Introduction to Crystal Elasticity and Crystal Plasticity
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Week-03
Lecture-05
Continuum Plasticity

Hello everybody. Today we will discuss the different state of the stress specifically, a 1-dimensional, 2-dimensional and 3-dimensional cases. So, first we will try to cover up that 2 dimensional stress state and then we will try to discuss the 3-dimensional state of the stress.

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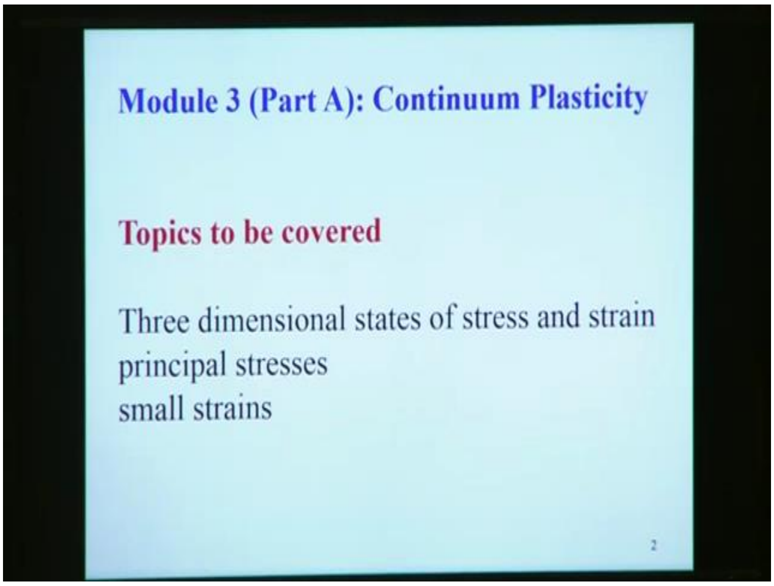

And how the principle stresses and principal strains can be calculated from a general stress state and finally we will try to discuss the formulation specific to very small strains. So let us start with this (st) analysis of the stress.

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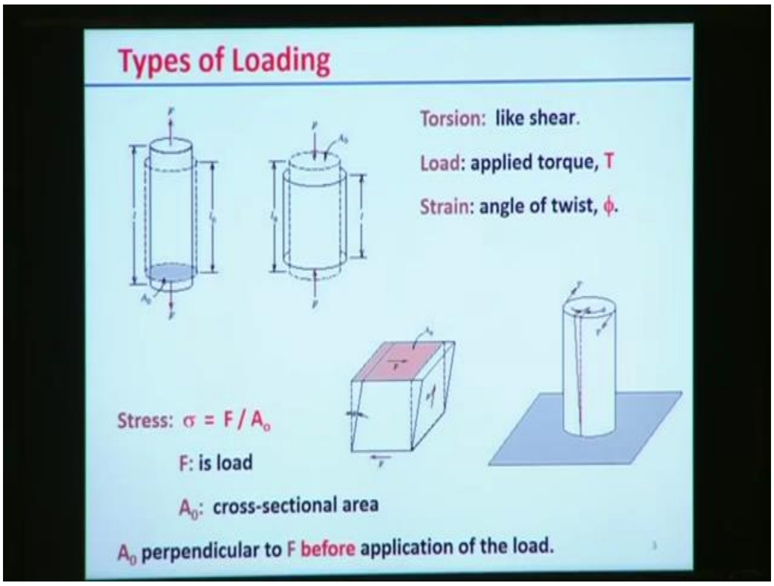

So the generation of the stress actually (ta) depends on the type of loading so the load can be applied at different direction when we try to estimate the load then we can consider the area over which the load is acting at different orientation and accordingly we can find out whether the existence of any normal stress or any shear stress, it will produce. Let us look into the different type of loading, if we look into the figure, first figure, we see that a cylindrical bar is specifically subjected to a load F and there is a sum expansion in 1 direction at the same time contraction in the radial direction.

So, this type of loading we can say the tensile loading. And if we look into the second figure, here if we applying the load but the direction of the F is just opposite, that means it is a compressive kind of load. So, this type of load actually produce some amount of the stress inside the body but we need to consider some representative volume from the body to find out the different stress state. Now if we look at another aspect the torsion also produce some amount of the stress.

For example, if we apply torque in this sample, maybe on the cylindrical element where 1 side is fixed, another side is open and it will try to produce some (am) amount of the strain as well as the stresses but here the strain is measured, the angle of twist, so that angle of twist is the what was the initial position and on this (s) face what was the (an) final deformation and the angle between the 1 specific, along the radial direction, that means 1, with respect to 1 radial what is the deformation over the circumference and that will measured by the angle of twist phi, so that is also measure of the strain in this case.

Just similarly we can, how we can define the application of the normal load and then we can find out the different kind of shear stresses here also. So in this case on this face there is application of the load and that load is parallel to the cross-sectional area, so in this case this angular form actually produce the shear strain and this is the amount of the strain in this case and shear stress we can define that load divided by the cross-sectional area.

So this cross-sectional area in this case which is parallel to the applied load so that actually estimate the shear stress and what was the initial case the stress is defined in this case that application of the load divided by a cross-sectional area but in this case cross-sectional area is normal to the applied load. So A0 is the perpendicular to the F before application of the load or maybe that A0 may be the parallel to the application of the load.

So depending on this whether, what are the nature of the stress, whether it is normal stress or whether it is shear stress it will produce or (nature), depending upon the type of the load whether it is produce the tensile load or whether it can produce the compressive load or whether it, whether we need to apply some torsion that actually decides the different type of stress and stress generation inside the representative volume of a of an specified element.

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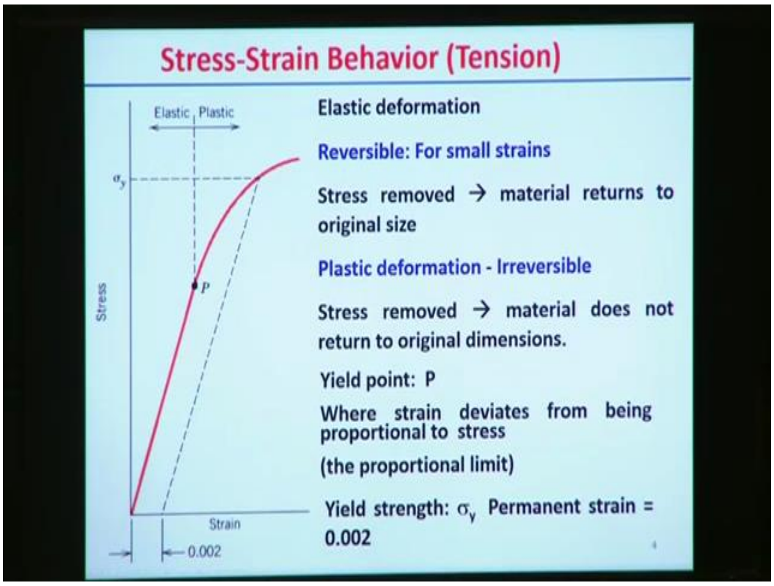

So now depending upon the type of load we can (dis) we can discuss that (al) this stress and strain behaviour with the application of the load. Let us look into this simple curve where it express the amount of the strain and correspondingly what is the variation of the stress with the application of the load that is represented by this graph. That is called simple stress strain behaviour with application of the tensile load.

So first up to point, suppose we starts from point O, so upto point P, the stress is proportional to the strain, so that means stress strain follow some linear relationship upto the limit P point and that corresponding P point actually decide the yield, yield stress value and this, upto this point the strain is deformation is actually reversible in the sense that if we remove the load at the pointy then it will come back to the original position though, so there may not be any permanent deformation in this case.

Now, corresponding point P we can define the yield, yield stress or maybe we can say that the response to the elastic limit for this specific material. So it is known the fact for practical material that the, this reversible nature of the stress or maybe exists with the very small amount of the strain. But maximum amount of the strain actually exist over the plastic zone, plastic deformation (zone) in the sense that when we cross the point P and the, with the further deformation of the material then it will produce the permanent deformation and that, that permanent deformation is not reversible, that is irreversible and that deformation is called as plastic deformation of the material.

So even for example if we remove the load at some other point beyond P, and then when it is then it will not come back to its initial position, so there must exist some amount of the deformation here. So that P is generally called the yield point that is the limit or that point actually decides whether stress is proportional to the strain or that means linear relationship between stress and strain exist or some non-linear relationship exist beyond that point. So up to that point elastic part and beyond that point that is generally defined in stress strain diagram that is the plastic part is there.

But this sometimes in the actual material, when we try to generate the stress strain diagram for a specific material so yield point is basically well-defined and it can be observed, there is a smooth transition of the slope at that point. So practically can identify that point and that means the yield point for a specific material. But certain cases that some other material or maybe very large group of materials, actually there may not be very transition zone from elastic to plastic deformation. So that transition zone is very difficult to identify when we simply look into the stress strain diagram of that specific material.

So in this case we generally consider the offset yield point, so that offset yield point is defined 1 (s s) permanent amount of the strain that is (point) point 002 so that point, and if we draw with (resp) from this point 1 parallel line of the initial slope of the curve then it will intersect certain point; suppose in this case this is the point, say point Q, so that Q actually defined in this case is the yield point of this specific material. So in this strength can be defined here in this case when it will try to produce the permanent strain point 002. So that is often called offset yield point.

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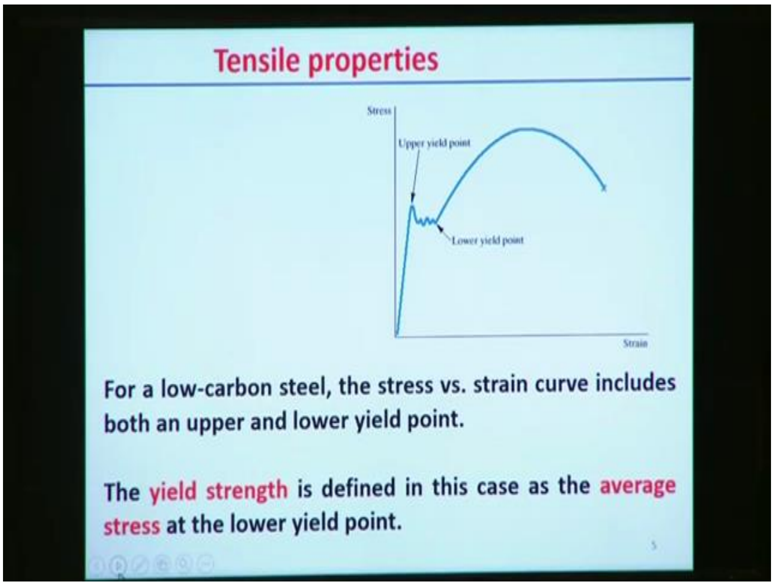

If we look into that most common material low carbon steel we can find out the typical stress strain curve and where we can find out that there is a 2 yield point, 1 is a upper yield point and the another is a lower yield point and this point is (cons) optimum point is corresponding to ultimate tensile strength and here it is the fracture point. This is the typical points we generally observe in the stress strain diagram. But in this case the yield strength is defined by the average stresses at the lower, lower yield point.

So, so there is, there is a 2 different yield point in this case, so generally we consider the yield point is the average value with reference to the lower yield point and then beyond that zone we can assume that this is the plastic deformation zone. So, low carbon steel up to, from here to here, its follow the linear (elasi really) linear relationship between stress and strains (stress) so that slope actually indicates the Young's modulus in this case.

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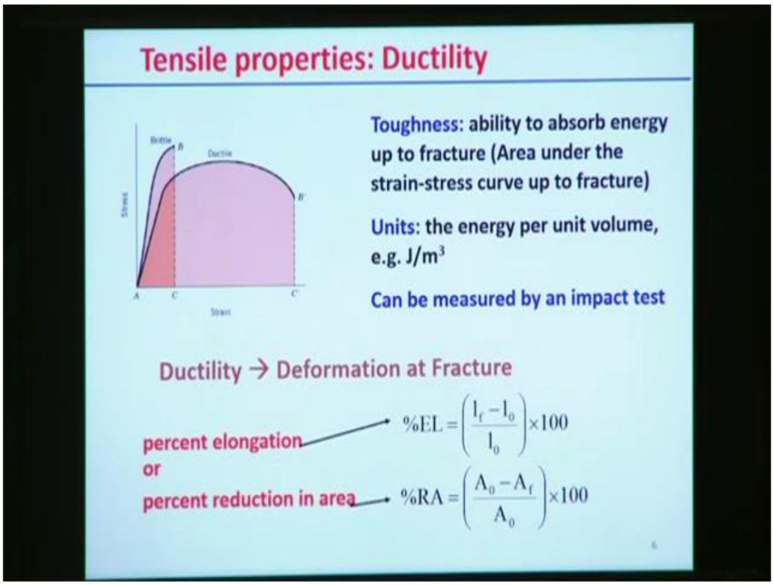

Now, if we do further analysis but if we try to evaluate the different tensile properties (fot) from the stress strain diagram also. First is that toughness; toughness actually ability to absorb energy before fracture, so (s) for example, that stress strain diagram for ductile material actually represented by this curve A up to point B dot, so at point B dot there is a fracture happened. So, the, upto that point what is the total amount of energy is absorbed by the material before fracture than that amount of energy is a measure of the toughness for the specific material.

So, physical representation of the toughness over the stress strain diagram for specific material is basically the area under this curve. So, the total area under this curve actually represents the amount of this toughness. So, that actually represents the amount of the energy absorbed before fracture, so, that amount of energy actually represented for unit volume. So, if it is possible to represents this stress strain diagram with some analytical expression is a constant of stress in terms of strain then simply we can, by integrating, suppose this is X axis and this is Y axis, and by integrating ydx over the limit then we can find out the amount of energy per unit volume in this case and that is equivalent to the toughness for the specific material.

So, this, but how we can measure this properties, this property can be measure 1 specific test that is called impact test; the measure the toughness measurement can be done. For example, if it is a brittle material so, the deformation level so plastic deformation zone or (deformation) total deformation is very less as compared to the ductile material. So in case of brittle material if this area represents the toughness, so, which is very small as compared to the ductile material.

So we can conclude in the sense that brittle material is generally having low toughness as compared to the ductile material and the physical interpretation is the, over the area and if we simply calculate the area under this stress strain curve for different type of material that will get the idea about the toughness of the specific material. So now, we can measure the ductility also, so ductility if we take the reference point deformation at fracture then we can simply measure the percentage, in terms of percentage elongation while in terms of percentage reduction in area, that actually the measure of the ductility.

For example, in this case the percentage of elongation can be measured, what was the final length and what was the initial length, with respect to the initial length and if we multiply by 100 then that actually indicates the percentage of elongation and that actually measure of the ductility in this case. similarly we can measure the ductility also in terms of reduction in the cross-sectional area.

So, A0 was the initial cross-sectional area, AF was the final sectional area, and A0 is the initial, and if we multiply by 100 then we can find out the, this is the change of the cross-sectional area, or maybe percentage reduction in the cross-sectional area that is actually the measure of the ductility. So normally that percentage elongation or reduction in cross-sectional area in brittle material is very less as compared to the ductile material.

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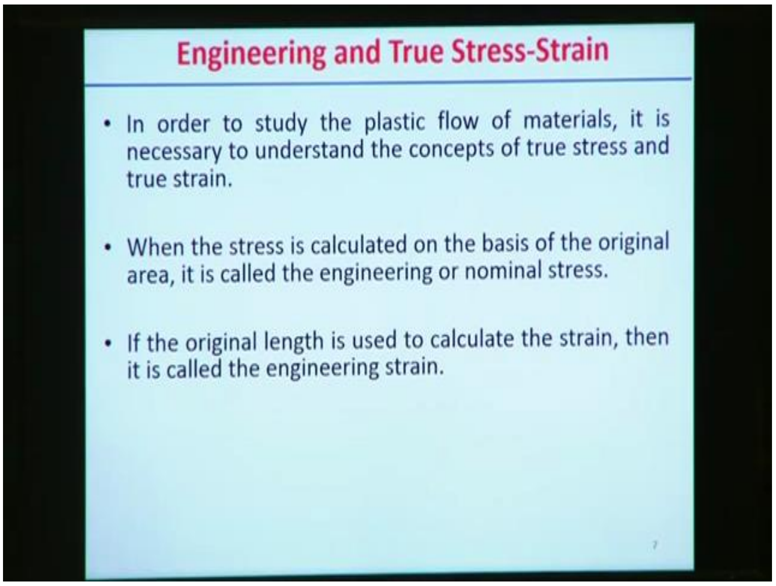

Now, the point is that is very much significant to analyse the engineering stress strain curve or true stress strain curve and where it is applicable (mo) mostly. So, in order to study the plastic flow of the materials, it is necessary to understand the concepts of the true stress and true strain, because true stress or true strain actually rely on the instantaneous deformation of stress value during the application of the load.

On other way when the stress or strain is calculated based on the original or initial cross-sectional area or with respect to initial length then that actually the measure of the engineering stress or strain or, or we can say that is the nominal stress or strain. So, let us look into how this engineering stress or true stress strain are related to each other, but it is necessary to mention that engineering stress strain is basically mostly used to analyse the (ma) material properties of the elastic limit but when we cross the elastic limit the, the plastic zone then true stress strain is more appropriate to analyse the different material properties or different characteristics phenomena of the stress strain diagram with the application of the certain amount of the mechanical load to the material.

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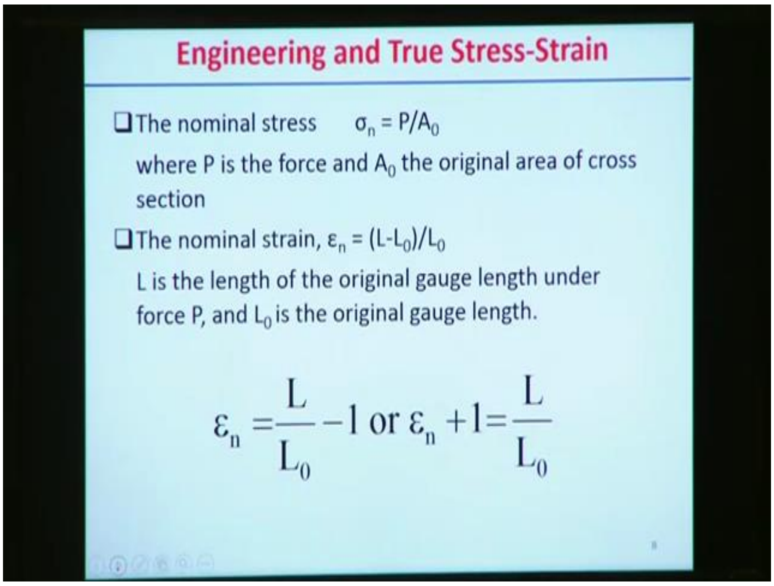

So, nominal stress here we define that, what is the application of load P and what was the original cross-section length, but if we define the nominal strain or engineering strain say that change of length, L is the final length and L0 is the initial length with respect to the initial length. So, that engineering strain can also be further correlated like that. So, L minus L0 by L0 that is equal to L by L0 minus 1, or Epsilon n plus 1 equal to L by L0, so this is the correlation between the strain with respect to the original length and the final length while you are considering the engineering strain.

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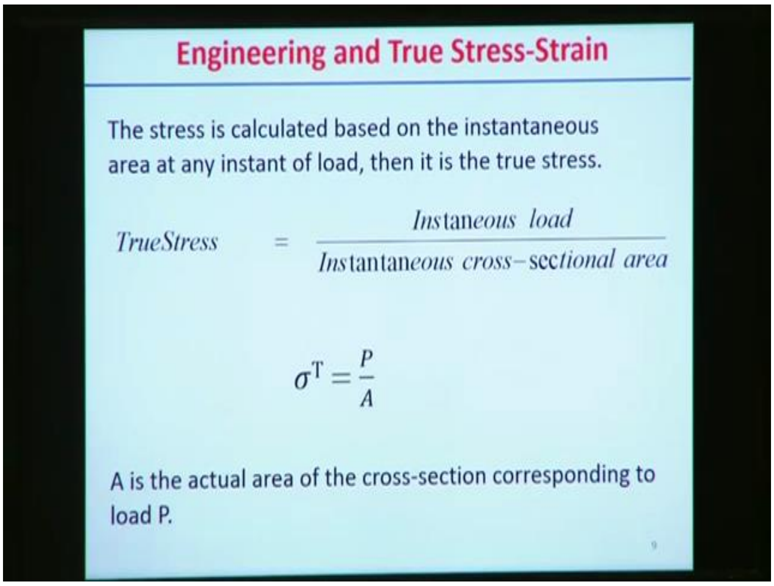

Similarly, when we try to find out that true stress value, the true stress value actually defined by the ratio of the instantaneous load with respect to (indus industaneus) instantaneous cross sectional area. So that means the Sigma T, that is a symbol we have used here to represent the true stress that is P divided by area, P is the applied load and A is the corresponding area at a specific point. Since during the plastic deformation there is a change of the cross-sectional area so it is justifiable to use that, what is the area of the cross-section at specific point and what is the load at the specific point that represents the true stress in this case.

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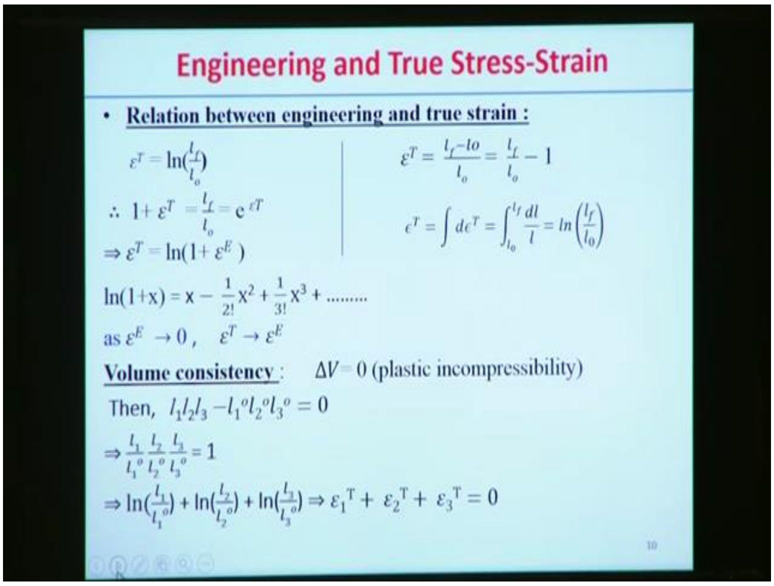

So now we try to find out that, what is the relation between the true stress and true strain? Let us look into this that this we have defined that true strain, sorry this would be (s) engineering strain. So, this would be engineering strain, so engineering strain is the change of the final minus initial length with respect to the original length and that is the relation. But, when we try to estimate the true strain we assume that there, 1 element from of length L and that this instance the increment of length is dl.

So at this point the increment of the strain d Epsilon, the elemental strain that is true strain actually, that can be defined by increment length with respect to at that point what is the original length. Now while you try to estimate the absolute value of the true strain in this, in this case, that we, we can do the integration over the elemental true strain and this is the integration form and that, assume that there is a deformation happens from (u) initial length, initial length to; suppose this was the initial length to final length, so that limit starts from initial length L0 to final length LF and finally it comes logarithm LF by L0.

So this is the expression of true strain in this case. Now we can further correlate this thing that ; so 1 plus then we can find out the LF by L0, LF by L0 which is equal to, equal to engineering strain plus 1, so 1 plus engineering strain is equal to LF by L0 that is equal to exponential true strain. So then it has finally come that true strain equal to logarithm of 1 plus engineering strain. So this, this is the relation between the engineering strain and true strain.

But if we further (e) expand these things 1 plus, logarithm 1 plus X equal to X minus 1 by factorial 2 X square; like this and if the amount of the strain is, tends to 0 then we can find out the true strain actually tends to engineering strain. So that means the small strain cases, the value of the engineering strain and true strain is almost equal, there is not much difference.

Now, we trying to find out the other correlation in, when you try to, in terms of the true strain or engineering strain let us look into that. So during deformation mostly there existence of plastic incompressibility, that means there may not be the change of the volume but change of the dimension of a specimen. So in that sense suppose the dimension was L1, L2 and L3 and that is the final dimension and initial dimension was L10, L20 and L30. So then, change of the volume, final volume minus initial volume is basically in this case, is upto 0 to maintain the plastic incompressibility.

So, if we assume that change of volume there is no change of volume, so we can further decompose the, in the terms of logarithm form then we can find out that, that each component actually represents the true strain. So, maybe true strain component, second component and true strain third (component) or maybe we can say X, Y and Z that is actually 0. So this is the relation when there is a (nay no) change of the volume during the plastic deformation, so that true strain actually holds good this relation.

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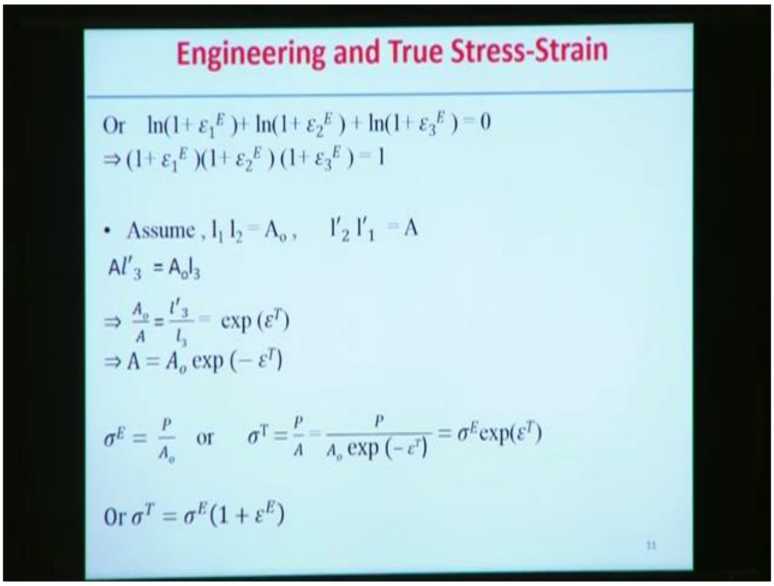

Now, similarly, true strain equal to logarithm of 1 plus engineering strain, so this in terms of engineering strain we can find out equal to 0, so that means this is equal to 1. So, that is the relation actually holds good in terms of the engineering strain when we (maintain) we assume that there is a no change of volume during plastic deformation. Now, we can do further calculation also. Suppose there is a cylindrical element and in that element first 2 dimension L1 L2 is equal to area A0 and L2 dot, L1 dot equal to A, that is the final cross-sectional area after deformation.

So this is, should be maintained volume consistency so there is no change of volume, so volume remains the same before deformation and even after deformation. But the dimension change, that means cross-section and another length dimension actually changes in this case then we can find out that A0 by A is L3 dot by L3 which is equal to the exponential of (engi) true strain and that A equal to A0 to exponential of minus E to the power Epsilon to the power T. So, that actually represents the (area) relation between the cross-section, initial cross-section and final cross-section is in terms of true strain.

So, we can do further calculation of the stresses; so engineering stresses is defined by the load divided by the original cross-section area or we can represents other way, the true stress, true stress is basically the ratio of the instantaneous load and the cross-sectional area at that point, then we can do (ec) we can put the expression of this cross-sectional area and we can find out that the relation between the true stress and engineering stress in terms of the true strain and, or we can find out that engineering true stress equal to in terms of engineering stress and engineering strain. So, this correlation between engineering stress and engineering strain is very much helpful to do the further calculation of this.

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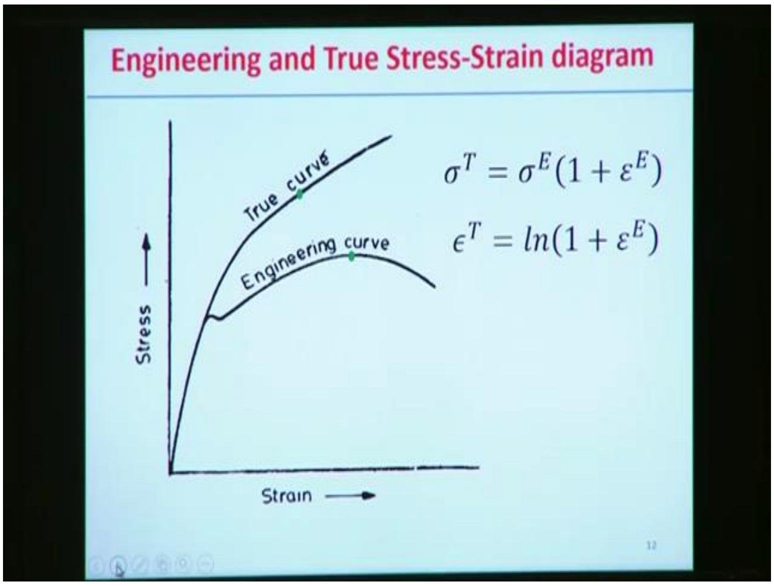

Now, when we try to represents the stress strain diagram of a specific material then we can obtain the engineering (most) most of the cases we obtain engineering stress, stress strain diagram in the sense that we are using the engineering stress and engineering strain which takes into consideration the initial or original length or original cross-section to calculate the stress and strain value in this case.

Now, this is the relation of the stress, engineering stress and (enginee) true stress; this is the relation we used the, between the true strain and engineering strain. Now if we put into that, up to the yield point there is a more or less the true strain part and engineering stress part is almost equal but practically this, this upto this common point, actually this is, this amount is the very less amount, so very less amount of the strain actually.

So, in this case engineering stress and (en) true stress and engineering (st) strain and true strain they are almost equal up to that point but with the when it just enters to the plastic deformation zone, so there is some (radiation) there is a sum deviation of the engineering stress strain curve and the true stress strain curve. If we look into that true strains, true stress strain curve is seen always, it is, there is an increasing nature, stress is actually increasing nature (we) when we consider the true stress strain diagram.

But in case of engineering stress strain diagram, this is the amount point that is called ultimate tensor strain and after that making happens and there is a reduction of the stress when we have finally, rapture happens or fracture happens at a specific point which is less than the optimum stress value. Since the true stress strain curve consider the instantaneous area, so with the, in the plastic zone with the further deformation of the material there is a reduction of the cross-sectional area and specific to, in the (())(29:01).

So, the, when there is a reduction of the cross-sectional area then the stress will be actually increases. That is why the true stress strain curve, always we getting the increasing trend of the stress value with the further deformation. Now, if the, this point actually represents the ultimate tensor strain with reference to the engineering stress strain curve. Now, from that point we can, but this point may not be the same strain level in case of true stress strain curve.

So, so there is a some deviation of the actual, that (act) actual ultimate tensor strain with respect to the engineering stress strain curve and that point is different in the true stress strain diagram because if we look into the relation between the engineering stress and true strain we found out that strain value, actually the strain value is logarithm of 1 plus engineering strain. So if this is the engineering strain value, so 1 plus engineering strain value and if we consider the logarithm scale, then actually true strain value is less than that of engineering strain value, so that is why this point is shifted to other side, lower side.

At the same time, if we consider the true stress value, the true stress value actually it is since the, this component is more than 1, so true stress value is always greater than the engineering stress value, so the, the stress value actually more this case. So this is the amount of the stress is more corresponding to that point and this is the amount of the strain corresponding to this less, corresponding to the ultimate tensor strain with reference to the engineering stress. So it is, by using this relation engineering stress and true strain or engineering strain and true strain we can predict the true stress strain diagram with reference to the engineering stress strain diagram.

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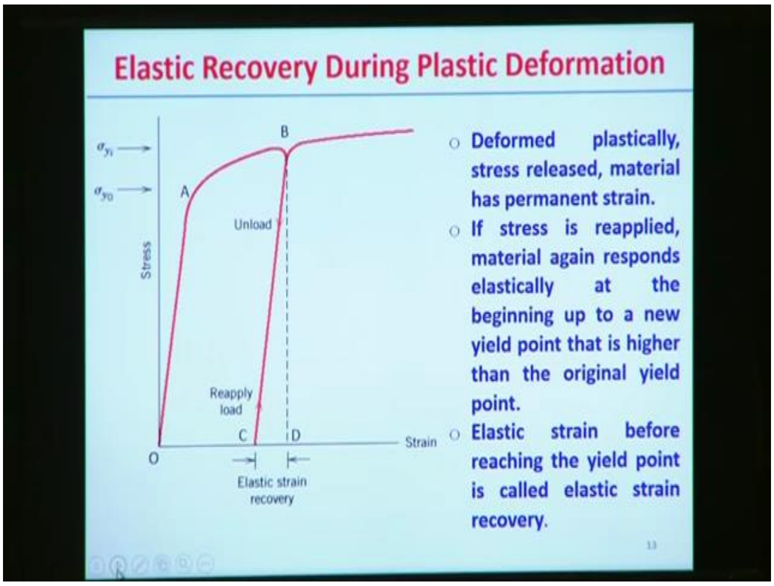

Now, if we, the, what situation arise something like that if we try to deform the material, or uniaxial deformation of a specific material upto certain point say point A, it follows the elastic deformation, beyond that there is a non-linear, if we see the non-linear shape of the curve and up to point B there is a non-linear deformation and the deformation actually enters the in the plastic deformation zone. So, but at the point B when there is a release of the load then the specimen will come back to the point C , so that point is, so at point C, but if we see from the diagram, so there is a some existence of the permanent amount of the strain.

So that, that amount of the plastic deformation actually exist within the sample that is not recoverable. Since it crosses the elastic limit and it enters the plastic deformation zone. Now, what way generally the theoretically this B comes to the point C but what are the way it is following. So in this case the, from B to C it is following the path which is the, parallel to the initial slope of AB, so basically parallel to the line of OA, in that path it will try to follow. So, to following this path now if we project P, actually when we deform up to point B, so up to OD is the total deformation or total amount of the strain.

But after removal of the load actually the material come backs to the point C, so there some amount of the recovery happens that is the amount of CD so that CD is basically called the elastic strain recovery. So this kind of behaviour is observed when material follow the mechanical deformation (s) nature, in nature like elastoplastic deformation. So that CD we can easily calculate the amount of the elastic recovery zone if we know the yield point values and we know the Young's modulus also, because Young's modulus (OC) OA, that is represents the slope and this path BC actually parallel to this slope.

Now again from the point C and if we deform the material then it will goes back to, it will, it will start yielding at point B, and then with further deformation it is follow the plastic (deform) path which is in the plastic deformation zone. So in the second time during the reloading actually the D point actually acts as the yield point in this case. So there is a change of the yield point during the second time deformation from point A to point B. So, that actually happens due to the strain hardening mechanism for this specific material.

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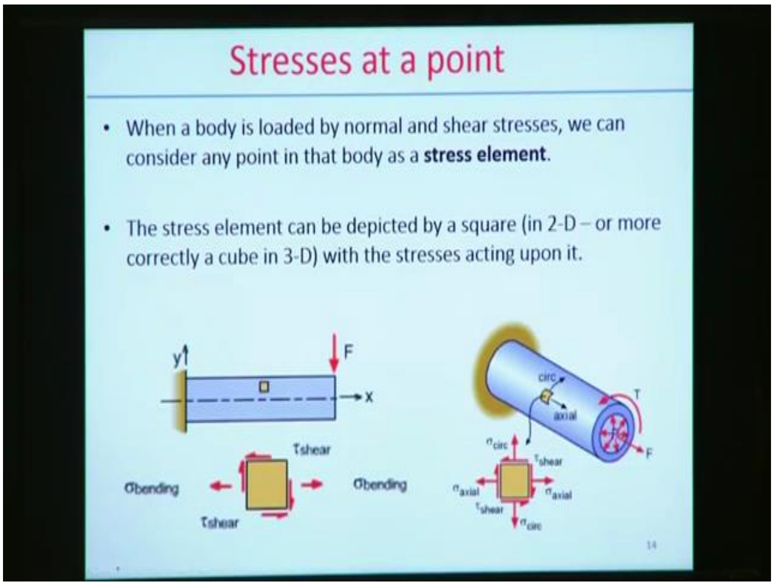

Now we coming back to from the 1 unidirectional stress (stress) stress or strain analysis, what is the difference between the engineering stress and the engineering strain. Now, we come back to that point of stresses at a point, how we can define the stress. So, when a body is loaded by normal or shear stresses we can consider any point in that body as a stress element. This stress element can be depicted by in 2-dimensional form or more correctly by cube in a 3-dimensional form with the stresses acting on it.

For example, if we consider this element there is a application of the load, this is a beam but we consider this a there is no variation of the any load or stresses along the normal to this beam, so this application of the load F, this is the direction and if we consider a small element (s) specifically in 2-D analysis we just pick up 1 square element and that square element we can represents the, what are the different stresses is acting on the, on that element. Here if we see with the application of the load, so there may be some normal stress will be acting, that is normal acting normal to the edge.

So, that, here in this case it actually produces the bending stress but it is normal to this edge but at the same time it will create the shear stress also that is acting parallel to the edges. So that shear stress we can define here. So with a combination of normal stress, that means bending stress and shear stress we consider this as a stress element and then we can further analyse the amount of the maximum stress or minimum stress like that. Similarly, this is around cylindrical element, it is subjected to a torque T and at the same time it is subject to a normal load F, T and F.

Now if we consider 1 element on the surface then it can be represented on this element that it is subjected to simply the axial load due to the application of the load F, but at the same time to do the application of the torque T it is subjected to some, or it will try to produce some amount of the shear stress. And how represents this element, it is like that we consider firstly a square element and within that element there is a axial stress is acting and at the same time circumferential stress also acting but at the same time there may be the shear stress on the edges.

So this is the description of a 1 body is subjected to different kind of loading and that we can represents this loading in terms of the small element and what are the different stresses is acting on that element, specifically the 2 types of stress is acting here,1 is the normal stress, another is the shear stress. So now we can further analyse the stress state.

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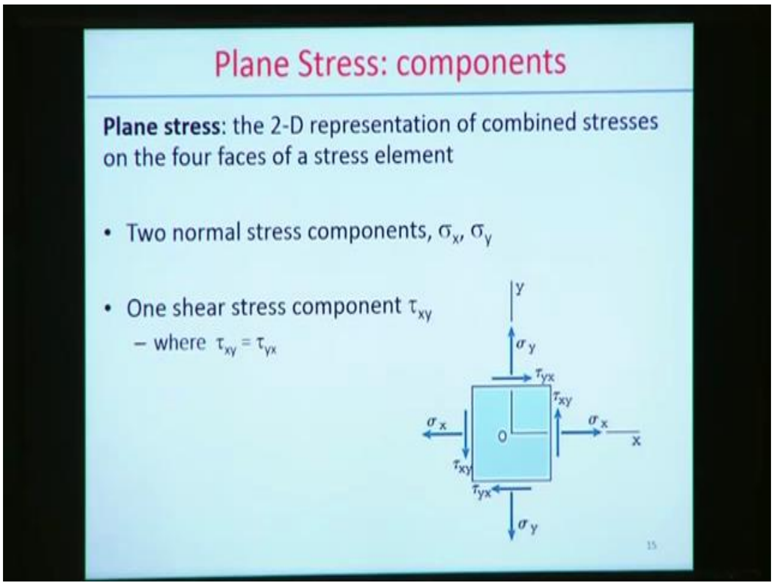

Now, we consider the plane stress condition for example the (third) stress in the third direction is basically (wi) 0. So, with the plane stress components this is the element and we define this as the X axis, this as the Y axis. Now, along the X axis, positive X axis the stress is defined by Sigma X which is considered with this configuration as a positive, similarly this direction this Sigma is (also) also considered as a positive in this case.

And similarly Sigma Y is acting along this (direction) positive Y direction so this is Sigma Y and this notation Tao XY and Tao YX in this case we can consider is as a shear stress and this is considered as a positive value. So, we further discuss about the sign convention with the positive negatives of the shear stress or (no) normal stress and the thing after few slides.

Now here is basically 2 normal stresses is acting along X and Y that is Sigma X and Sigma, 2 different normal stresses and the 1 shear stress components acting because we are assuming there is a not rotation of the element so Tao XY equal to Tao YX so convention of this is like that with the direction as well as how we are writing the Tao XY and this is the Tao XY and this is also Tao XY and other cases Tao YX, and this is the Tao YX. So, but in this case Tao XY equal to Tao YX. So, this is the 2-D representation of the stress state.

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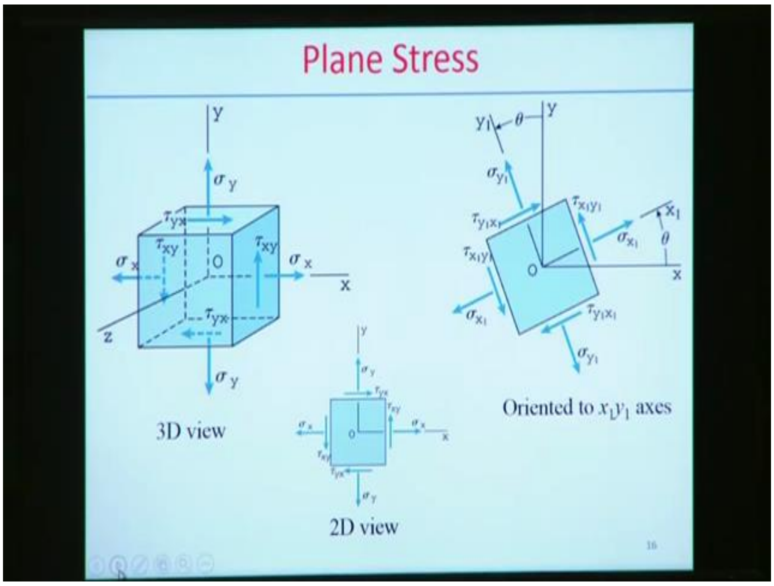

Now, what will happen if we try to represent the stress state with a certain specific angular certain oriented . So, this is the 3 dimensional stress state, first figure 3-D view if we look into 3-D view then it will be very obvious (s) normal stress is acting on this face and shear stress is acting parallel to this face Tao XY, so in the sense Tao XY. So that means it is the second Y is the, that represents the witch Y direction, positive Y direction and it is a and other, other component is the X, X.

Similarly, Tao YX, so it is acting along the YX the second component along the direction X and this is acting over the face. And similarly the Sigma X is acting normal to the (oppo) sorry Sigma is the another; the Sigma who is acting normal to the Y axis. So, from, this is the typical 2-D view that, and now we will try to explain that if the, there is a orientation with (re) with respect to X axis (angul) angle from the Theta and then (it) it will define the new axis with the X1 and Y1; so then with respect to the X1 and Y1 axis what are the different stress that means normal stresses and shear stresses and how we can correlate with respect to the original axis, that means Sigma X Sigma Y and Tao XY.

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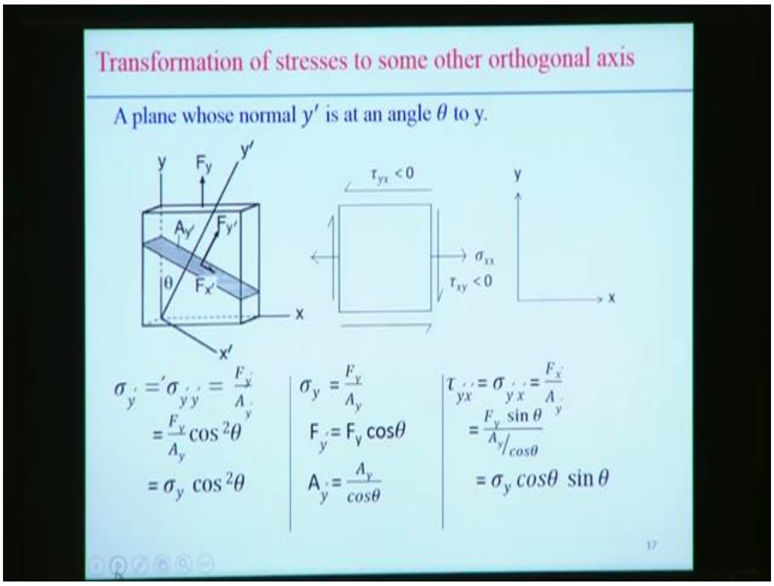

Now, if we take the transformation of the stresses (or) the taking (wi) simple system how this transformation can be done with some orthogonal axis. Suppose, the 3 dimensional view is the, this is the element and FY and along the direction Y so there is a acting of the load is FY and similarly normal to the X axis the loading, load is FX. Now we consider a plane whose normal Y dot is at angle Theta with respect to Y.

So, we consider 1 plane but that plane is basically parallel to the Z axis. So we are not considering the, any variation or component of the stress along the Z direction. So, that plane we define in such a way that Y axis, Y axis actually Y axis transformed to the Y dot (s) that Y dot actually normal to the, normal to that plane. So, and that plane, that Y dot and Y is making an angle of (from) Theta. Similarly, we transform the X, X dot which is another orthogonal axis with respect to Y dot as X dot.

So, we define the different stresses here; first we define the stresses here Sigma Y, Sigma Y equal to FY by Ay. So, that was the original so that is a (s) stress. Now, FY dot, that means transformation of the force is respect to the, or along the Y dot direction, that FY dot can be represent as FY cos Theta, similarly the area of that plane AY dot can be represented AY by cos Theta. Now, the normal stress along the Y dot direction, Sigma Y dot or maybe we can say Sigma Y dot Y dot is equal to FY dot by AY dot; that means along that Y dot direction what is the load or what is the force and with (resp) what is the normal area with respect to the Y dot axis.

So, if we put that FY dot here, FY cos Theta, divided by AY by cos Theta, similarly it is find the FY by AY cos square Theta, and FY by AY we already define that is the Sigma Y that is with reference to the initial position, so Sigma Y cos square Theta. So (in) the, geometrically if we look into all this things so it is possible to transform from 1 (co) 1 component of the force that the new stress value, so that stress value is defined with respect to Y dot axis in terms of the Y axis and that angular relationship between Y and Y dot in terms of Theta, so this is the expression.

Similarly, we can find out the shear stress as well also. So, Tao Y dot X dot, that means what is a shear stress is acting on this plane, so on the defined plane; that means it is definitely acting the, parallel to that plane. So, in this case or notation we can use this Tao Y dot X dot which is similar notation of Sigma Y dot X dot; that is equal to the area acting normal that along the X dot direction. So, along the X dot direction that means FX dot the component is here FX dot and what is the area AY dot, that is the area. So, FX dot is actually FY sin Theta, similarly A dot Y, AY dot is AY by cos Theta then we can find out that shear stress is (ac ac) actually Sigma Y cost Theta sin Theta.

So, with this transformation or some (geometry) geometrically here we can find out the, what is the stress state at the, some other axis or some other axis system which is analogous to the or with reference to the initial axis system. So, here now (h) how we can define the sign convention, shear stress and normal stress here; so if it is defined as the X and Y axis like that. Now that normal (sh) if Sigma X is greater than 0 then we can (find) we can consider this as a positive, but if we consider the sign convention Tao XY in this case, this is the, this is the representation of Tao XY.

So in this case it is positive sorry in this case it is negative value, but we can define in such a way that Tao XY, so it is, so direction is like that, it is the positive X; this side is the positive X but here, opposite to the Y so negative Y direction, that is why this direction actually produce the negative value of the shear stress, that is the representation of the shear stress, if it is possible to, so the shear stress is like this and then we can say Tao XY is greater than 0, that means (this) in this symbol actually represents the positive value of the shear stress.

Similarly we can find out this is also (nega) this direction also represent the negative value but at the opposite side, this actually represents the positive value of the shear stress. So with a axis system we can define the we can decide whether it is a positive shear stress or the negative shear stress or what is the positive normal stress value or negative normal stress value with the, this sign convention.

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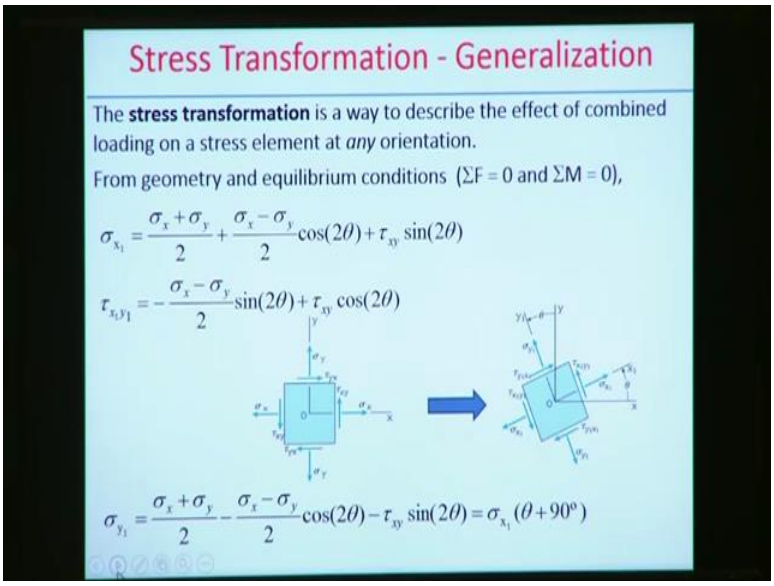

Now we come to that point, stress transformation in general form. So stress transformation is a way to describe the effect of the different combined loading conditions and on a (s), combined loading condition that actually act on stress element and, at any orientation. So, at any orientation if there is a application of the load that we can represents the 1 stress element and within the stress element, different orientation, we can find out the new values of the stress or strain with, if we know the (a) actual geometric orientation of the element.

So, from the geometry and equilibrium conditions, so further we can, if we follow that sation of F equal to 0 or moment equal to 0, so that is actually represents the equilibrium condition and from the geometry this geometric condition, we can (find) further find out the stress value at the different axis system. For example, in this case, this is the initial (defa def) definition of the stress state on an (a) on an square element, the Sigma X, Sigma Y Sigma and Tao XY and Tao YX.

So in the sense here we show the direction (in) in such a way that all, actually represents the positive value. There is no negative value in this, with this specific representation. Now, suppose there is a orientation at angle Theta and that will produce the 2 orthogonal axis system, X 1 and Y 1 with the Theta and on the X 1 and Y 1 axis there is a (represent) of the, representation of the stresses like X 1, Sigma X 1, Sigma Y 1, Tao X 1Y 1, Tao X 2 Y 2, like that.

So, in this case the, on the new axis system or the transform axis system, we can find out the Sigma X 1 equal to in terms of the Sigma X, Sigma Y the angular form and in terms of Tao XY. Similarly, we can find out the Tao X 1 Y 1 in terms of the Sigma X Sigma Y and Tao XY, so we use the geometric configuration and that means we know the angular what is the orientation angle Theta and we can find out the Sigma Y 1 also.

So this angle from now (i i) if we crosscheck it in such a way that suppose Theta equal to 0, so when Theta equal to 0 then all this expression should come back to the position like that, that equal to the what was the initial position. So this position will come back again when Theta equal to 0. Now if we put it cos Theta, cos 2 Theta, cos0, 1 this is 0 so if we add this thing so it becomes Sigma X. So, that is true also.

Similarly Tao X1 and we can investigate also simply putting Theta equal to 0, it will Tao X1Y1, it should be equal to Tao XY. Similarly Sigma Y 1 will be equal to Sigma Y if Theta equal to 0. So, this is just cross check it with the expression.

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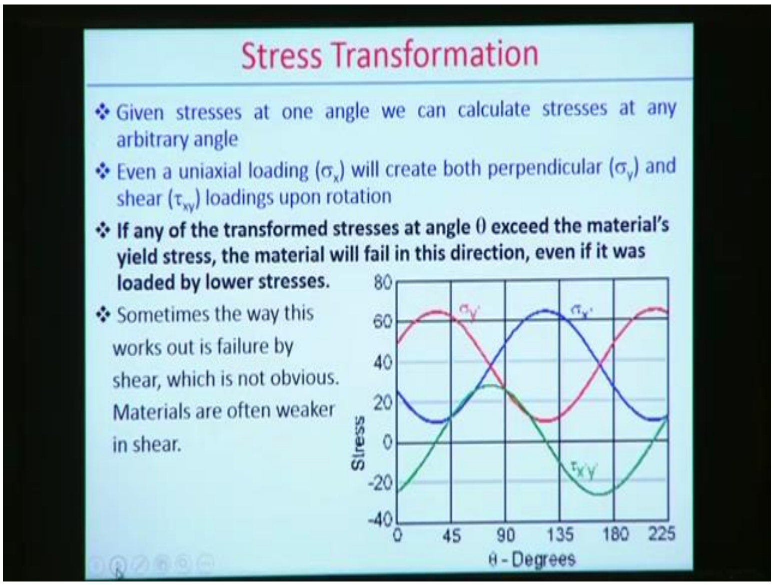

Now further we can analyse this thing that, given stresses at any angle, we, can we calculate the stresses at (arbi) any arbitrary angle, so that means we can define 1 angle Theta and then with that angle, with the orientation of the Theta and we can define 1 orthogonal axis system and we can represents the different stresses also. Now, if we consider that any arbitrary angle Theta, now how the stress and strain actually varies in this case.

So even if we consider the uniaxial loading condition so if there is a application of the only uniaxial loading condition that means there is only single stress state there, Sigma X. Now, if we try to find out the state of the stress in any angular form which is Theta not equal to 0 degree, then we can find out, then we (to) then we will be able to find out that Sigma Y and shear also acting on this element. Now if any of the transformed stresses at angle Theta exceeds the material's yield point and then the material will fail, so even it was loaded by the lower stresses also.

So, we need to analyse the variation of the stresses at different angular form. Let us look into that figure, if we see, this is the variation of the Sigma Y, red, Sigma Y and if we see at different angular form Sigma Y, similarly we can (va) find the variation of the Sigma X also and Tao X dot Y dot, so that X dot Y dot actually represents the new system of the axis and that is defined with respect to the orientation Theta with the initial reference value, that means initial X and Y axis.

Now if we see, there is a repetition of the stress strain, so this point again repetition of the stress strain after, hopefully after 180 degree and similarly we can find out this thing. So this is the individual variation of the 2 normal stress component as well as the shear stress component. So it is also, important to know that what maybe the with this variation there must exist some certain maximum amount of the stresses or minimum amount of the stresses and location, location in the sense that at what orientation this, all these stresses exist; that is (())(53:13) to investigate further looking into this variation of the stresses with a 1 specific angular form.

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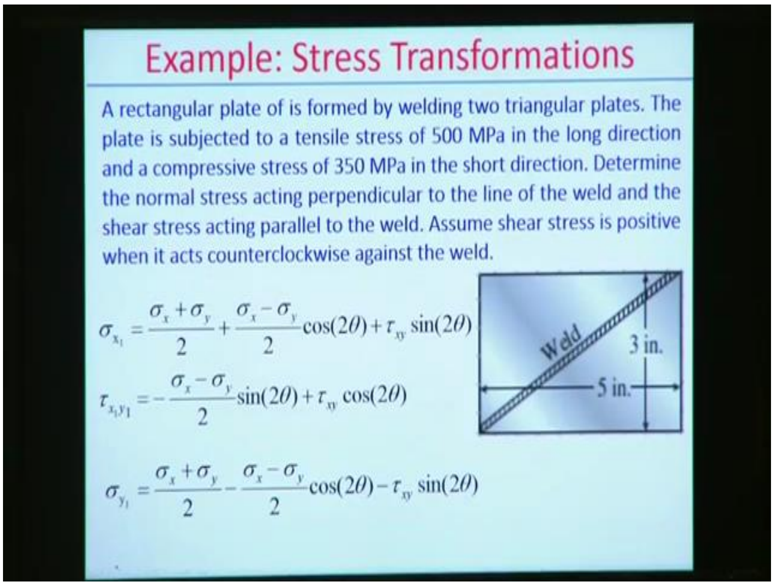

But, before that we look into 1 example that (s) or the stress transformation, how we can, is at, practically necessary when you try to analyse some problem. So, this problem actually define that; a rectangular plate is formed by welding the 2 triangular plates. So, this is 1 triangular plate, this is another triangular plate and the dimension is given, and weld it and then we can find out 1 rectangular plate. Now, we, it is necessary, the plate is subject to the tensile stress of (6) 500 megapixel in the long direction means this direction, 500 megapixel is stresses acting and the compressive stresses is acting on other direction that is 350 mega Pascal.

Determine the normal stress acting perpendicular to the line of the, of weld line. So, if we define this as a X axis and this as a Y axis. Now, it is something like that, the weld joint (in) actually exists along this. So it is having some angular form, angular orientation Theta. Now, we need to find out the normal stress acting perpendicular to the line. So, now we need to find out normal stress perpendicular to the line and the shear stress acting parallel to the weld. So, this may be the shear stress.

So, if this is the axis system is X 1 and this is axis system is the Y 1, so this is X 1, this is Y 1, then we need to find out the amount of the stresses with the orientation angle Theta. So, Theta can be calculated with the dimension is, that 3 inch and 5 inch, from here we can easily find out the Theta and then we can find out the new axis system X 1 and Y 1. So, Sigma X 1, Sigma Y 1, and Sigma X 1 Y 1 simply using the transformation equation in terms of Theta and in terms of the initially defined Sigma X and Sigma Y.

But, here we need to find out shear stress Tao XY but it is mentioned that the normal stress acting perpendicular to the line. So, that means that actually represents Y axis. So we need to find out the Sigma Y 1. So Sigma X 1 is may not be needed in this case. So this problem actually explains that transformation of the axis is helpful to find out the or to solve the different kinds of problems.

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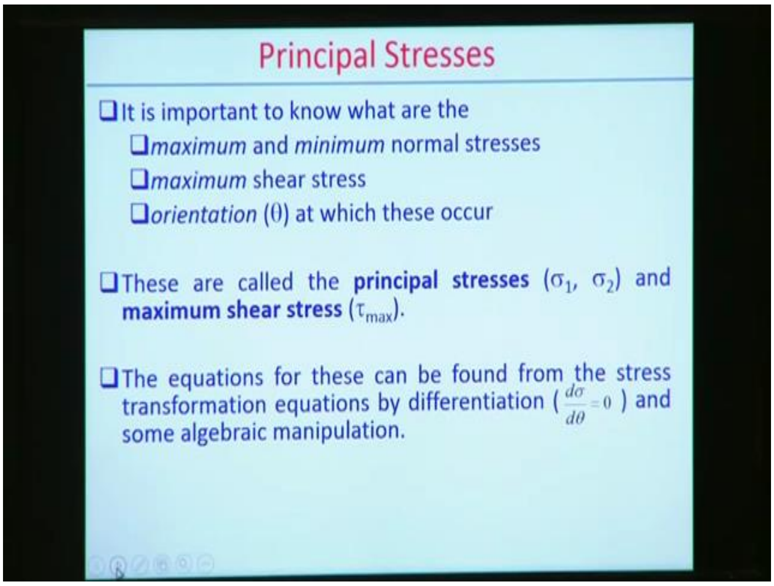

Now, since we observed that there is a variation of the different stresses with respect to the orientation angle Theta then there must be it important to know (what) what maybe the maximum or the minimum normal stresses exist, what are the maximum shear stresses exist and orientation over which, or at which this maximum or minimum stresses occurs or maximum shear stress exists.

So, all these maximum stresses, the maximum normal stresses actually called as the principal stresses and that with a specific orientation that is defined by the Sigma 1 and Sigma 2. And maximum shear stresses we can define the Tao max, if we analyse the variation of all these stress state in 2 dimensional case. Actually the equations for these cases are found out from the stress transformation (eq) equations by differentiating that d Sigma by d Theta, Theta is the angular form equal to 0. From here we can find out and with some algebraic manipulation we can find out all this maximum and minimum stresses values.

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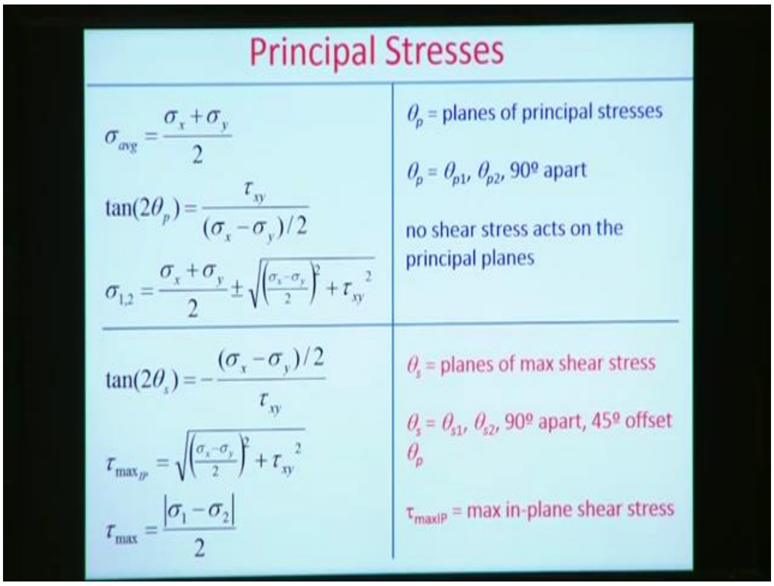

So, what are the, or that is specifically the, all the principal stresses, so what are the principal stresses. all these cases, let us look into that part. So first is the Sigma average value, Sigma average value we can (de) simply define the Sigma X plus Sigma Y by 2. Now, tan 2 Theta P here, so Theta P actually the planes of principal stresses. So (what) Theta P, planes of principal stresses actually the 2 planes exist, 1 is the Theta P 1 (or) and another is the Theta P 2, but they are 90 degrees apart, (and) but no shear stresses acts on the principal stresses, no shear stresses act on the principal planes.

So, we can define the principal plane over which the principal stresses acts but on that plane there does not exist any shear stress value. So, with that we can the manipulation, we can find out that Sigma 1 2, that means principal stresses 1 and principal stress 2, we will be able to find out Sigma X plus Sigma Y by 2, plus minus of Sigma X minus Sigma Y by 2 plus Tao XY Square. So that (wef) that Theta P actually find out that Theta P actually calculate the orientation (o of) orientation of the axis over which the principal stress exist and that is with respect to the initial stress state value or that is, that means with respect to the XY.

So, we can find out the principal stresses in this case by using this formula and the angular, the planes of the principal stresses we can define from here also, if we know, the (stress) initial stress state. Similarly, we can find out the planes of the maximum shear stress, the maximum shear stress on the planes exist at the Theta S1 and Theta S2, again there the difference between Theta S1 Theta S2 is the 90 degree but it is 45 degree offset with respect to Theta P; Theta P is defined by the planes of the principal stresses.

So, here and, so the, Theta S can be calculate from here, normal stress state and the (se) initial stress state and maximum, the Tao max IP is the maximum in plane shear stress, that can be defined here with this formula and finally the maximum amount of the shear stress is the basically the magnitude of the difference between the principal stresses and (50) and 50 percent of that. So these are the principal stresses value in case of 2-dimensional stress state and all this formulation we can find out that amount of the principal stresses and there orientation with respect to the initial define of the axis system.

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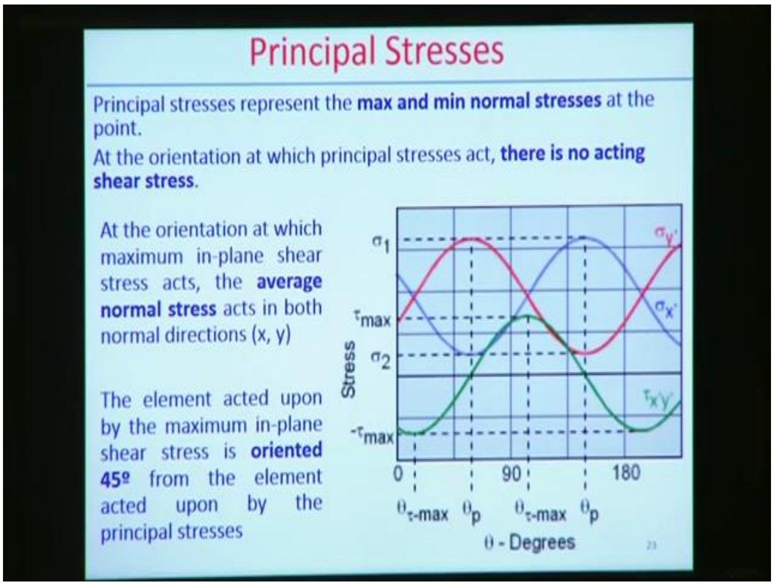

Now, principal stresses represents the maximum and minimum normal stress that is fine, at the point, at the orientation at which principal stresses act, there is no shear stress. So, from the graphical form, form we can find out the variation of the different normal stresses and the shear stresses, we can find out that; 1 point that actually indicates the principal stress value the maximum normal stress value and on this maximum normal stress value the shear stress is equal to 0.

Similarly, there is a another phase of 90 degrees, so we can, this point also principal stresses exist but shear stress equal to 0 at this, at this point and 1 principal stress that is minimum stress value, so that also exist Sigma 2 in this case that 2 values here, 1 and another case is the 1. So, if we look into that at the 2 different orientation Theta P, another orientation Theta P which is 90 degrees apart, so along that orientation the principal stress, principal normal stress exist in the sense the maximum stress and the minimum stress, here also maximum stress and the minimum stress exist at the at orientation of the Theta P.

But, what are the amount of the shear stress value? So, if we look graphically the shear stress is, exist at this point, maximum amount of maximum shear stress but the amount is the positive and it is the (nega) magnitude is the same but (max) maximum value, but . So on the, (a) at the orientation act which the maximum in plane shear stress act, the average normal stress acts in the both normal direction. So, corresponding to the maximum shear stress value and this is the average value, normal stress value actually acts at this point.

So, but here the orientation with respect to that Theta P (is) is basically the, so Theta P the orientation is 45 degree with respect to, with reference to the Theta P, so, at the in plane maximum stress. So the element acted upon by the maximum in plane (s) shear stress is oriented 45 degree from the element acted upon by the principal stress value.

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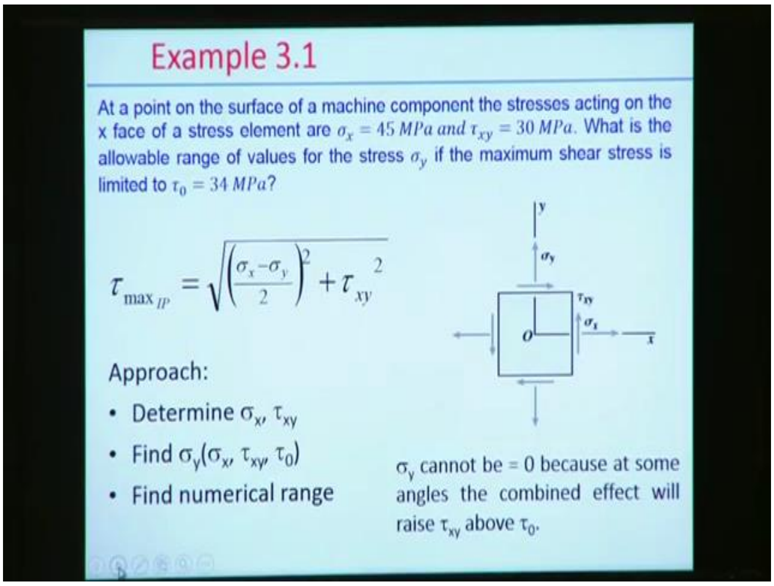

Now we will look into the few example problem to bring the clarification of the application of the all the stresses or how we can find out the maximum (min) minimum principal stresses. Now, if this example; at a point on the surface of a machine component the stresses acting on the X face of a stress element Sigma X equal to 45 mega Pascal and Tao XY 30 mega Pascal. Now, what is the allowable range of the values of the stress is Sigma Y if maximum shear stress is limited to 34 mega Pascal.

So now we know that maximum in plane shear stress, this is the expression and we have the Sigma X value and we have the Sigma X and we know the shear stress value Tao XY acting on this case. Now, we need to maximum allowable shear stress limit also given. So, from here we can find out the Sigma Y in terms of Sigma X, Tao XY and Tao 0 and we need we can find out the numerical range of the Sigma Y by the knowing values of the Sigma X and Tao XY. But, to be noted that Sigma Y cannot be 0 because at some angles are the combined effect will raise the shear stress value above, above 34 mega Pascal, that is the limiting case of the shear stress value. So, here we can find out the limit of the stresses value.

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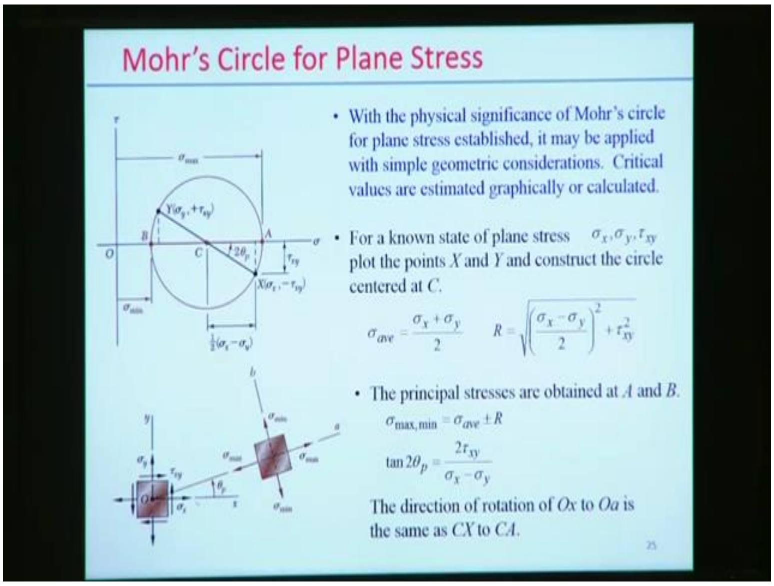

Now we look into that Mohr’s circle which is very basic things for the stress analysis of, specifically is very much useful for the 2 dimensional cases. So, physical significance of the Mohr’s circle (up) for plane stress is established, it may be applied with a simple geometric consideration. Critical values are estimated graphically or can be calculated. So, this is the typical values of the typical picture of the Mohr’s circle. Here the Mohr’s circle, we, how we can draw the Mohr’s circle.

So, we define the axis system and origin O, this, this origin so assume that along the X axis the stress is represented, along the Y axis the this (nor) shear stress is represented here. Now, here we need to find out for the first the centrepoint and from the centrepoint we can define along the radius r, so over the radius we can draw 1 circle, so that, on that circle, that point OA actually represents that, with reference to the origin O, the OA actually represents the maximum values of the normal stress and OB is actually represents the maximum (va) minimum values of the normal stress and along the direction this (diameter) this, so maximum shear stress is along the direction that is the equal to the radius in this case.

Now, stress state, how we can represents the stress state 1 specific (s) suppose we have defined the X, X Y axis system, I think, . Suppose here the initial stress state we defined, Sigma X, Tao XY with this (con) with this notation or with this configuration. Now, at the Theta P, so with the orientation there exists the principal stress state. So now, how this principal stress state can, can be represented or can be extracted from the Mohr's circle?

Now, the point X is basically the represent the coordinate of point X on the, on this coordinate is basically X equal to Sigma X, the normal stress value and minus of the Tao X also, negative values of the shear stress because the Y axis actually, this side is the positive Y axis, that represents the shear stress value opposite side is the negative shear stress value. So this is the simply coordinate, the Sigma X (mou) Sigma Y and (this) this angle actually represents the Theta P.

So, since here is the angle Theta P, but when you try to represents the Mohr's circle over the 360 degree, so here the Mohr's circle will try to represents the 2 kinds of the angle. So, 2 Theta P and this point X is the stress state X and Y point actually (re) represents the Y component Sigma Y and corresponding the shear stress value. Now, with this configuration, so that is the, so we can say that the, we can say that X Y actually represents the initial stress state and when it is oriented to certain (an) angle form Theta P, so at that orientation Theta P, the stress state actually represents the maximum values of the shear stress and minimum values of the sorry maximum values of the normal stress and (ma) minimum values of the normal stress.

So with the orientation of the Theta P, and that Theta P is, we draw 1 Theta P angle and that we present the maximum stress state or maximum principal sorry principal stresses on the line, on the axis actually O Sigma, that axis system. So physically, and this X point, the coordinate point indicates this amount is basically Sigma X and this amount represents the shear stress value. Similarly this amount represents the Sigma Y and this amount represents the Tao XY.

Now, from the node state of plane stresses Sigma Y, Sigma X and Tao XY, plot first we need to find out the, plot the points X and Y and construct the circle centred at C. So, first is the how we can find out the centre at C, the average value Sigma average equal to Sigma X plus

Sigma Y by 2. And then, radial distance can be represented by Sigma X minus Sigma Y by 2 square plus Tao XY square. So, in this case that C is basically from here to here, the centrepoint that is actually represents the average (s) average stresses value normal stress value and that (presents) represents the point C.

Now, with respect to the point C we have, we need to define, we can define already the X and Y point and now with reference to the point C we can draw 1 circle. When, we drawing 1 circle, then Sigma Max upto OA actually represents the amount of the Sigma Max and OB Sigma min in this case. Now, the principal stresses are obtained at A and B, where Sigma Max and Sigma min which is equivalent to the average value that means OC plus minus OA, that means average value plus minus the radius in this case.

So, tan 2 Theta P easily calculate from the coordinate of the (s) stress state. So, tan 2 Theta P is the twice Tao (x) sorry, Tao XY, this amount, divided by Sigma X. So this, this distance actually represents the half of Sigma X minus Sigma Y. So, Tao XY divided by half of Sigma X minus Sigma Y that is, twice Tao XY by Sigma X minus Sigma Y, that represents the tan (theta) 2 Theta P, that means this represents the stress state at this point.

So the directional rotation OX, with respect to this figure the directional rotation 2OA, 2OA is the same as CX to CA. So, from CX to CA in the Mohr's circle but which is practically, it is from OX to OA, OA. So this is the typical Mohr's circle description.

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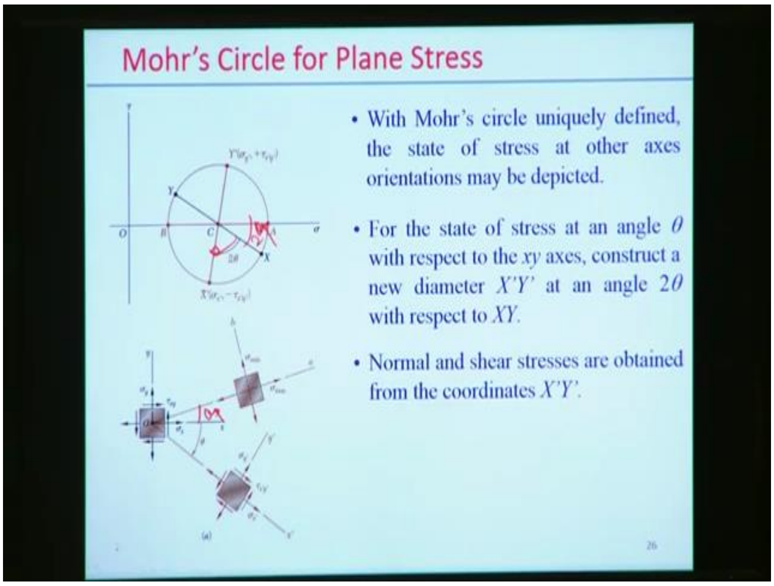

Now, Mohr's circle uniquely defined, the state of the stress at any other axis but maybe other orientation, for example, Mohr's circle can, can also be drawn if there is a initial orientation of angular form Theta with respect to X axis. Now this is the (initial) with, with (resp) some other axis system, say X dot Y dot, now, with respect to that we need to find out what is the maximum stress and, means minimum stresses that can be represented by the Mohr's circle as well also.

So in this case the initial stress state is (def) so, XY actually represents the stress state with respect to the axis systems small X small Y, the stress state. Now, but X dot Y dot we need to represent the stress state to draw opposite angle 2 Theta here and then X dot and we define the X dot and Y dot. And finally give the principal stress exist with respect to Theta P then their angular can, can be twice Theta P and then accordingly looking to the coordinate finding the radius we can draw the circle, Mohr's circle in this case.

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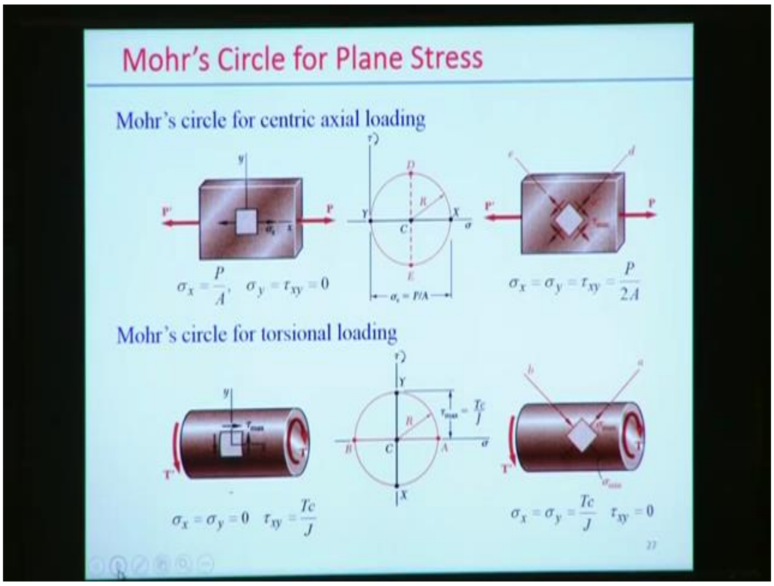

Now this is the Mohr's circle for the simple axial loading, representation of the Mohr's circle for axial loading, that Sigma is really, only 1 stress, I think one kind of loading is there so only 1 (ax) along the X axis there is only one stress Sigma X, but Sigma Y and the shear stress is absent here, then Mohr's circle can be drawn like that, for the stress state it is, that radial distance CY that is Sigma X Sigma Y and Tao XY equal to P by, P by twice P by 2A.

So, basically Sigma X equal to P by A, Sigma Y (())(73:35), when there is a existence of the maximum stress value, then it is the half of the initial value, so P by 2A, but in this case the maximum Sigma X Sigma Y and Tao XY all are same. So, similar way we can find out the stress state here, and first we need to find out what is the CY using the formula and then radial distance, C is the centrepoint and radial distance if we look into that, maximum value is there, but minimum values 0 is this case, minimum value there is 0 and shear stress value which is equal to the radial distance, but, the CD is the radial distance or, maybe radius, so that radius is the half of the Sigma X value. So that are the maximum shear stress value in this case.

Similarly Mohr's circle for the torsional loading also; so there exist only the, the no normal stresses, only the shear stress component J. So, in this case the shear stress component the state of the stress can be defined like that; Sigma X Sigma Y and Tao C, but maximum values Sigma X and Sigma min the, at this point the Tao XY equal to 0. So, shear stress maximum equal to Tao Sigma G but (miximum) and minimum value with respect to the origin is the same in this case.

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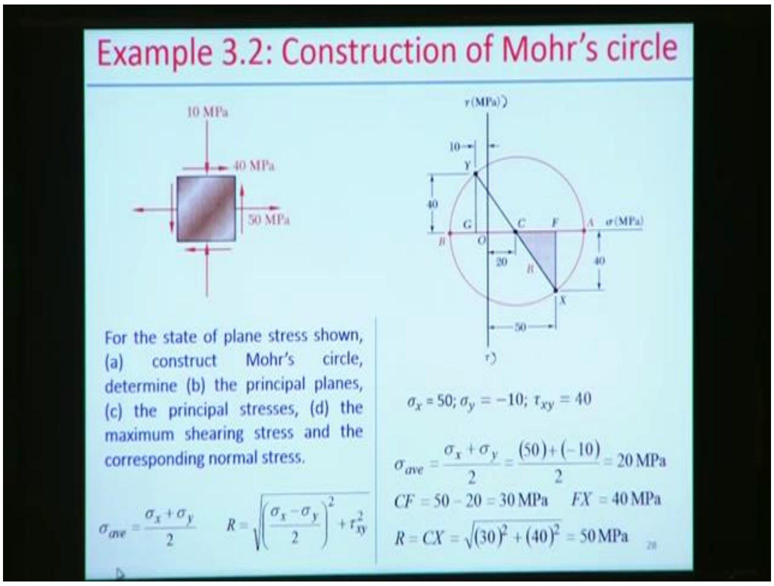

So, construction of the Mohr's circle, just try to look into this thing that, suppose this is the stress state and first we try to draw the Mohr's circle from the stress state that, we define axis Sigma and Tao and in this case stress state Sigma X equal to 50 Sigma Y equal to with the notation, Sigma Y that means compressive stress is there 10 mega Pascal and here with the, with the notation the shear stress is positive 40. Now, average values of the, average, average stress value Sigma average equal to 20 mega Pascal from here we can find out the stress state at the X.

So since the Sigma X is given 50, so the X can be drawn on this certain point, the distance 50, but centre point is 20, so CF can be calculated as 50 minus 20, 30 and FX is the, at this point the state of the initial shear stress state is equal to 40 mega Pascal. So FX equal to 40 Mega Pascal from that we can find out the R, CX 50 mega Pascal and then we construct the circle. Then we can find out that maximum shear stress value, or principal stresses in this case.

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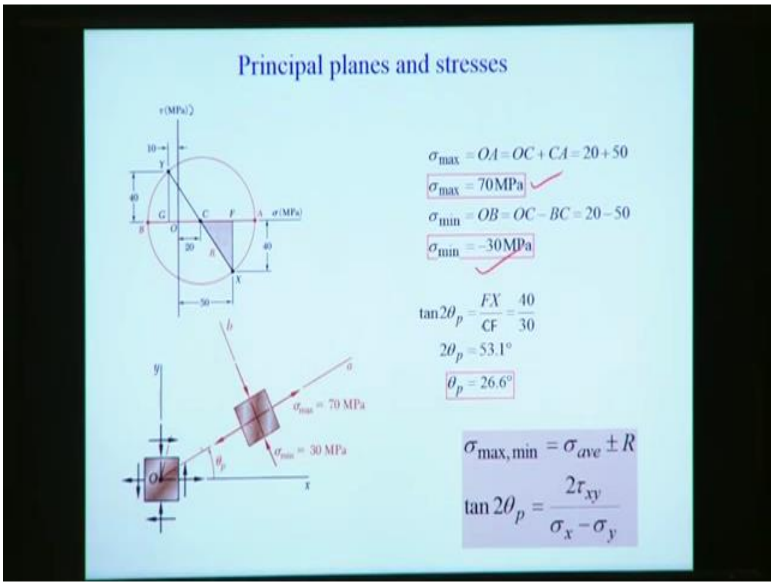

So, maximum shear stress (value) maximum stress value is basically OA, OA is calculated as 70 mega Pascal and minimum equal to OB, so OB equal to, I think, minus 30 mega Pascal and tan 2 Theta P, so this angle is the 2 Theta P, is basically from the, we can find out the geometry 40 by 30, so Theta P is equal to 26 point 6. So, this is the; so first looking into the average stress value and then find the radial distance and looking into the stress state we can find out the stress state point X and Y and from that point we can draw the circle by identifying the centre point.

So this is the way to construct the Mohr's circle from the stress state. So, now principal stress exist Theta P and here if we see that maximum and minimum because minimum stress is the compressive and, and the maximum stress is the tensile, so this is the representation and that exists over the angle Theta P, so which is 26 point 6 degree here.

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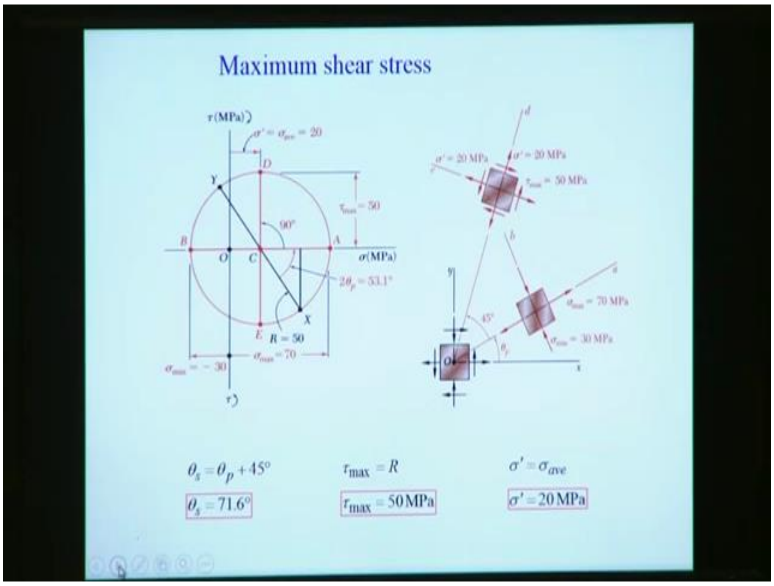

But, maximum shear stress value, we can calculate in this way that; so this is the (s) stress state, here is the principal stress exist so no shear stress but with respect to the offset of 45 degrees with respect to the Theta P at that point there existence of the maximum shear stress value, but in this case, the, from the Mohr's circle how we can (const) find out this thing that maximum stress state value actually represents the CD, and, because this is the radial distance.

So, CD is actually 50 50 mega Pascal and since XY was the initial stress state, X and Y we define at the initial stress state and 2 Theta P along that direction along the, this axis represents the maximum principal stresses value, and further 45 degree offset, so that means here 45 into 2, 90 degrees, so offset at this point actually represents, the point D actually represents the maximum stress value. So, we can easily find out that Theta's is equal to simply Theta P by 45 degree, so the, and Tao Max equal to R here, 50 mega Pascal and stress value at this point Sigma dot equal to average value, 20 mega Pascal.

So, so far we have discussed that 2 dimensional stress state and (in a) and what is the difference between the engineering stress and true strain or relation between engineering strain and true strain, where it is applicable and finally we come back to that 2 dimensional stress strain and representation of the stress state using the Mohr's circle and how we can analyse or how we can find out the principal stresses or normal stresses and shear stresses and this all are the basic analysis of the stress in case of 2 dimensional.

So this transformation of the stresses actually useful for the further in the plasticity analysis or further problem, (when we) when we try to analyse the crystal elasticity or crystal plasticity problem. So, hopefully this class will give some basic idea of the stress analysis and now next class I will try to analyse the 3 dimensional stress state on specifically the more important thing is that the transformation of the stress from 1 axis to some other axis. So, thank you very much and for your kind attention.