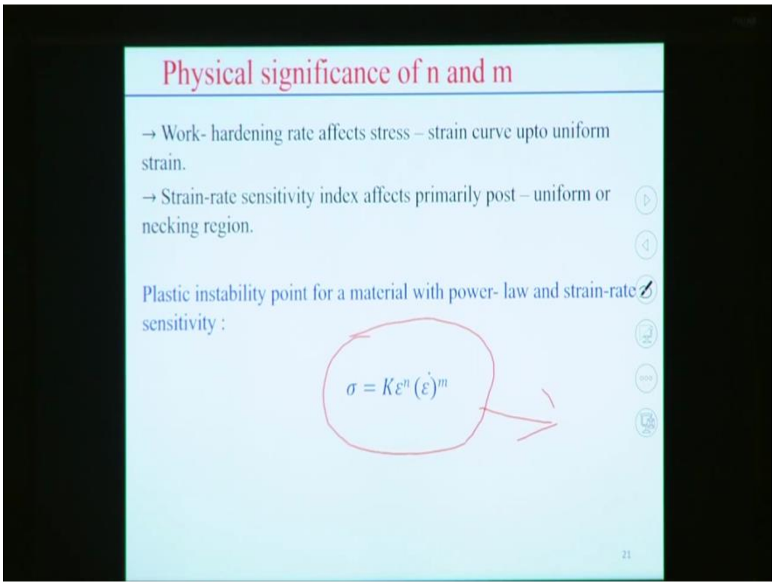

Now physically what we mean by n and m, because we have used two concepts n and m, uhh to explain the different plastic behavior of the material. So basically n actually represents the work hardening effect and that effect is more significant up to the uniform strain that means before start of the necking point. Up to that point n is very much significant to analyze in this case.

But m is the strain rate sensitivity index that is more significant to analyze basically primarily the post uniform jump that mean the necking region. So we can say in general, n can be important parameter before start of the necking and m can be important parameter after start of the necking. Because m the plastic deformation, when you try to predict the flow stress that is more effectively represented when there is a effect of the velocity that means equivalent to the strain rate.

But there may be the combine effect of the (())(60:15) and strain rate, so if we consider the combine effect of the strain uhh stress and strain rate then we can represent the, again modify the Power law like this. So K Epsilon to the power n and strain rate to the power m. So this is the typical expression of the stress while we are considering both the effect of the strain hardening and strain rate.

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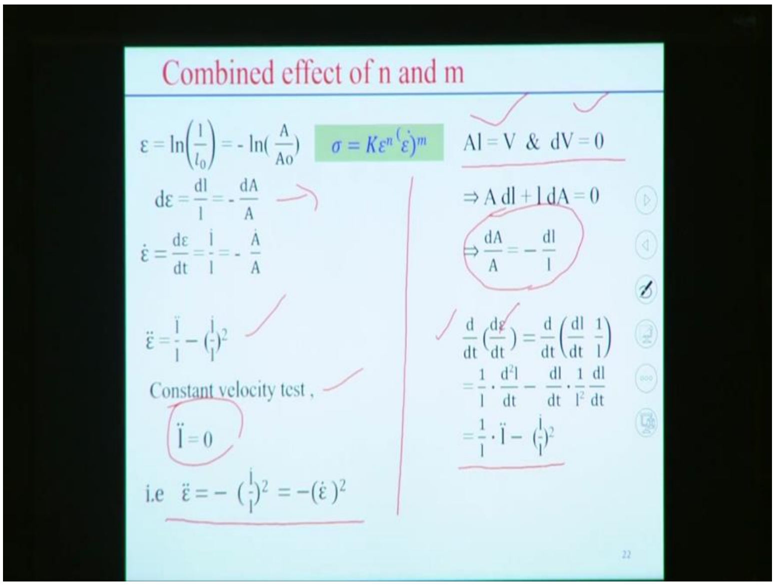

I am trying to explain the combine effect of n and m by looking in to this phenomena, if we see that different expression for that and let us look into that their significant of that. So first strain is a straight way Epsilon equal to m minus ln A by A0. So this is a straight forward calculation I am trying to represent and Epsilon dot actually represent the strain rate, which is length and cross sectional area is divided like that, sorry related to that like that.

So cross sectional area and length is volume and change of the volume is equal to 0, so from here we can find the relation between the length and the cross sectional area. Here, now this is the further relation, what is d by dt, d Epsilon by dt, sorry so basically this is strain rate, d Epsilon by dt. So here we can find out simple law of the using the simple derivative and you can find out this is the relation.

A significant point is there, similarly you can do the second derivative, the Epsilon double dot with respect to time, we will get this expression but constant velocity test, so basically we generate typical test of the uhh sample by keeping velocity as a constant. So in that case we can neglect the acceleration, so that mean l double dot will be 0, by considering this we can find out expression of the Epsilon double dot equal to this way.

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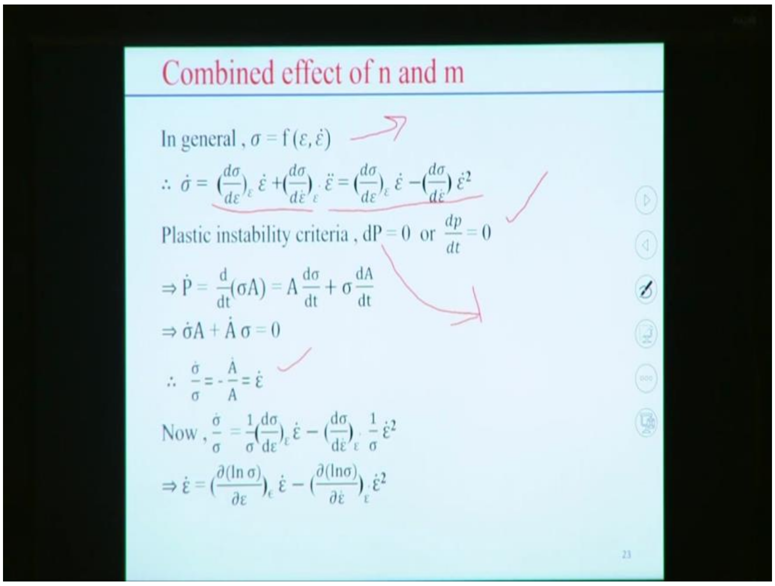

Now if we see in general the stress is a function of strain and strain rate because we are considering the combined effect of the both strain rate sensitivity index as well as the strain hardening coefficient. So then in that sense while this is the stress so derivative sigma dot dl with respect to the time, so we can, sorry d sigma dl, we can find out that derivative here in terms of partial derivative form and we can use this using the previous relation, we can find out all this expression.

But when you try to predict the plastic instability condition there is a change of the load equal to 0 or dp by dt equal to 0, in this case then you can find out the p dot equal to d by dt sigma, so accordingly we can find out this is the condition and we put it here and then we can try to find out sigma dot by sigma equal to in terms of other parameters, then we find out that Epsilon dot equal to this sigma by d Epsilon.

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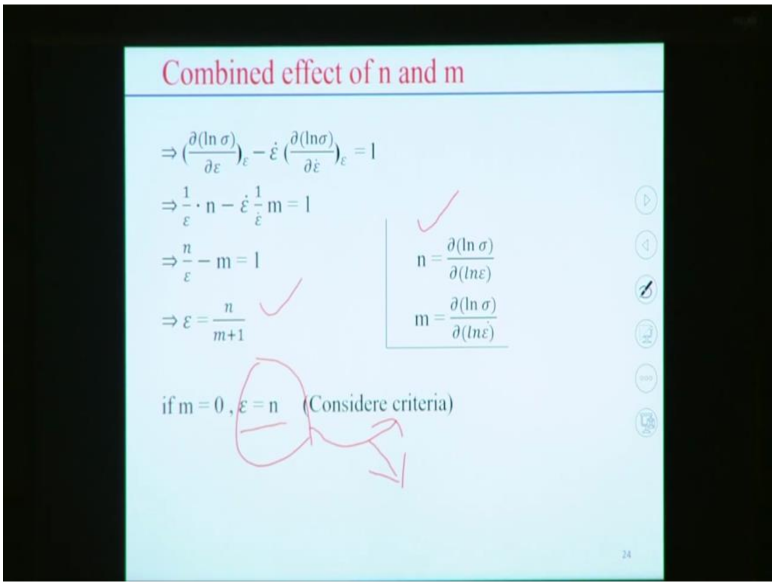

Now further this equal to 1 and here if you see that n equal to slope on the logarithm scale between the stress and strain and m is the slope on the logarithm scale between the stress and strain rate, simply putting this value we can reach this condition, sigma equal to n by m plus 1. So this is also plastic instability condition while we are considering the combined effect of whole strain and strain rate.

So this is the if m equal to 0 then E equal to Epsilon so this is, this condition actually for considerate criteria. So this criteria is often used and of course Epsilon equal to n and this criteria actually comes from when we are assuming the relation between stress and strain by the Power law.

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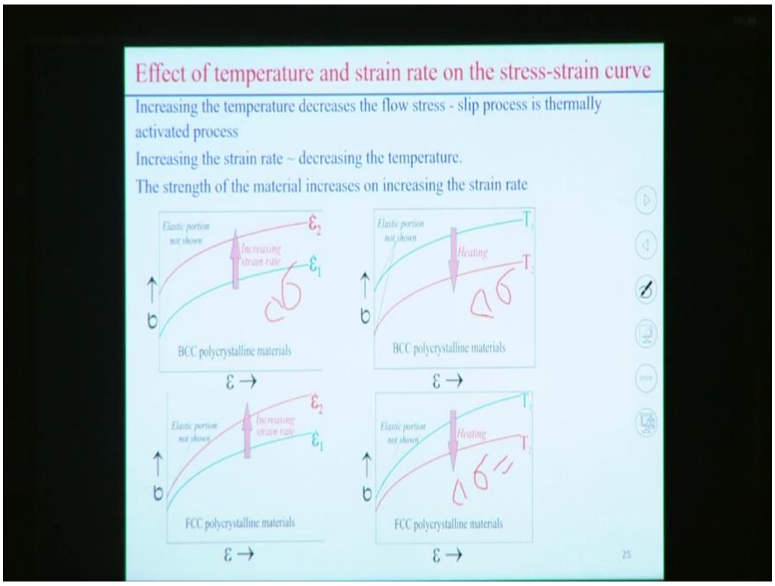

If we see the typical effect of the strain rate on the stress-strain curve actually the increasing the temperature decreases the flow stress value because it actually initiate a slip process and the slip process is actually thermally activated process. Similarly increasing the strain rate is having the similar effect of the decreasing the temperature or vice-versa. So decreasing the strain rate and increasing the temperature also give similar effect. So we try to investigate of simply looking into what are the different type of material and how it (())(65:46) in respect to the temperature and strain.

So first we look into this figure for BCC polycrystalline material, typically the stress-strain curve is like that, so stress and strain rate effect one specific, sorry, yeah specific strain rate and if we increase in the strain-rate then second curve actually represents the relation between the stress and strain.

So if we see that if there is a increment of the strain rate keeping as a temperature as a constant, for BCC polycrystalline material there is a increment of the stress level and that increment actually follow almost uniform flow of the deformation stage. Similarly for the same material if we consider that effect of the temperature for a specific strain rate then we see that increase in the temperature decreases the strain level throughout the strain (())(66:56) that gap between two different temperature the stress level is almost constant.

So in this case basically with respect to temperature delta sigma, so difference between the stress level throughout the deformation process is almost similar and that happens in case of BCC polycrystalline materials. But if you look into FCC polycrystalline materials if you see that having similar effect, so increasing the strain rate, the stress level actually increases but decrease in the temperature but increase in the stress level of vice versa increase in the temperature there is a decrease in the stress value in case of FCC polycrystalline material.

But in this case the increment of the stress throughout the strain value is not constant so there is a continuous increment on this thing. So this is the basic difference, the effect of the temperature and strain rate in case of BCC and in case of FCC materials.

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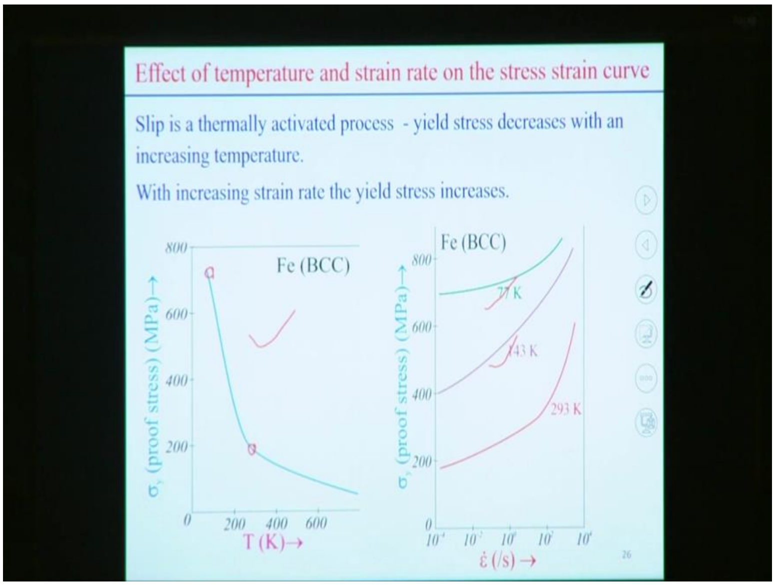

This is the typical representation for BCC materials, there is true stress or we can say flow stress value or sometimes we can use the yield stress value with increasing temperature. So initially at low temperature the (())(68:22) is very high, but within (())(68:24) of the temperature initially there is a high heat of decrement of the its value where as the temperature difference, after what it decreases to very low value at very high temperature.

But if you look into the for BCC materials effect of the strain rate would be seen that there is a constant improvement of the uhh yield stress value of true stress value in case of BCC material and BCC 290 Kelvin already so room temperature and this the low temperature, below room temperature 143 and 77 Kelvin. So in all the cases the increment of the strain rate that actually yield stress value in general decreases, sorry in general increases irrespective for a fix temperature value. But this effect is more when the temperature is high as compared to the low temperature, 77 and 143 Kelvin in this case.

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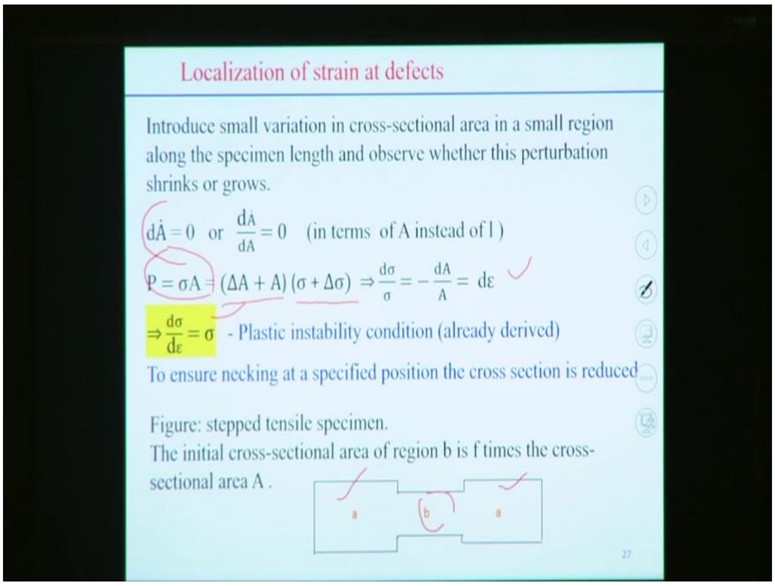

Now sometime it was found that if can, pre-existence of some kind of localization of the stress or pre-existence of defect one specific sample that actually imitate the necking process, so sometimes initially we make the samples not from uniform cross sectional will be intention (())(70:02), low cross section for specific part so that we can ensure the necking should start and that actually facilitated the different experimentation of the plastic behavior of the specific sample. But mathematically how we can predict this necking phenomen or how can represent, what is the effect of this necking over the stress-strain diagram.

Mathematically we can see the introduction of the small variation in the cross section area in the small region along the specimen length and observe whether this reaction actually further shrinks or grows with application of the load. Intersect that we say d A dot equal to 0 or we can say that d A dot by dA equal to 0. So here we are in terms of A instead of l that means we are representing in terms of aerial cross section instead of l.

So physically we try to look into that what is the reduction of the area what is the change of that at that point. So we will investigate it then we can mathematically we will introduce the instability condition at that point. Suppose load is represented by this P equal to sigma into A so that uhh that sigma A assume there is a variation of the sigma, sigma plus delta sigma and there is a variation of the cross sectional area delta plus A.

So from here we can derive that actually D sigma by sigma equal to minus dA by A and finally we can find out that this is equal to this sigma, that condition if we have already derive that consider as a plastic instability condition. So to ensure looking at a specific position that cross section is sometime reduced so it is necessary to analyze if there is a change of a cross sectional area actually how it influence to the stress-strain behavior. For example we consider one part having two different cross sectional area A and in between there is a cross sectional area b, so cross sectional area b is less than A.

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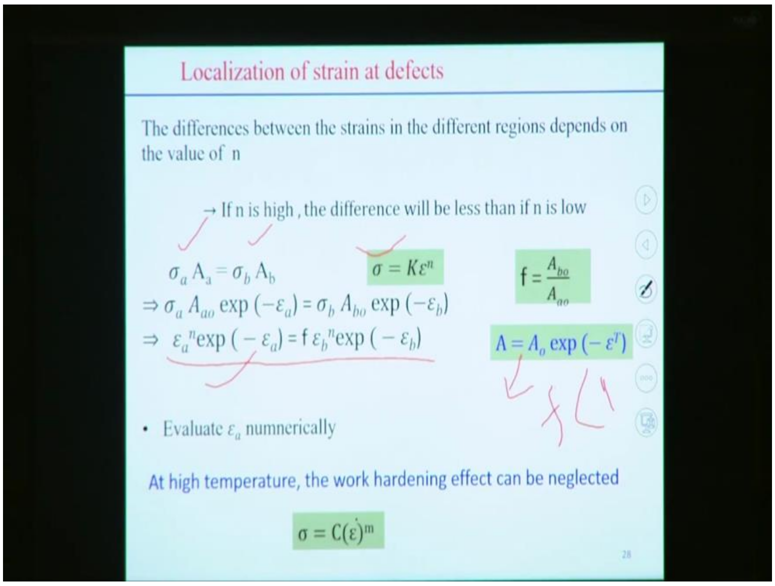

Now the difference of the between the strains of the different regions depends on the value of the n, that means strain hardening effect here we will consider. If n is high the difference will be less than if n is low. So let us investigate is mathematically how we can do that. Assume that there is application of the load f in the part, so that f will be transmitter throughout the n section, so at section A this is the load and at section b this is the load.

So that is the stress into at that time, at that point what is the cross sectional area and this is the stress and the cross sectional area at that point. Now uhh A Ad can also be expressed in terms of, suppose this is the relation that between the initial cross sectional area and some intermediate point what is the cross sectional area when there is a continuous deformation for the process.

So that relation we have already derived and it is connected with the true strain here. So similar expression we use A to, Aa 0, so Aa 0 actually represents the initial cross sectional area before the application of the load and Aa actually represents the current cross sectional area. So similarly we can do further for the cross sectional area b also then we can find we can use the relation stress relation using ithe Power law relation between the stress and strain, sigma equal to K Epsilon to the power n if we put it we can find out this expression.

So here if we see that in terms of Epsilon A in terms of Epsilon B, so we can represent these things and where f is the ratio of the initial cross sectional area, F Ab0 by Ab0. So it is obvious that b0, section b will consider as the less cross sectional area as compared to n. So f is basically less than 1 and this we can do the sensitivity analysis in the sense if the ratio f is less than 1 if given f is 99 percent, that means we consider f 0.99.

So the stress deformation behavior will be completely different from cross section A and B. Then we can do the further analysis by looking into this but here what is the, we need to evaluate Epsilon A or Epsilon B that is through numerical process. So simply (())(75:29) method we need to follow to find out the Epsilon A.

Similar kind of treatment can also be done specifically at high temperature when there is a effect of the strain rate is more significant. So in that case we need to consider sigma equal to C Epsilon dot to the power m, we can do the similar or we can do the similar treatment so similar way we can instead of sigma A here, so we can use the Epsilon A dot to the power m, so in terms of m we can find out the similar kind of expression.

So this is all about one dimensional stress-strain and what are the effect of the strain hardening coefficient at the same time what are the effects of the strain rate sensitivity index and what are the combined effect if metal behavior in such a way that both the strain hardening and strain rate, both are significant for the mechanical behavior of the material. So we can do all these analysis uhh from this expression what I have discussed just last, almost last one hour.

So next class I will try to analyze the basis plasticity theory, yield criteria, how we can find out and that will be very specific to the 2 dimensional or 3 dimensional stress strain. So thank you very much for your kind attention.