Introduction to Crystal Elasticity and Crystal Plasticity
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Week-02
Lecture-04
Elasticity

Good morning everybody.

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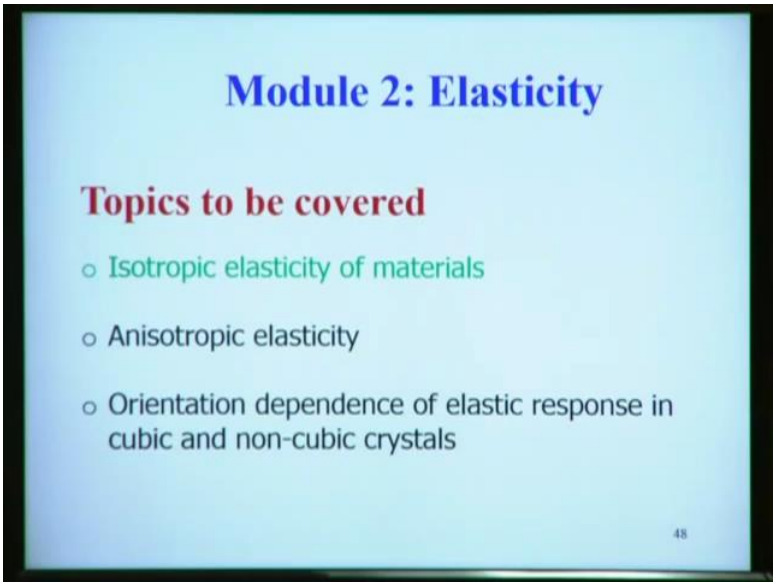

So let us start the second part of the module 2 that is anisotropic elasticity. So, last class we have covered isotropic elasticity in case of 2-D, 3-D and specifically we have tried to derive the different elastic constants when the material behaves within the elastic limit. So, but practically if we look into that crystal structure of a specific material due to the orientation of the different atoms at different positions, it actually holds good, some amount of anisotropic behaviour. So in this case it is also necessary analyse the anisotropic elastic behaviour of different crystal structure.

So this module I will try to cover first with a anisotropic elasticity for different crystals and second part will be the orientation dependence of elastic response in case of cubic and non-cubic structure.

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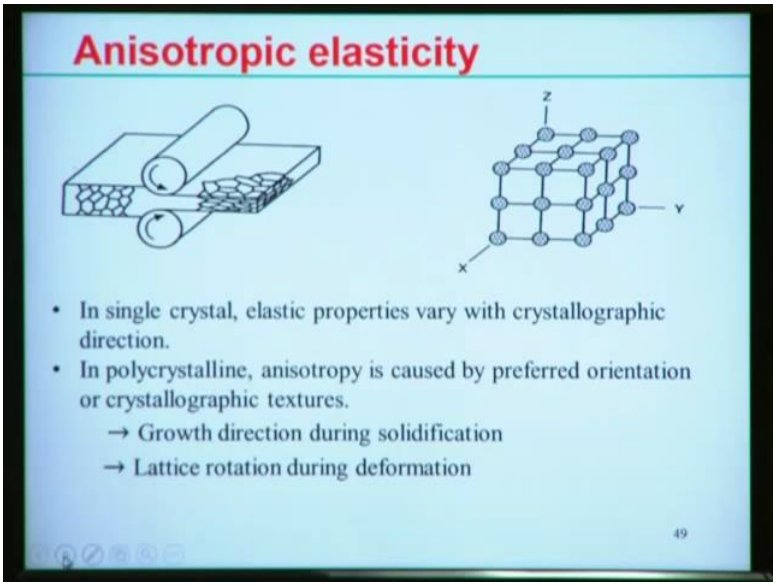

Actually, look into that picture often where the anisotropic comes practically. So first thing is the, in a single crystal the elastic properties actually vary depending upon the (elastic) crystallographic directions.

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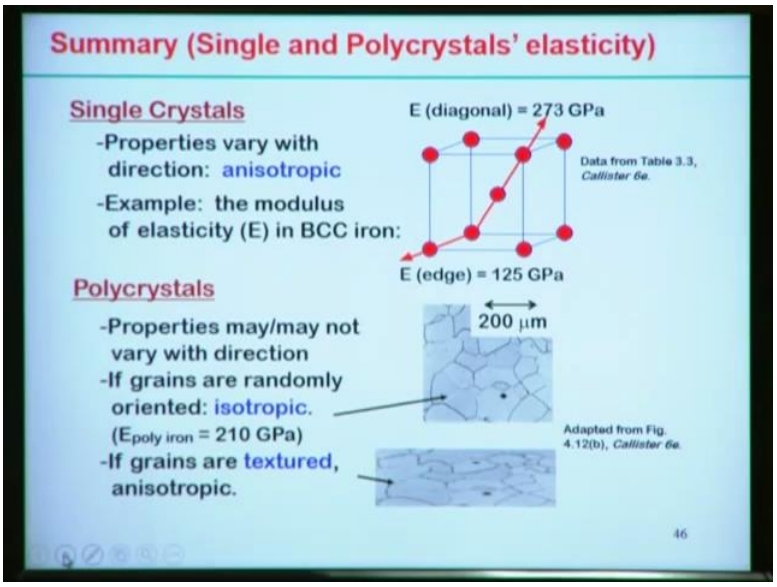

So, last class we have discussed these things, if we go back to our last slide if we see that single crystal structure and if we measure the Young's modulus along different directions we found that along the body diagonal, the Young's modulus is 273 Giga Pascal and along the edge the Young's modulus is 125 Giga Pascal. So there is a use difference of this elastic constant in a single crystal of iron.

But if we look into the polycrystalline material same properties may vary within the each grain but if we consider the aggregate, that means several crystals together and we can average out all these things properties, so in this case the properties can be isotropic. But, specifically if the grains are textured then we can find out that anisotropic properties of the crystals.

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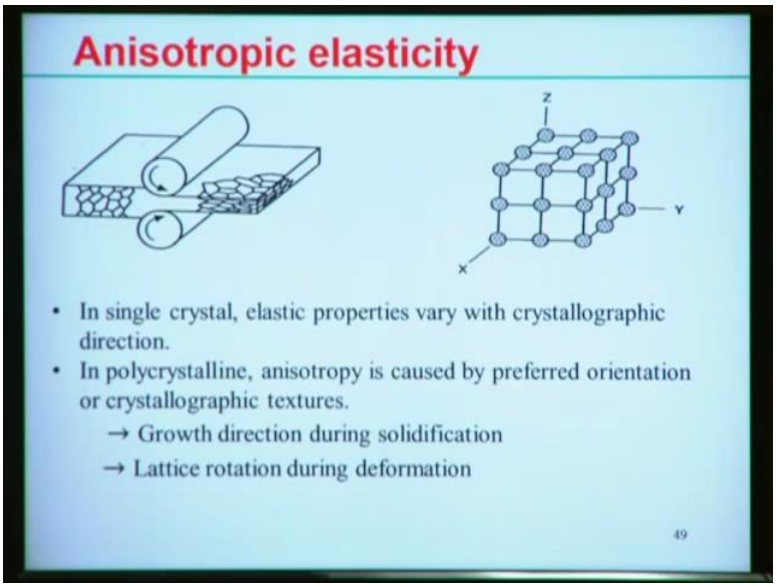

So, in case of polycrystalline the anisotropic actually caused in the, by the preferred orientation of (crystallo) crystallographic textures. For example, growth direction in the solidified structure, that actually behaves or maybe it is desirable to consider the anisotropic analysis. And second is the lattice rotation during the deformation that is in, mostly in case of metal folding process. So these 2 cases, the anisotropic behaviour is most important to a analyse.

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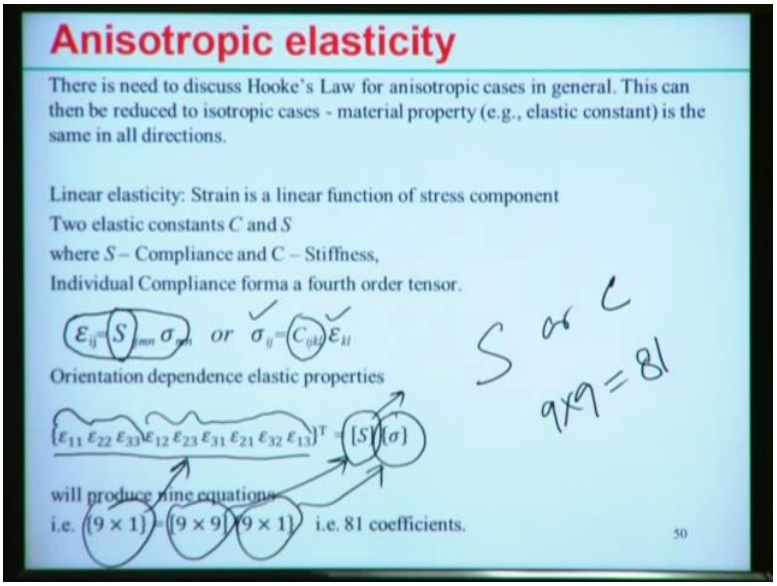

So, of course there is a need to discuss Hooke's law also in case anisotropic, like what we discussed in case of isotropic elastic properties. So then it is possible to reduce to perfectly anisotropic to isotropic cases depending upon analysis of the (symmet) symmetry exist within the crystal structure itself. So all this material property i.e. that means some constant term we can introduce here to analyse the anisotropic behaviour of single crystal material.

Linear elasticity we have already discussed, strain is having a linear (re) relationship with the strain, and that relationship can be complied with 2 different constants, specifically that is called compliances and the stiffness. So that constant term actually analogous to the relation between the stress and strain. If we look into that relationship between (strain) stress and strain, then we introduce the compliance which is typically formed by S.

But at the same time when we relate between the stress and strain, in that case we introduce the constant term that is called stiffness, specifically, if we consider the all possible anisotropic in this case stress or strain, tensor having 9 components. So, in this case, this constant term, either S or C can be introduced as a matrix form into 9 by 9. So total 81 number of components will the come into the picture.

But practically, to find out all these (constant term) constant term is really difficult. So, we can take the advantage to analyse the symmetric properties of different crystal structure and it is possible to reduce this huge number of elastic constants term. So let us look into how we can analyse all this anisotropic constant terms. So, if we look into, in a matrix form, we use

the S term and this is the column vector that is the strain component, and if we find out that there are normal strain components 3 and other 6 are shear strain components and right-hand side is; so to bring dimensionality, so this column factor is having the dimension 9 by 1 and then we introduce the compliance term S should have total dimension 9 by 9 and the stress also is having 9 by 1 elements.

So, when we try to bring relationship between stress and strain or components of the stress and strain, we introduce the either depending upon whether stress is the right-hand side or strain component is the right-hand side began introduce either S, in terms of S that is called compliance or in terms of C that is called stiffness, so in. Now, we will try to analyse that how this can be correlated in case of 3 dimensional strain.

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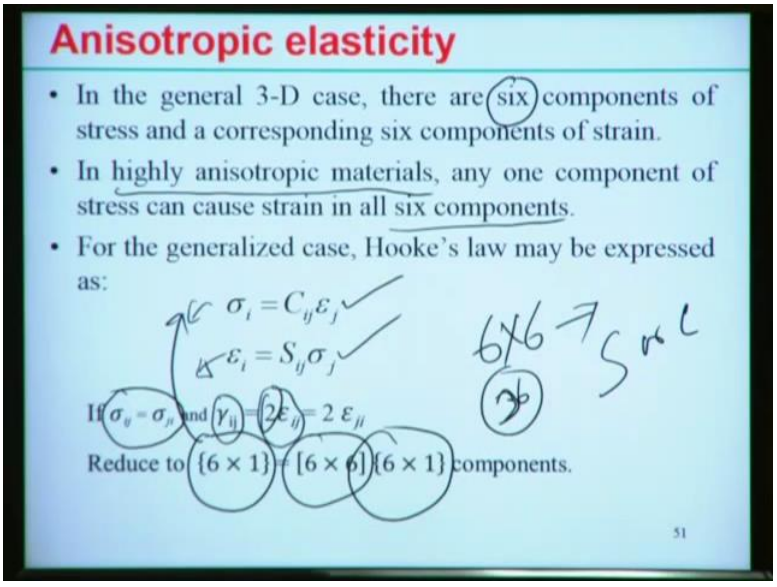

So, in general 3 dimensional state we represents the stress or strain components total 6 looking into the symmetric nature of the stress tensor or strain tensor. So when there is a component of the 6 and then we can introduce that 6 by 6 constant term, either compliances or stiffness, that means either S or C in this case, because in case of highly anisotropic material actually 1 component of stress can cause strain in all 6 components. So, this, it is desirable to bring total 36 constant terms in this case.

So stress can be represented in this way and strain can be represented in this way or in this case stress or strain are having the column vector and strain also having the column vector of 6 components only. So accordingly we can decide the dimension of the column vector

dimension of the matrix. Now, if Sigma IJ equal to Sigma JI, that means its actually produced the symmetric matrix and the shear strain can be represented in terms of the normal strain component which is on a specific plane the transformation factor is considered as a 2 (here).

So in this case the shear component, we can adjust when you writing the individual component and try to enter within the (s) linear system of the equations and we can accordingly adjust by looking into whether in terms of shear strain or we are introducing in terms of the normal strain component.

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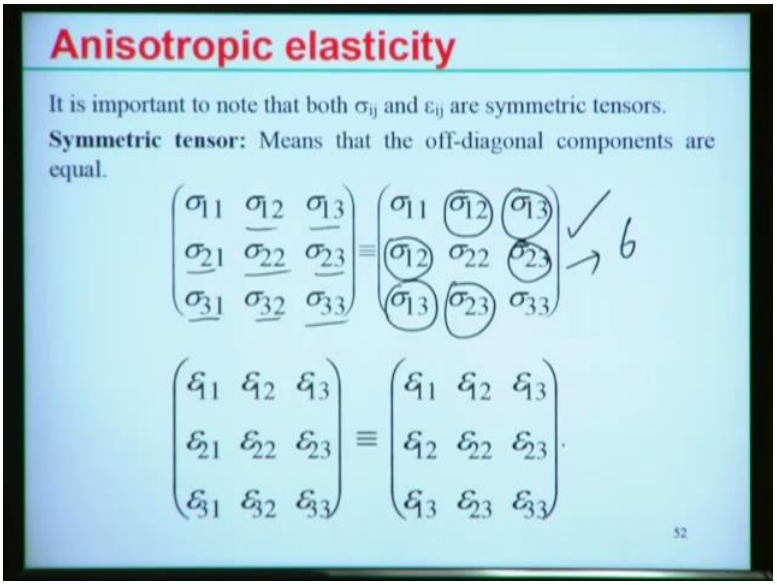

Here, that, if we look into that symmetric tensor, we represents that Sigma 11, then Sigma 12, Sigma 13, Sigma 21, Sigma 22, Sigma 23, Sigma 31, Sigma 32 and Sigma 33. So, in this case it is having 9 components but when it is symmetric tensor, then this component and thats component remains same, so, 13 that means Sigma 12 equal to Sigma 21. Similarly Sigma 3 equal to Sigma 32, like that we can represents the symmetric tensor, but here it is having 6 components and in case of strain also we can represents the similar way, that here also it is having the 6 components.

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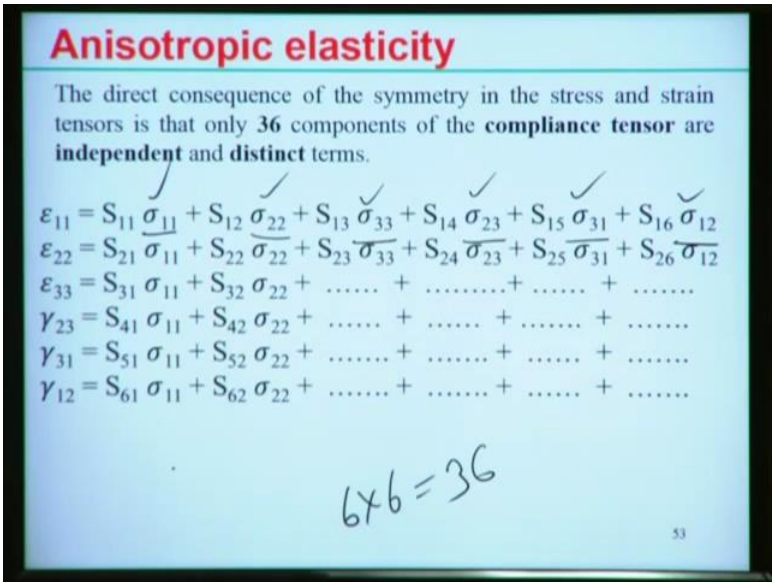

Now, we represents that the linear system of the equations of the stress and strains in such a way there, that Epsilon 11 we can represents that Sigma 11, Sigma 22, Sigma 33, Sigma 23, in the consecutive way Sigma 31 and Sigma 12. So, by following these things, so first 3 components are normal stress components and next which is the shear stress component. So, 3 normal stress and 3 shear stress and accordingly we add or multiply 1 individual terms by the (plastic) compliances term (that that) by that S term.

So in this case we are getting the 6 equations and with the 6 equations and with the 36 number of, 6 by 6 that means 36 number of compliances term. So that it 36 compliances tensor are independent and also having distinct terms. So let us look into that things how we can reduce the compliances terms as well.

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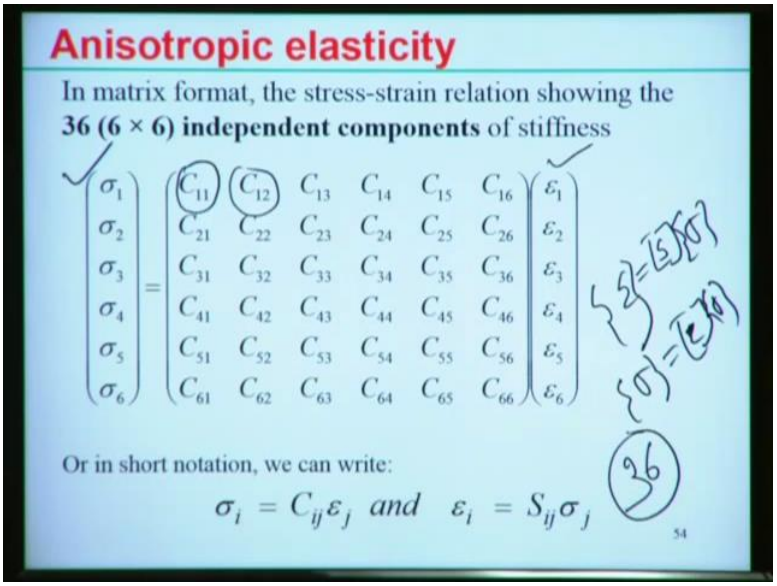

So, in matrix form the stress strain relation showing the 36 independent components of the stiffness. So, like in compliances terms we can represents the stress strain relation in terms of stiffness also, instead of writing S here we can write C, C 11, C 12 and this way and this actually represents the column vector of stress, this actually represents the column vector of strain.

But, what was in case of (compli) in terms of the (compli) compliances we (ha), we have written strain equal to S into Sigma, but here in terms of stress, we are writing the stiffness term that means C term. So, both way we can write this equation either in terms of S or in terms of C. But definitely all the, these 2 cases we are having total 36 independent constant term.

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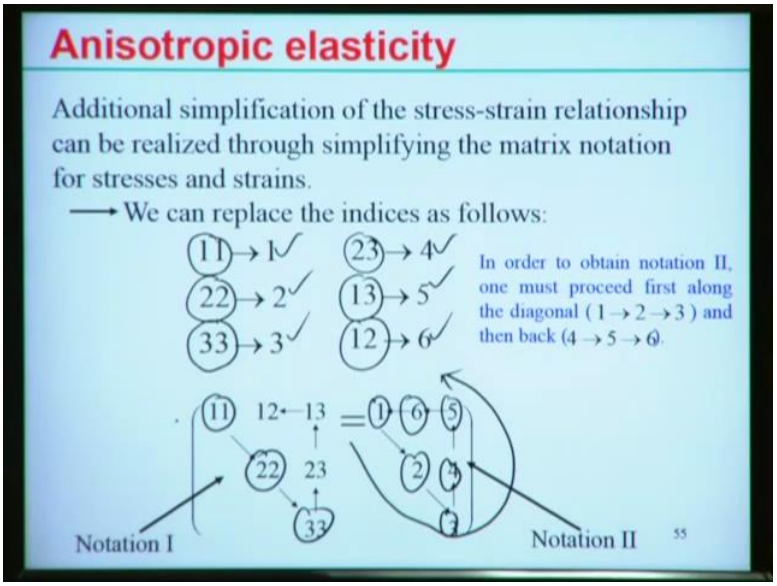

Now instead of writing stress tensor in terms of 111 to 22, 23 like that we can represents, or we can follow certain (s) note, or certain notation and we can simplify the equation in such a way like that, 11 term can be represented as 1, 22 term can be represented 2, 33 term as represented as 3. So that means first 11223, the 3 normal stress or maybe strain component in terms of the 1, 2, 3, but shear component maybe in terms of 23, 13 and 12 that is 4, 5 and 6. So, that is corresponding to either shear stress or shear strain.

So, if we look into this matrix the notation is like that, first we write 11, 22, 33; so basically we are moving diagonally with terms like 1, 2, 3 and then terms is like 23, 13 and 12; 4, 5 and 6. So, specifically we are following some specific notation, first along the diagonal and then back side and then we found out 1 specific sequence like this to write the notation in terms of single term, will be 1, 2, 3, 4, 5, 6 instead of writing the notation, 11, 12, 13, like that. So, actually this type of notation is helpful for the (analysis) further analysis of this kind of anisotropic problem. So this (c) this (is) this notation all this thing is the specifically, specifically the simplified (())(15:52) of the representation for the further calculation of this (an) anisotropic cases.

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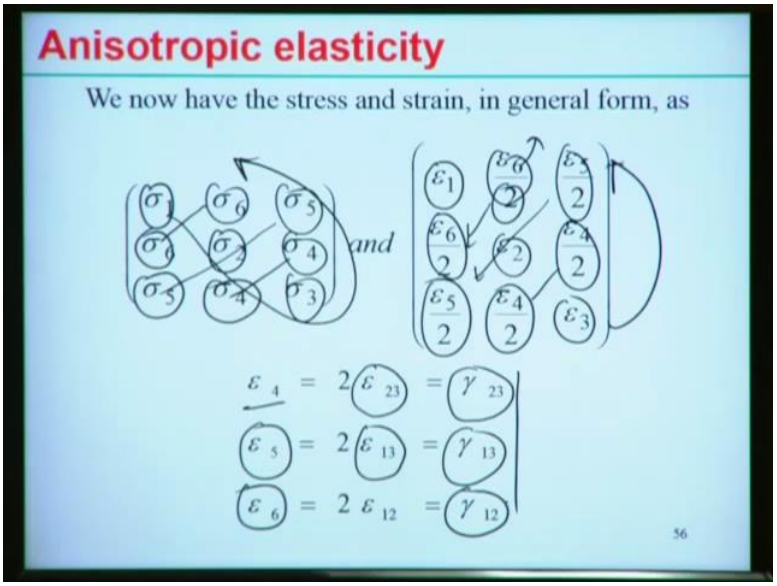

Now, then, looking into that notation we can represents the stress component like that Sigma 1, Sigma 2, Sigma 3, then 4, Sigma 4 here, Sigma 5 here and Sigma 6 here, like this and due to the symmetric nature of this stress matrix, here it is (si) Sigma 6 corresponding to that and corresponding to that is Sigma 5, corresponding to the, it is Sigma 4. That is, that means the, with respect to the diagonal, so both side is having similar component (ats) according to the symmetric (na) nature of this stress.

So, similarly, of course when we try to represents the strain matrix, in this case also be following the similar notation, and we find out Epsilon 1, Epsilon 2 and Epsilon 3, then 4, Epsilon 5, Epsilon 6 and due to the symmetric nature it is Epsilon 6, symmetric nature it is 5, and it is 4. So in this case also we (see) we see that we have retained that Epsilon 6 by 2.

So actually that position is for shear component and we, if we multiply the shear (com) component, the factor here then we representing this terms of the normal strain component and that 2 is because of the factor converting from the shear strain to normal strain. So, here also Epsilon 5, Epsilon 6, Epsilon 4 all the shear component but in terms of the normal strain component, we have introduced within the matrix itself.

So, this is the typical relation, Epsilon 4 is basically 2 times of Epsilon 23 and we can use direct notation gamma 23. Similarly Epsilon 5, 2 times of Epsilon 13 and which is corresponding to the gamma 13. And Epsilon 6 is corresponding to the gamma 12. So this all

are the shear component we have retained, this is the, in this way to make the calculation simply, simplify calculation.

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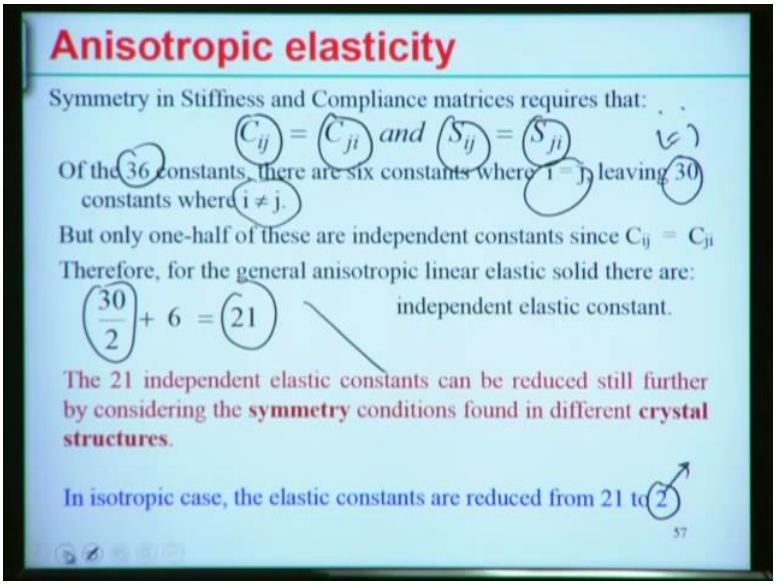

Now, although we observed that there is a symmetric nature of the stress or symmetric nature of the strain, we have considered depending upon the problem itself but at the same time symmetry in the stiffness or compliances matrix requires that similar kind of relation that Cij equal to Cji or Sij equal to Sji. So, in this case if we consider the symmetric nature of the compliances matrix or stiffness matrix and then it is possible to reduce the total number of constant terms here, so let us see.

So, out of 36 components, there are 6 constant terms when i equal to j, specifically i equal to j that, in this case there are 6 components and remaining 30 component which is related to i not equal to j. But, we can find out what is the total number of linear elastic or (sorry in) in case of linear elastic solid the, how many independent elastic constants are there, that is 30 by 5 because 30 was the, when i not equal to j, so 50 percent of that and when i equal to j, actually i equal to j exist along the diagonal itself.

So, diagonal element is having total number of 6, and apart from that, either upper or lower triangle part, from there we can find out total 15. So detail 20 number constant terms that are independent constant terms can be possible to bring out, when we consider the symmetric nature of the elastic or compliance terms. So, in this case 20 independent elastic constants can

be reduced again further by considering the symmetric conditions for the different crystal structures.

So, we will investigate gradually for the different crystal structure and how we can reduce this, this total number of constant terms in this, in this types of crystals. But we can say that this total number of elastic constants can also be possible to reduce up to 2 and in case of isotropic properties of the material. So, in this case, because isotropic case only there exist to number of constant terms, we will subsequently investigate this phenomena, but let us look into that first the how we can bring down the number of constant in case of different crystal structure.

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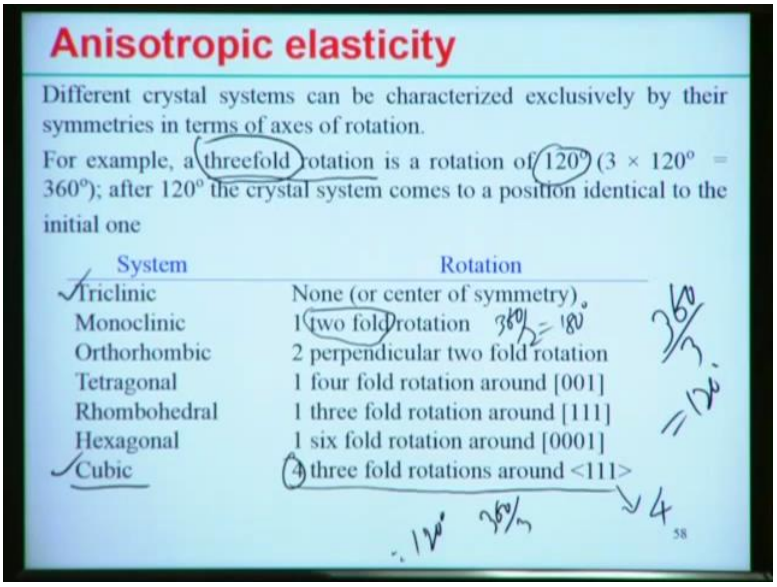

So, first we need to look into that symmetric nature of the different crystals system. So, first system (is) this symmetric nature we can explain by the axis of rotation, but if we look into 1 example that a threefold rotation, for example threefold rotation is considered as 1 symmetric, symmetry if rotation is 120 degree. For example, total 360 degree and threefold rotation 360 divided by 3 that means 120 degree. So, after 120 degree of the rotation the crystal systems comes to a position, who is, what was at the initial position.

So that is, that is the (())(22:17) of the analysis of the symmetry. So here it is written as threefold rotation, so threefold means at the if we rotate total 360 degree, so 3 times it can come to the initial position that means it is having the threefold rotational symmetry.

Similarly, different crystal structure or different crystal systems specifically having the different type of symmetric nature.

Let us look into that, first is a triclinic. In triclinic, the, if we know the triclinic, the crystal system and the n1 of the edge range or lattice parameter are equal with each other and n1 of the angles between the edges are not equal with each other, so there does not exist any symmetric nature. So, Centre of symmetry does not exist actually in case of triclinic. If we look into the next crystal structure monoclinic so in this case we can find out the 1 twofold rotation, so 1 twofold rotation means, the twofold rotation means 360 by 2 that means 180 degree. So, with the rotation of 180 degree, the crystals, the arrangement of the atoms are (s) same to the initial structure.

So, that is a twofold rotation in case of monoclinic. Orthorhombic, similarly having 2 perpendicular twofold rotations, tetragonal is having 1 fourfold rotation around 001. So, 1 with respect to the specific direction 001 that means 001 generally represents around the axis, so it is having 1 fourfold rotation. So fourfold means, every 90 degrees rotation there is a symmetric, symmetry exist in this type of crystal system. Tetragonal, okay, rhombohedral also having threefold rotation around 111, hexagonal is having 1 sixfold rotation around 0001 and cubic is having 4 threefold rotations around 111.

So let us look into that cubic structure, the 4 threefold rotations around 111, so actually this 111, it represents the direction of the body diagonal but in case of simple cubic structure if we look into that there may with the possibility of the 4 such body diagonals. So, that is the, this represents actually the family of the direction so this family consist of the 4 different directions, that is 4 different body diagonal in a cubic structure.

So that is why it is having 4 numbers of, and having threefold rotations; threefold rotations means it is 360 by 3 that means every 120 degree rotations with respect to that 1 body diagonal, then there exist 1 symmetric nature of cubic crystals system. So, looking into all this symmetric nature we can reduce the total number of elastic constant during the analysis.

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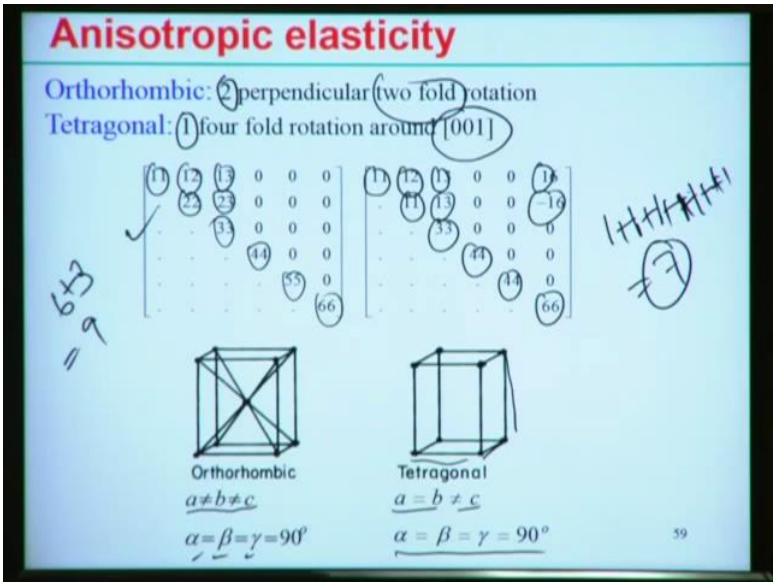

So, first look into that part that is orthorhombic. Orthorhombic is having 2 perpendicular twofold rotations. If we consider this symmetry then we can represents the C or S, that means compliances or stiffness matrix like this. So here the constant term is like that 11, 12, 13, 22, 33, 44, 55, 66. So basically there is 1 2 3 4 5 6 diagonal elements, so that means either compliances or stiffness constant terms and other, there are other 2, 12, 13 and 23, so that means another 3. So, total 9 different constant terms is used to represents the compliances matrix or (s) stiffness matrix in case of orthorhombic crystal system.

Similarly, if we try to so before shifting to that, may be if we look into the structure, the orthorhombic structure, the typical orthorhombic structure is a not equal to b not equal to c, but the angle alpha, beta and gamma they all are 90 degree. So, with this structure we can find out that 2 perpendicular twofold rotation and with the symmetric measure the can find out that this total 9 number of constant terms actually exist (in) in case of the compliances or stiffness matrix, in, for the analysis of orthorhombic crystal structure.

Similarly, we can look into the tetragonal crystal structure. Here we can find out that a equal to b, is typical structure as there will be 2 lattice parameters, maybe this edge and this edge is equal but that is not equal to c, so maybe this is not equal to that a or b. But alpha, beta, gamma all are 90 degree, so all the, with respect to the origin with all the edges are 90 degree. So this type of structure is having 1 fourfold rotations around 001. So, that is 001 with respect to 1 axis. So, if there exist this type of symmetry and then the stiffness or compliances matrix

terms can be represented this way; say 11, 11, so that means 1 plus 33, 1 plus 44, 44, 1 plus 66, 1.

So, this 1 2 3 4 and apart from that there is a 12, 13, so plus 1, plus 1 and this is also 13 and that is another 1, 16, 16, plus 1, so 1 2 3 4 5 6 7. So that means total 7 number of constant terms actually exist in case of tetragonal (or) or maybe if we analyse the symmetric nature of the tetragonal structure or tetragonal crystal system. So that, find out that, so the message from here is that if we further analyse the symmetric nature of the different crystals then we can bring down the total number of constant terms.

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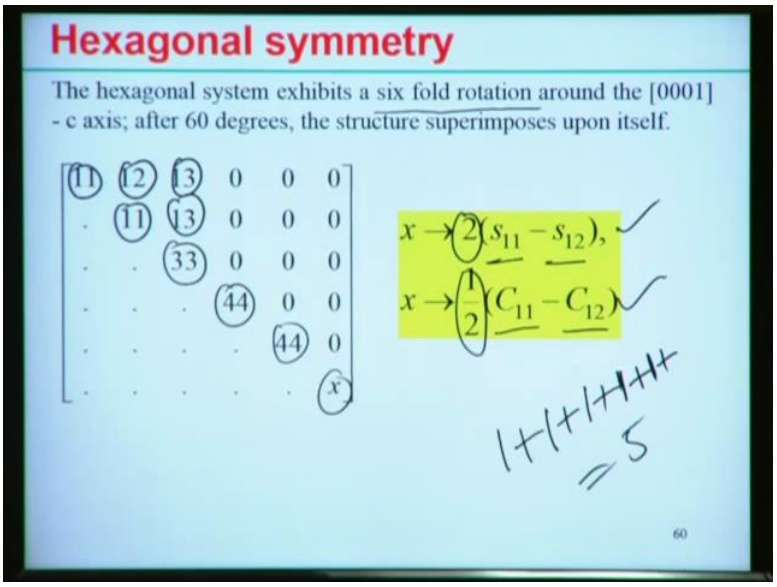

Now, if we look into the hexagonal symmetry, that hexagonal crystal structure I think it is, it is obvious that there exist a sixfold rotation. Sixfold rotations, that means after (s) around the 0001 the direction that is, that means (ac) actually that is corresponding to the c axis. So, we know that hexagonal crystal structure is having 4 axis system that is a1, a2, a3 on the base, and along the Z direction or along the C direction there is another axis. So, with respect to that axis there is having sixfold rotation that means every 60 degree rotation, it brings the symmetry in structure.

So, with this symmetric nature of the hexagonal structure, so let us look into how many constant terms are there in this case. So, 11, 11 here; so 33, 44, 44 and there is another x. So, 11 is 1 type of 1 type, 3 is the another type, 4 is the another type and x, if we look into the x, the typical expression of x, see if we look into that S 11 and in terms of S 12 or in terms of C,

C 11 or C 12. But in this case here the 12 term is there, so we can bring plus 12 but 13 term also there; that means the x is in terms of 11 and 12.

So, we are having total number of constants here 1 2 3 4 5, because the (a) x is represented in terms of 11 or 12. So total 5 number of constants is defined in case of hexagonal symmetric structure and but if we look into that other, but it is a, this is also that compliance system symmetrical also symmetric with respect to that and if we look into that, the compliances and stiffness term, the factor 2 is used in different way.

Actually this factor with the 2 or inverse of that 1 by 2, that depends on the whether we are using the compliance system or whether we are using the stiffness term here in this case. That means, basically, it depends on the relationship either with respect to the stress strain or just reverse strain in terms of stress. So accordingly this constant term varies in this case this is to be noticed here (())(33:04) very careful (with) using this (compliance where) representing the equations either in terms of compliances or we are representing the equations in terms of the stiffness matrix.

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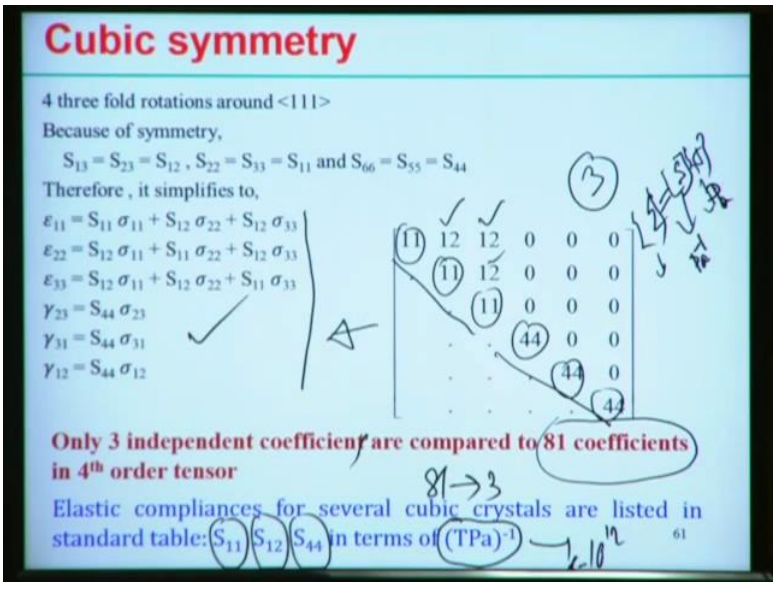

Now, we shift to the cubic symmetric structure. So, cubic symmetric structure already menti1d that, cubic (symmetric) it is having 4 threefold rotations around 1 1 1, that means that represents the body diagonal and (())(33:33) to the body diagonal, so it is having fourfold rotations and if we look into that symmetry then we find out that S 13 maybe in terms of the

compliances, S 13 equal to S 23 equal to S 12. Similarly, S 22 equal to S 33 equal to S 11 and S 66, S 55 and S 44 all are equivalent.

So looking to that we can represents the system of equations like this, Epsilon 11 in terms of the Sigma 11. So, this is the system of equations and in this case the compliances or stiffness matrix is represented like this. So 11, 11, 11 so this all are equal the diagonal; 44, 44, 44 is all are equal and the 12, 12 and 12, so basically 1, 2 and 3 different elastic constants exist in case of cubic symmetry.

So, with respect to the, the (s) symmetric nature of the compliances or stiffness matrix we can write the linear system of the equations in terms of either strain or in terms of either stress and if we solve it accordingly we can find out the different constants or maybe vice versa, the different constant values are available and we can use the different compliances or (stiff) stiffness terms for a specific problems and we can (find) correlate with respect to strain or with respect to stress.

So, thing is that, that this stiffness matrix or compliances matrix use and we can convert this in terms of the system of the equations or vice versa as well during the analysis of the (s) different type of crystal structure. So, it is obvious that only 3 independent coefficients are bring down in case of cubic symmetry as compared to 81 coefficients we consider the maximum anisotropic or (4) in case of fourth order tensor. So it is possible to reduce the total number of constant term from 81 to 3 in case of cubic structure when we consider all the symmetric nature of the, of this cubic crystal systems.

So, if we look into that the cubic crystals systems is having the 3 different constant term, S 11, S 12, S 44 or C 11, C 12 and C 44. So in case of cubic symmetry for different material generally this 3 constant terms is all tabulated form, we can get and we can use from the standard table, the values and to solve the different kind of problems, but if we look into the, the constant term here, the unit of the constant term is basically in terms of the inverse of the Pascal, let us investigate this thing.

Epsilon is basically here the S (())(37:01) in terms of Sigma, so basically the strain is dimensionless quantity and stress is having the unit of in SI it (p) Pascal or Newton per meter square or this thing. Similarly to make it dimensionless (the) the unit of S is just reverse of the stress unit. That means if this is the unit of stress is Pascal then S would be, to the power inverse, that is why be careful to note it down that S 11, S 12 and S 44, all this constants

terms are represented in terms of the inverse of the Pascal or maybe Tera Pascal; Tera means 10 to the power 12.

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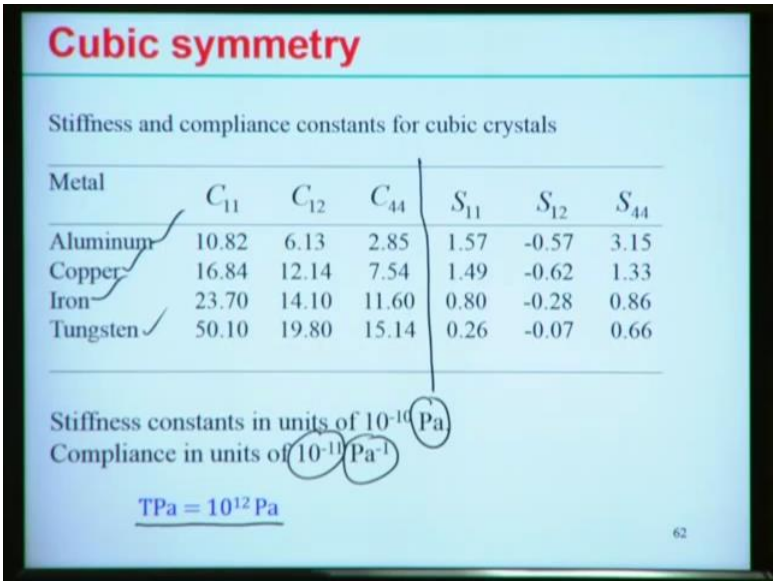

Here is the, the standard table for the cubic symmetry on different cubic structure for different materials the stiffness and compliances terms, different constant terms are represented for cubic crystals. If we look into that in (terms) metal that aluminium, C 11, C 12 and C 44 we can use all these constant terms or we can use compliances term S 11, S 12 and S 44 and for aluminium, copper, iron and tungsten these are the standard values obtained but we should be careful looking into the, this units of this thing.

Stiffness constants is the unit of Pascal in this particular table and compliances in terms by the multiplier due to the minus 11 but inverse of the Pascal. And Tera Pascal equal to 10 to the power 12, the multiplying factor Pascal. So to use the numerical value from the standard table, we should be very careful to look into the different units is given for the compliances terms and units for the stiffness term.

Now after the cubic symmetry, next stage is the in case of isotropic symmetry of how we can correlate in case of isotropic symmetry or (s) basically for isotropic material behaviour, how we can use this constant term. So for isotropic materials and basically we already menti1d that for anisotropic for individual single crystal behave anisotropic or if there exist some tensor structure that also behaves (in an) in a anisotropic nature, but in case of polycrystalline

(m) material when we try to aggregate under (ra) randomly oriented grains that we can consider in an aggregate having the isotropic behaviour.

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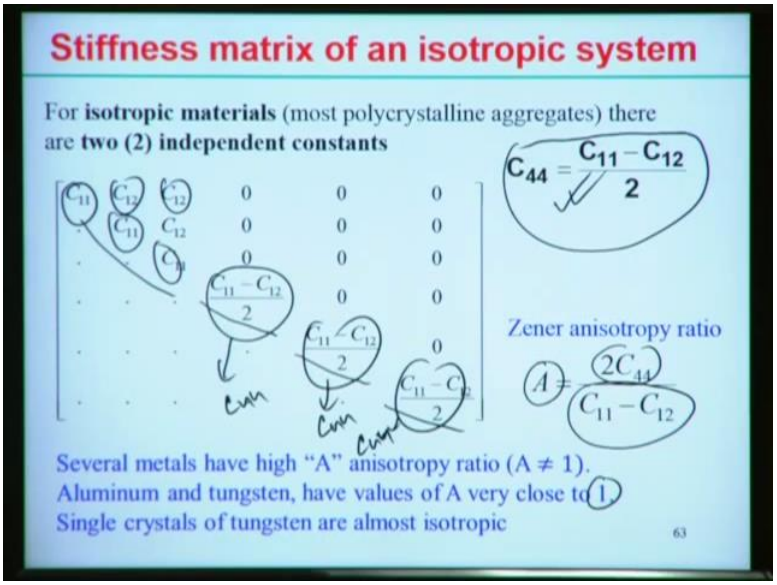

So in this case we represents all this compliances or stiffness terms or that matrix into, in terms of only 2 constant terms. So how we represents the compliances or (tiff) stiffness matrix isotropic behaviour. Here we are having C 11 and we are having these constant terms, half of C 11, 12, minus 12 by 2. So other is the C 12, so only having 2 different constant C 11 and C 12 to represents the isotropic behaviour of the materials here.

Now if we look into that, actually C 11 minus C 12 by 2 equal to C 44, this relations actually exist when the material is the isotropic. But this sometimes, this ratio that is called Zener anisotropy ratio A ease this ratio, is used to analyse the (isotropic) anisotropic nature of the material, for example if it is perfectly isotropic then this relations is true, but if it is not perfectly isotropic then we introduce, we can introduce this constant term, that ratio. So several metals having the high A value, that represents the anisotropic ratio. So (specifically) definitely for anisotropy, A should not be equal to 1.

But aluminium and tungsten practically having the values A, actually that ratio, it is very close to 1. So single crystals of tungsten are almost isotropic, that means A is almost equal to 1 in this case. So, that ratio actually, to represents the anisotropic, but in case of actual isotropic material this relation holds good so instead of writing this term we can write C 44

here. So this are the typical (elastic) compliances or stiffness matrix in case of isotropic material.

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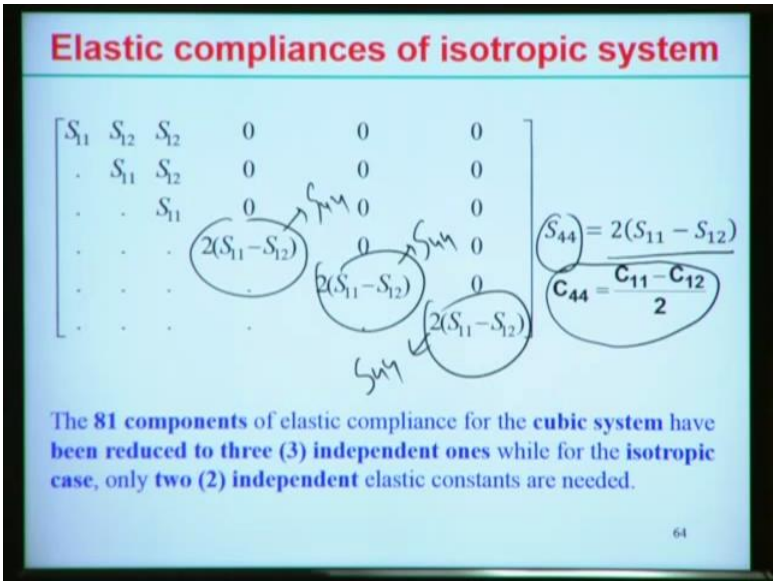

Now in terms of (ace) similar way we can write the compliances matrix in case of isotropic system. So here we just observe that the relation between the S 44 and S 11 and and 22, here the multiplying factor is 2 but in case of (s) in terms of the stiffness, the C in (s) terms of the half of that. That should be carefully noticed and all this terms it can be represented by S 44 because that is holds good in case of isotropic behaviour.

So, in case of isotropic behaviour though there is a, we started with the (as of) with the (80) 81 number of components or 81 number of constants the stiffness or compliances matrix but we did this to cubic system up to 3 independents, then finally in case of isotropic case we can reduce the constant terms ultimately 2.

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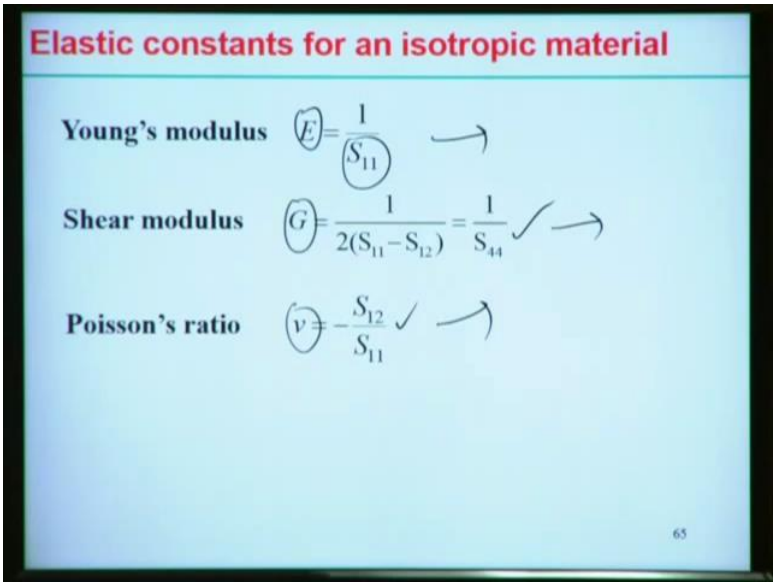

So now, if we, we have already defined the different (ela) elastic properties or different elastic constants in case of that is, for example that Young's modulus, shear modulus and the Poisson's ratio. So that Young's modulus, shear modulus and Poisson's ratio, that can also be represented in terms of this elastic compliances or the stiffness (elast) stiffness term.

So here the, in terms of compliances E can be represented 1 by S 11 and G is the inverse of 1 by 2 into S 11 minus S 12 that is equal to 1 by S 44 and Poisson's ratio is the S 12 by 12. So actually from the standard table available so using that constant term diagonally we can evaluate that this 3 different elastic constants for example, Young's modulus, shear modulus and Poisson's ratio can also be obtained.

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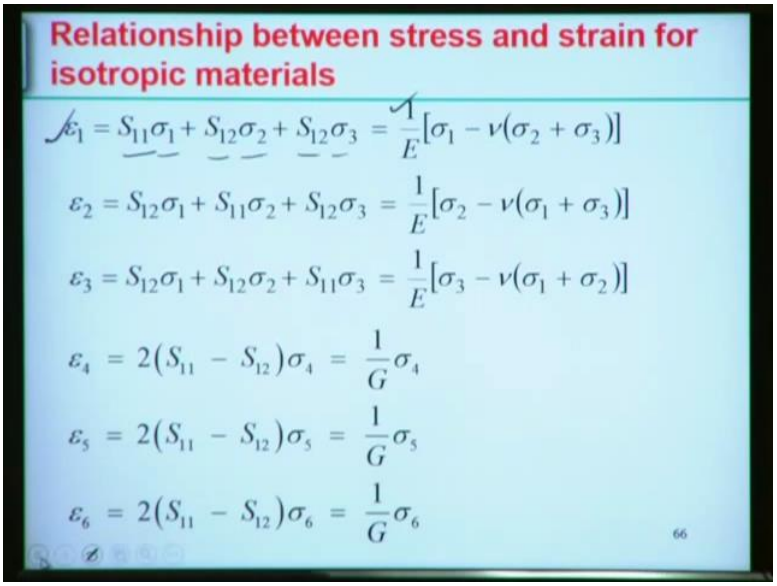

Now we, if we try to reinvestigate this things in terms of the compliances terms the Epsilon 1, the, if we go to the linear system of the equations the Epsilon 1 is the S 11 Sigma 1, S 12 Sigma 2, S 12 Sigma 3. So now if we put into the relationship between the elastic constant and the compliances term we can find out that S 11 is, was equal to 1 by E, so that we have retained and Sigma 1 comes S 12.

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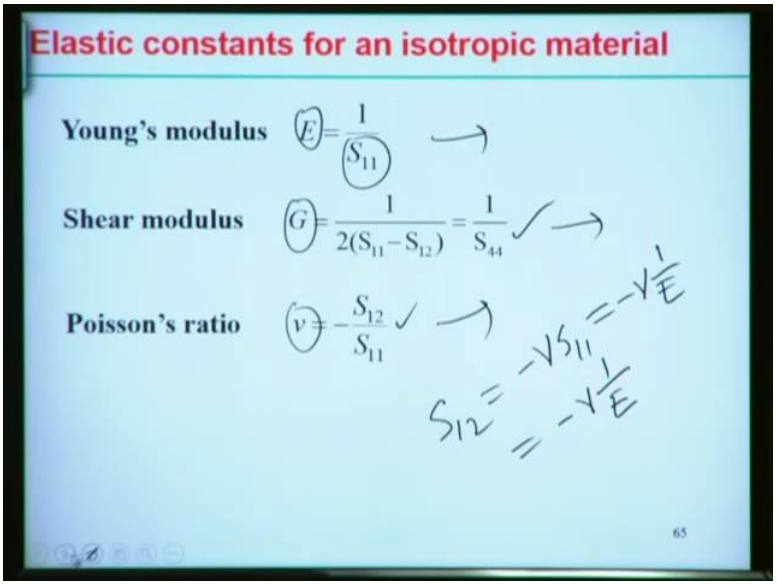

So, S 12 we can find out that S 12 equal to minus S 11 that is equal to 1 by E. So basically S 12 equal to 1 by, Nu by E.

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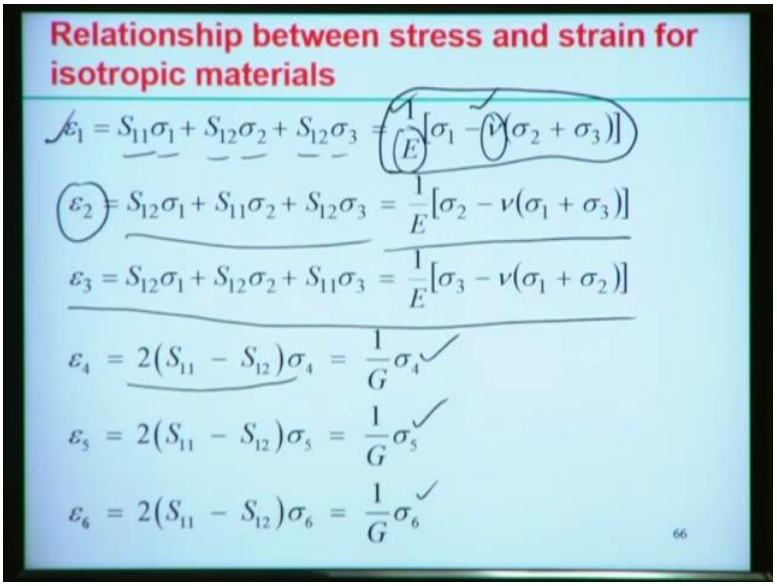

So if we put that relation then we can find out this relation that is true while we derive the, in terms of elastic constants and due to correlate in 3 dimensional stress strain, what are the strain or stress relation in terms of the elastic constants E and Nu. So in this relation we used only 2 elastic constants here. Similarly Epsilon 2 can be represented this or that is actually comes from that relationship, the, or of this compliances or stiffness terms and similarly Epsilon 3 can also be represented like this.

So Epsilon 4, Epsilon 5, Epsilon 6 in terms of the S 11, S 12 or in terms of the S 44 we can represents that is G, shear modulus in terms of shear modulus Epsilon 5 and Epsilon 6. So these are the 3 dimensional stress strain in, within the elastic limit, that different strain component or stress component are, can be correlated with respect to the elastic constants but at the same times if we know the compliances terms or stiffness terms from the analysis of the anisotropic behaviour of in case of different crystals system. And this constant term can be, to evaluate this thing, this constant term can also be used to estimate the different elastic constants term.

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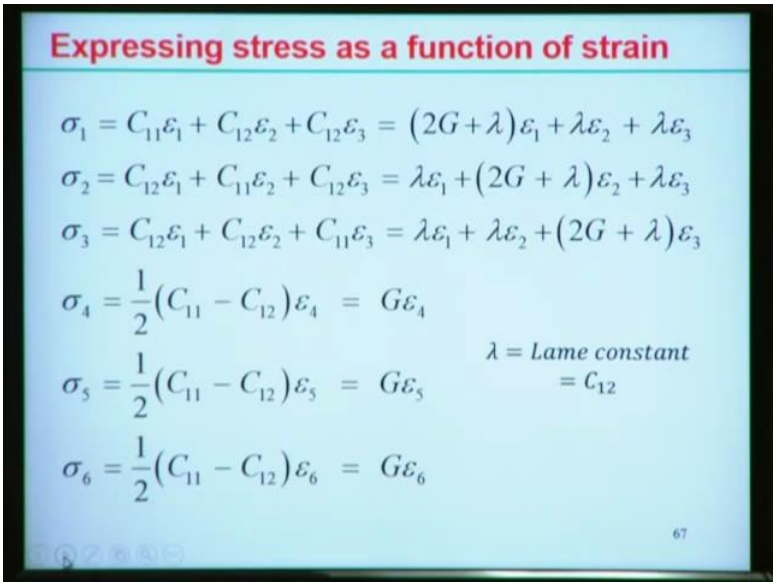

So, similarly we can represents in terms of the stiffness that stress (rela) related to strain so all similar way we are representing here.

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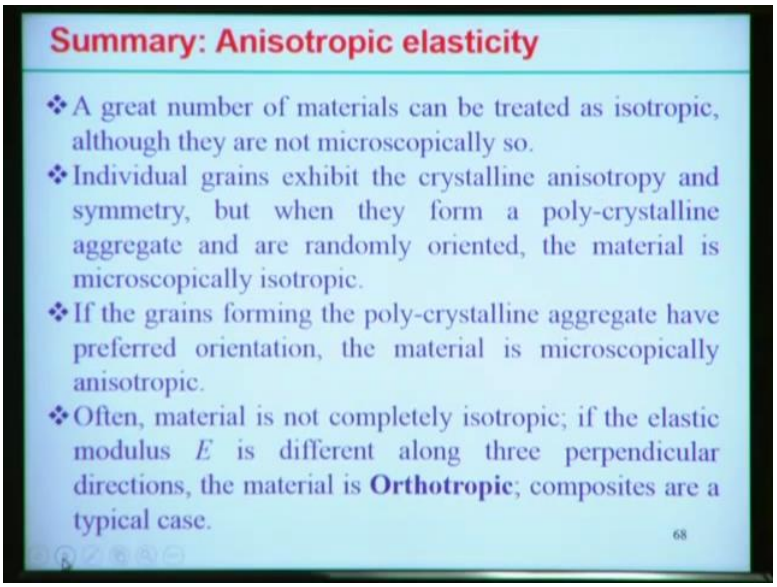

So, now the summary or point, typical practical points, when you try to analyse the anisotropic where it can be applicable in practically. First things that, a great number of materials actually can be treated as isotropic although they are not (microscopal) microscopically so. So, what does it mean that materials can be treated isotropic without this, but this isotropic behaviour is possible when we not, we are not focusing on the individual single crystal structure.

Rather we are trying to focus on the specific microstructure that is also aggregates of the several (crys) grains and then in that case we can use the isotropic behaviour, we can analyse the further. But if the individual grains exhibit that crystalline anisotropic, that is definitely true and symmetry but when they are form a poly-crystalline material or poly-crystalline aggregate, they are (an) the grains are randomly oriented, then the material can be considered as a isotropic behaviour, microscopically isotropic.

But if the grains forming the poly crystalline aggregate having some preferred orientation, the material is microscopically anisotropic. So in this case if there exist some preferred orientation then we should consider the anisotropic behaviour for this specific materials. But often material is not completely isotropic; that is true, if the elastic modulus E is different along 3 perpendicular directions basically irrespective of the directions the elastic modulus are different, the material is orthotropic. So composites are the good example of this type of anisotropic behaviour.

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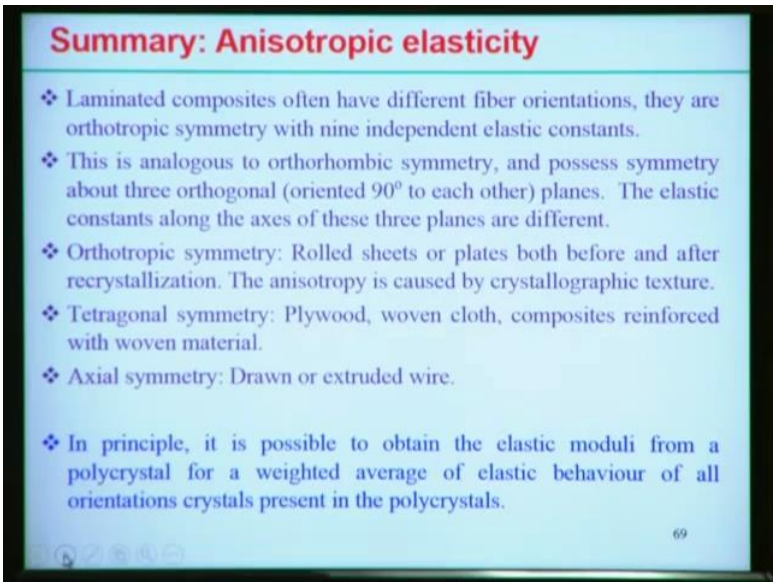

Now while we apply all this anisotropic for specific situation, for example laminated composite have different fibre orientations, so they are the orthotropic symmetry with 9 independent elastic constants. So actually the, this is analogous to the orthorhombic symmetry and the possesses symmetry about the 3 orthogonal planes. The elastic constants along the axes of 3 planes are different.

So this is the 1 case laminated composites having some kind of nonsymmetric structure or maybe anisotropic behaviour can also be considered in this case but orthotropic symmetry generally understand or we observe the rolled sheets or plates before and after the recrystallization, the anisotropy caused by the specifically crystallographic textures in this specific rolling or process. So, we can consider this anisotropic as the (iso) isotropic orthotropic symmetric behaviour in this case.

Tetragonal symmetry we observe in the plywood, woven cloth, composites, reinforced with the woven cloth or we can find out the tetragonal symmetry. So accordingly when we try to the anisotropic behaviour began introduce the constant term looking into the tetragonal symmetry of this specific structure. Axial symmetry we observe in case of extruded or drawn wire, so what this is the specifically (ty) this specific type of metal forming processes.

So in principle is possible to obtain the elastic modulus from (polycryst) polycrystal or a weighted average of elastic behaviour of all (orient) orientations crystals present in the polycrystals. So, point is that if we try to produce the (())(51:58) or value of the elastic modulus sometimes we can introduce the weighted average method considering the elastic modulus of individual directions or to analyse the (s) in case of different polycrystalline structure.

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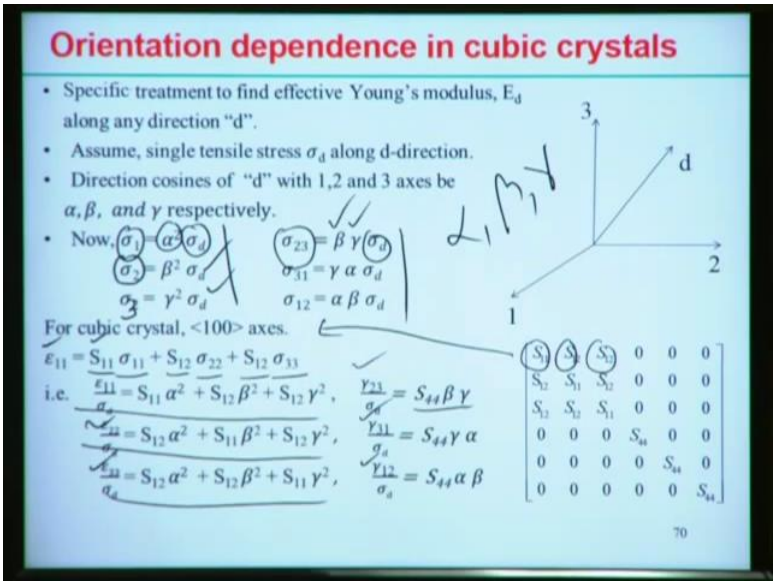

Orientation dependence in the second part now we start, that is orientation dependence of cubic crystals; how we can find out the orientation dependence of cubic crystal structure? So,

specifically 1 specific direction the simplest way what maybe the elastic properties in 1 specific direction that we analyse that way. Suppose 3 axis system we define, 1, 2 and 3 are the, represent 3 axis system and 1, we are interested to find out the Young's modulus along the direction; for example along the direction d.

So in this configuration, first as the single tensor stress Sigma d is acting along the direction d and if we find out what is the, so along the direction Sigma d is, stress is acting and if we know what is the strain along this direction d and the ratio can be represents E as a Young's modulus, 1 specific direction at d. So to do that with the specific configuration, first consider the direction cosines alpha, beta and gamma with respect to the 1, 2 and 3 axis and the direction cosines is defined with reference to the axis d along the 1, 2 and 3 axis; for example alpha, beta and gamma is the 3 different direction cosines.

So in this case when we try to analyse the transformation of the stress with respect to 1 axis to the another axis if we look into that part then we can find out, the Sigma 1 that means stress along 1 direction 1, simply can be represented by alpha square into Sigma d. So that alpha square equal to that direction cosines with respect to axis 1, which is the alpha into alpha. Similarly, Sigma 2 can also represent beta square Sigma d; so this is a stress component along the axis 2 with (res) with reference to the stress at direction d. Similarly (si) Sigma 3 can also be treated like that.

Now shear component Sigma 23, so that component 23 with reference to the stress along the axis d is simply multiplying by the 2 direction cosines beta and gamma, Sigma 31, gamma alpha; similarly Sigma 12 is the alpha beta, all direction cosines we use and we took the reference as stress along the direction d, we can represents the (d) individual component (shear) normally components stress and the shear component and along the different axis 1, 2 and 3 axis.

Now, for a cubic crystals along the X, along the direction 100, we say any 1 of the axis, the strain can be represented with the compliance system S 11, Sigma 11, S 12, Sigma 22, S 12, Sigma 33. So that linear (li) that linear relation or that system of equations actually comes from the matrix notation when we, using the compliance system (in) in case of cubic crystal structure.

Here we see that, the, in that matrix there is a S 11, S 12 and S 12 and other components are 0. So from here we can directly use here, we can find out the relation of the strain and stress

in terms of the compliance system. So, similarly we can find out the system of equations 22, Epsilon 22, Epsilon 33; similarly gamma 23, 31 and 12. So here if we see that putting the value of Sigma 11 or here we can say Sigma 11 is equal to Sigma 1 here. So, that is alpha square Sigma d, if we put that, then we can find out this 3 relation.

Similarly, the shear component equal to in terms of S 44, beta gamma. So, point is that we are representing different strain component in terms of the direction cosines with respect to 1 specific direction d and the stress value along the direction d so, all in terms of the direction cosines and Sigma d. Now, if we try to resolve the stress, now we need to find out what is the strain component along the direction d.

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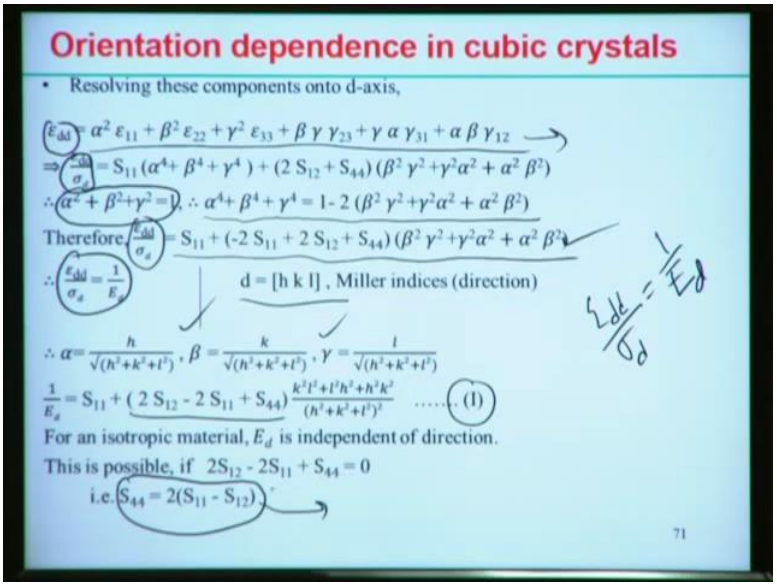

So, along the direction d the (strain) strain component can be represented in these but these actually comes from the, the transformation of the axis with respect to 1 axis to another axis, if we look into that formulation and we can find out this is the component of the strain along the (s) 1 specific direction. So this transformation of the stress with respect to 1 axis to another axis, that will be discussed in the next module, but we use this, using that transformation of the strain (comp) component with respect to 1 axis to another axis, then we can find out the Sigma dd in terms of the direction cosines and component along the 3 different direction 1, 2 and 3; normal strain component as well as the shear component.

So, from here we can find out that Sigma dd, that means strain along d direction is in terms of the elastic constants, S 11, S 12, S 44 and in terms of the direction cosines but (root) from the

direction cosines holds good relations that alpha square plus beta square plus gamma square equal to 1, if we put it then we can find out this alpha to the power 4 plus beta to the power 4 plus gamma to the power 4 equal to this relation. And now we put this expression here and we can find out that Sigma dd by alpha d in terms of S 11 and the direction cosines.

Now, this Sigma, actually Epsilon dd by Sigma d, so strain along direction d, stress along direction d, is basically indicates the Young's modulus along the direction d. So that Young's modulus along the direction d is represented in terms of the compliance system S 11, S 12 and the direction cosines. So, further formulation or further analysis of direction cosines in terms of the Miller Indices can also be d1.

Suppose, Miller Indices of direction d is h, k, l, so in this case the direction cosines alpha can be represents h by the magnitude root of h square plus k square plus l square, similarly beta similarly gamma. So if we put all this expression we can find out that Young's modulus along direction d is represented by equation 1 where if we know that 1 specific directions the Miller Indices; so (in) in this case we can find out the (yo) Young's modulus, (1) with that specific direction using this formula.

Now, for an isotropic material, if Ed is independent of direction, then we can find out that independent of direction, if it is only possible, if this term is 0 because that (hdkl) that, that term, it can, it cannot be never 0. So, if this is the 0 then we can find out this relation, S 44 equal to twice, 2 into S 11 minus S 12; so thats holds good also in case of isotropic material. That we have discussed for the isotropic material how we can modify from the cubic structures to the isotropic symmetry we use this (rela)(we we) we can use this relation to find out or to reduce the further number of constant terms here.

So here it is also other way we can find out that, this expression that isotropic material properties hold good in case, if this term equal to 0 and that brings the condition is 44 equal to twice into S 11 minus S 12. So, all this expression can also be possible to represents in terms of the (sti) stiffness constant.

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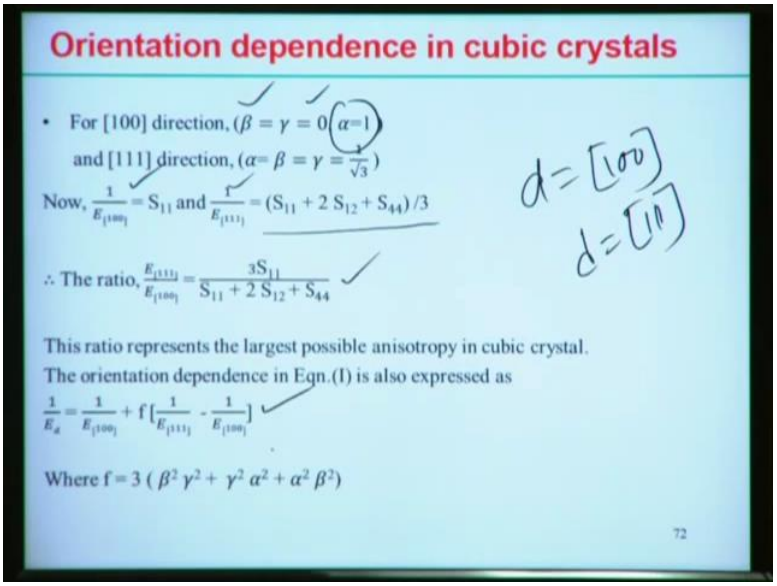

Now let us look and investigate this thing 1 specific direction; let us for example 100 direction. So 100 direction is basically axis of the cubic crystal, so in this case that direction cosines maybe with respect to 100 so alpha, first direction cosines (())(62:28) 0 that is equal to 1 and, but beta and gamma that actually represents that 0. So similarly, in case of 1 1 directions where the directions cosines alpha, beta, gamma equal to 1 by root 3.

So, if you will try to put the (())(62:51) formula then 1 by E suppose direction d equal to 100, in this case we can find out 1 by E 100 equal to S 11 because here, h, k, l equal to 1 or upper term equal to actually 0 so 1 by E 100 is basically S in terms of S 11 only. Similarly if we put d equal to 111 direction, then we can find out 1 by Ed is equal to in terms of the S 11, S 12 and S 44. So that ratio E 111 by E 100 can also be represented by this. So this ratio actually represents the highest, largest possible anisotropy in cubic crystal.

So the orientation dependence of equation 1 can also be expressed in terms of this 1 by d, 1 by E 1 in terms of the (ore era) elastic modulus in direction, 1 specific direction 100 or 1, 111. So, with respect to that we can find out thus, this correlation where f equal to this expression in terms of the direction cosines. So we can use this relation also further to analyse the anisotropic behaviour at different directions or basically elastic modulus.

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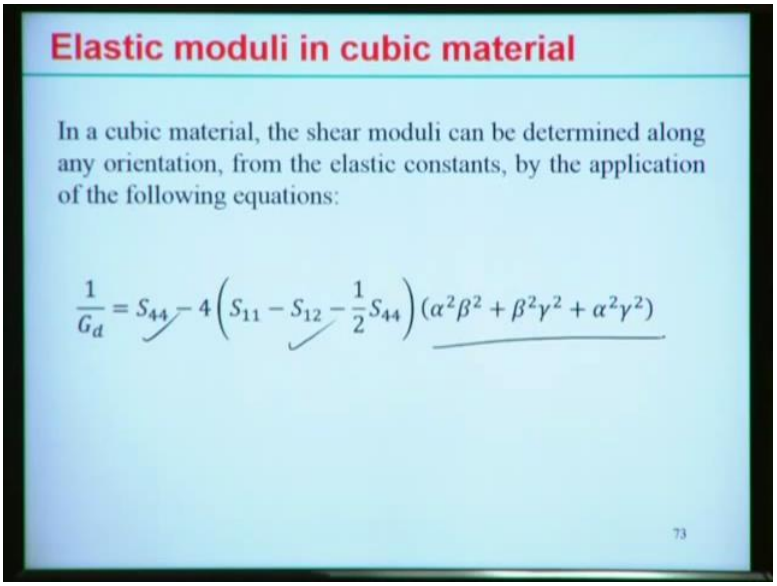

So similarly, in case of cubic material the shear moduli can also be determined 1, in 1 specific direction or 1 specific orientation from the elastic constants as well by the application of the following equations. So in this case the 1 by Gd, G is the shear modulus, and in terms of S 44, S 11, S 12 and this and the direction cosines, so that can also be derived from the 1 specific direction in case of anisotropic elasticity.

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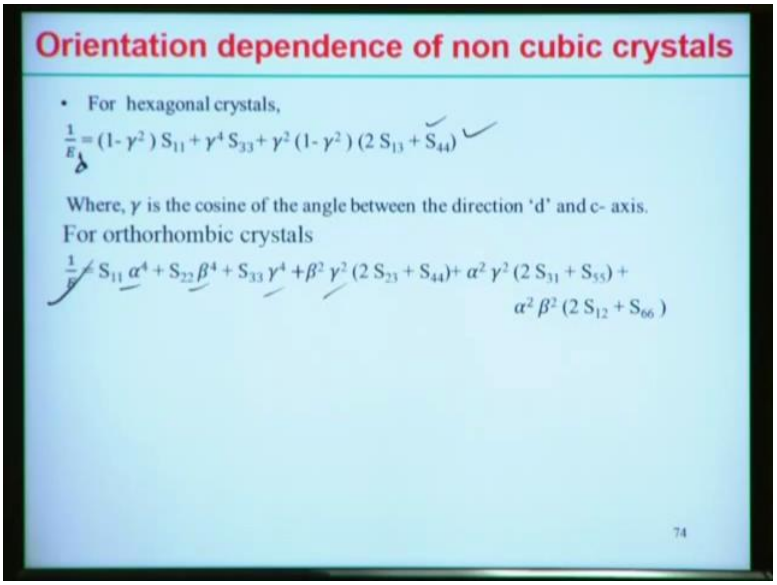

Now, if we look into the non-cubic crystals, then for (ex) specifically for hexagonal crystals these are the dependence direction dependence of hexagonal crystals in terms of the different constant term S 11, S 33, S 13 and S 44 and others are the direction cosines but if you (s)

other means here we use only 1 direction cosines gamma that is the cosines of the angle between the direction d and the C axis.

Similarly for orthorhombic crystals we can use this relation, so direction depending on elastic modulus terms of the, constant term S 31, S 55 S 12, S 66 and others are the, Alpha, beta, gamma are the direction cosines in case of non-cubic crystal or for example here the orthorhombic crystal. So these are the way to find out the any, the elastic constant terms, Young's modulus or shear modulus. When we interested to 1 specific direction whether it is cubic crystals or whether it is non-cubic crystals we use, need to use all this expression for that.

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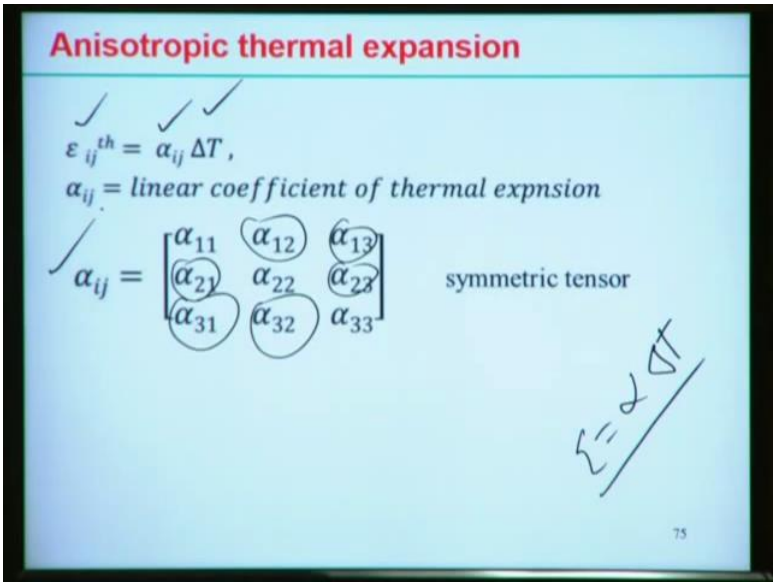

So, similar way, we can (find) explain the anisotropic thermal expansion. So if it is a linear case then we can find out the strain is 1 direction simply, what is the coefficient of thermal expansion and the temperature difference; so in this case it is maybe significant when there is a 1 direction or if it is a, there is a existence of the isotropic (elastic) isotropic properties exist then we use this relation directly.

But if there is a anisotropic behaviour also exist in case of, for example linear coefficient of thermal expansion are different and the different orientation of the crystals, then in general we can use the, analyse the thermal strain by using this formula alpha ij into Delta T so that Epsilon ij; so basically in terms of the matrix we are representing the linear coefficient of

thermal expansion having all the components like stress, we (re) we represents what way the stress tensor or the strain tensor.

So of course, (in) in this case it is also the symmetric tensor, if we consider symmetric tensor in this case we use the alpha 12, alpha 21 is the same, alpha 13, alpha 31 is the same, alpha 23, alpha 32 is the same. So this is the symmetric matrix for the linear coefficient of the thermal expansion is the similar way we analyse the (s) stress or strain component.

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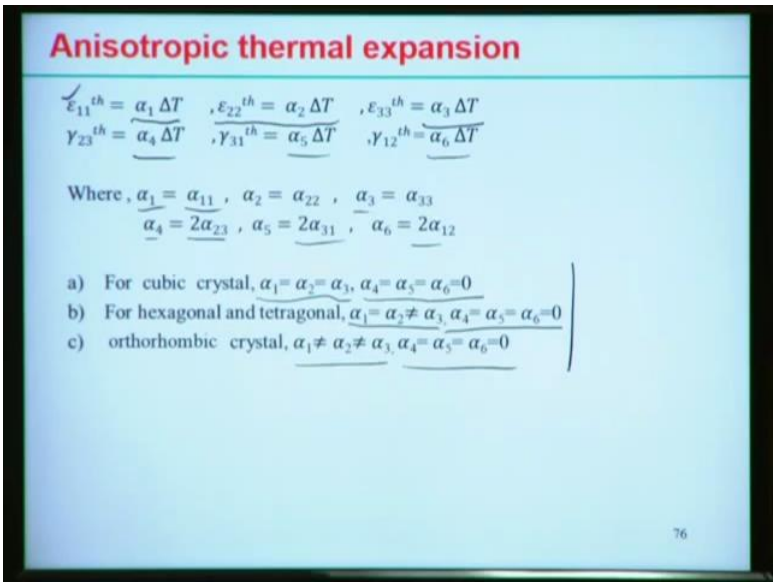

So these are the simply notation we use the anisotropic thermal expansion for different cases. So, for example alpha 1 Epsilon 11, thermal is equal to alpha 1 Delta T, 22 is alpha 2, alpha 3 we use that alpha 4, alpha 5, alpha 6. So for the symmetric matrix we reduce into the 6 components of the thermal expansion coefficient and where the relation between this alpha 1, alpha 11 so (s) this actually relations (ors) bring similar to what we use the notation for the, from 112 to 1, that we did in case of symmetric stress or symmetric strain.

So similar way we can introduce here. So since the always the shear component so we are multiplying by the factor 2 here to that what way we introduce in case of linear strength and the sorry normal strain and the thermal shear strain. Similar way we introduce that effect in terms of the expansion of the (th) coefficient. So in case of cubic crystals these are the relation actually hold for the symmetric nature.

For the hexagonal and tetragonal we use these relations, and for orthorhombic crystal we use this relation to analyse the thermal strain and also of course all this difference in the thermal

(s) first we try to bring the different constant terms using the coefficient of the thermal expansion that actually tells about the anisotropic nature and the different (o) orientation but again by looking into the analysis of the symmetric nature of different crystal structure we can further reduce this constant term to less number of constant term in case of coefficient of the thermal expansion.

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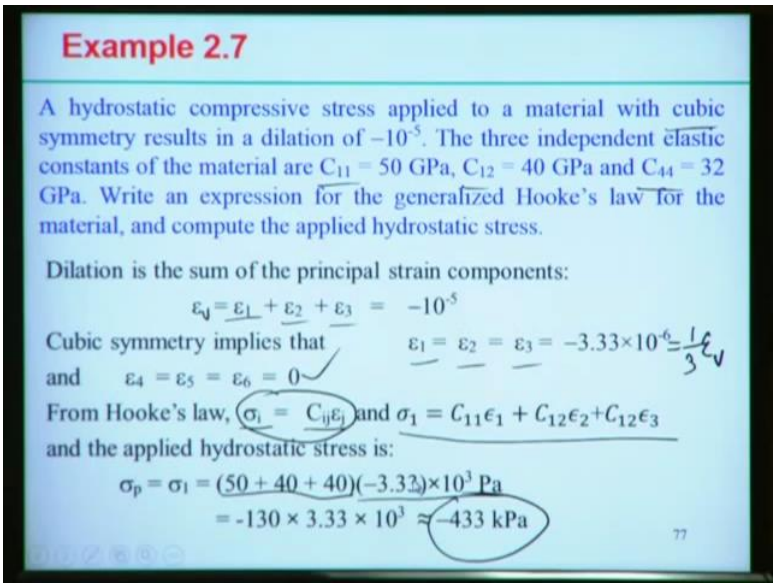

So this was all about the theoretical part of the orientation dependence of crystal structure. Now we will try to shift typical examples that will help to understand the theoretical part of the, of this subject or, or specifically this module where we are analysing the anisotropic and isotropic elasticity. So first, look into that first example; the hydrodynamic compressive stress applied to material with cubic symmetry.

So here we mention that cubic symmetry results in a dilation of 10 to the power minus 5, that means the volumetric strain is here 10 to the power minus 5. The 3 independent elastic constants of the material are given, C 44. We need to generalise the Hooke's law for the material and we need to compute the applied hydrostatic stress. So, definitely the volumetric strain is the sum of this thing, and since it gives the hydrostatic stress, maybe due to the cubic symmetry implies that Epsilon 1 equal to Epsilon 2 equal to 1 third of the volumetric strain here.

And then other shear component will be 0 due to the symmetric nature of this things. Now if we apply the Hooke's law here, we can find out Sigma i equal to Cij into Epsilon j, or similar

way we can use the other equation also in terms of the compliances but here the data given in terms of the stiffness; that means C 11, C 12 and C 14 C 44 is given. So we use this relation and we can find out the system of the equation is like that, Sigma 1 equal to C 11, Epsilon 1 plus (c) C 12, Epsilon 2 plus C 12, Epsilon 3.

Now, if we put all these constant terms here and Epsilon 1 equal to Epsilon 2 equal to Epsilon 3, that means the strain component, then we can find out the 4, minus 433 kilopascal. So hydrostatic pressure here is the 433 kilopascal here. So, this way we can use that different elastic compliances or stiffness term to solve different kind of problem.

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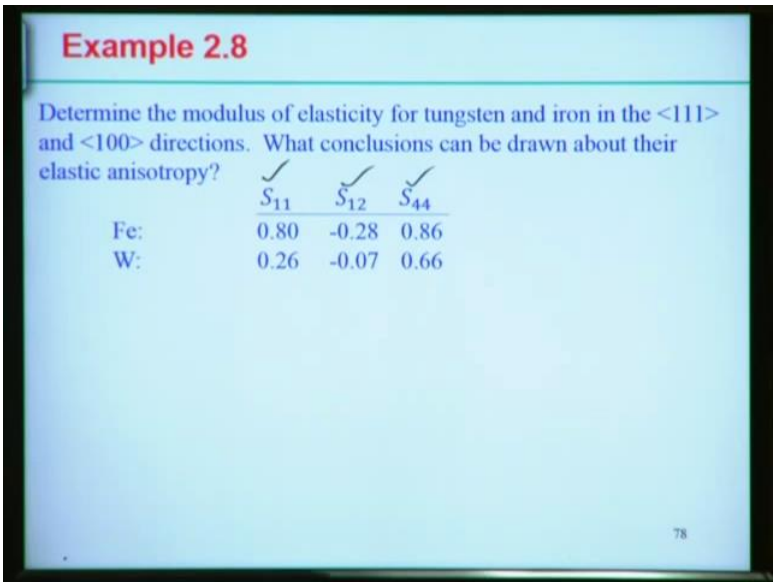

Let us look into another problem, that we need to determine the modulus of elasticity for constant and iron 1 specific direction 111 that means along the body diagonal and 100, so along the axis of the crystals what conclusion can be drawn about their elastic anisotropy. So here it, if we see that S 11, S 12 and S 44 is the standard values (i) available are with this problem itself. So we need to analyse all this in terms of the compliances, not in terms of the stiffness.

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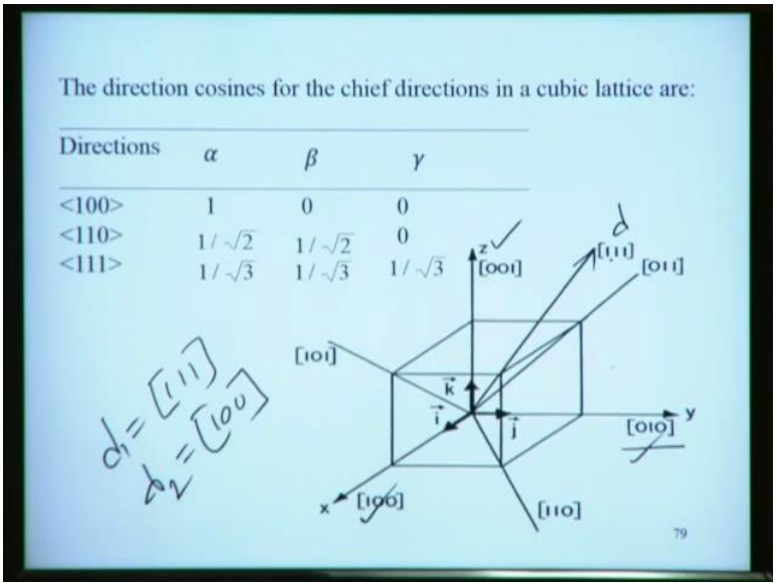

First, we assume that (who) there is a direction d here, first 2 cases 111 and another direction is 100 because these 2 direction we need to find out the elastic constant terms. So, if we consider d as 111, so suppose d as 111, this direction and we have defined that other 2 directions of the crystal axis systems, the X, Y and Z so 1, X 100, 010 and 001. Now, with respect to that we need to find out what is the direction persons with respect to 111.

Here if we try to find out with respect to 111, the direction coastlines alpha, beta and gamma are, we can easily evaluate and because this direction cosines can also be find out that (o uh) with respect to 2 different axis between this and that, 100 and 111 we can use that (wu).

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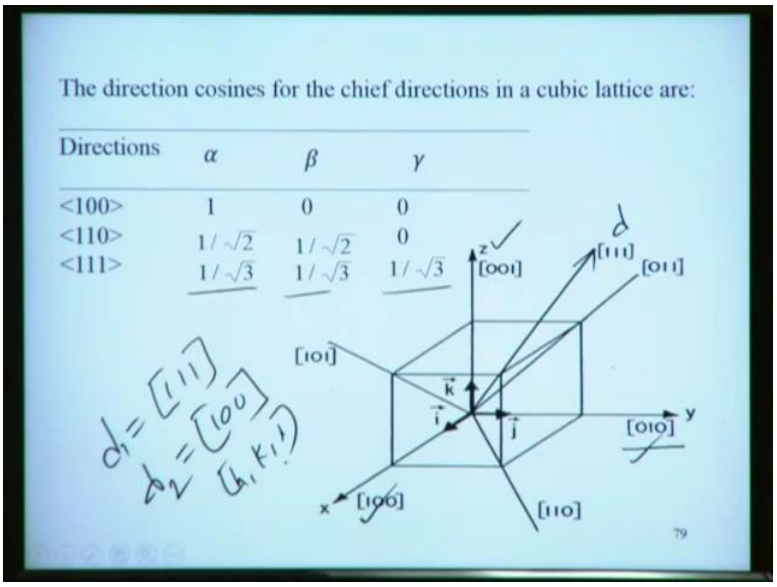

So we have used h1, k1, l1 is the 1 direction and suppose h2, k2, l2 is another direction, so direction cosines can also be find out like that; h1h2 plus k1k2 plus l1l2, (())(75:19) of h1 square, k1 square plus l1 square into h2 square, k2 square plus l2 square. Now from here we can find out the direction cosines between the 2 axis.

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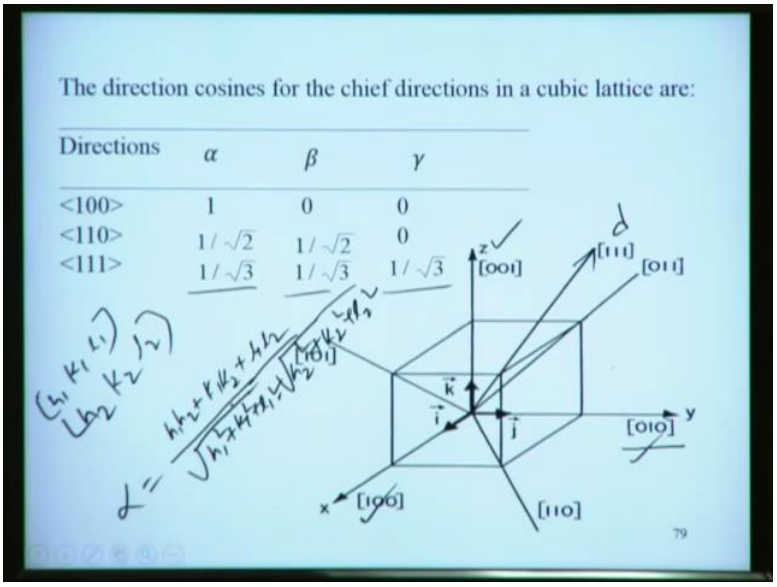

So these are the way to find out the direction cosines between the 2 directions we can find out these are the values of (alpha, beta) alpha, beta and gamma and with respect to 100 axis we can find out and with respect to if if again d equal to 100 that also we can find out and we need to find out 110 also if necessary. So we find out this template table or direction cosines with respect to 1 axis.

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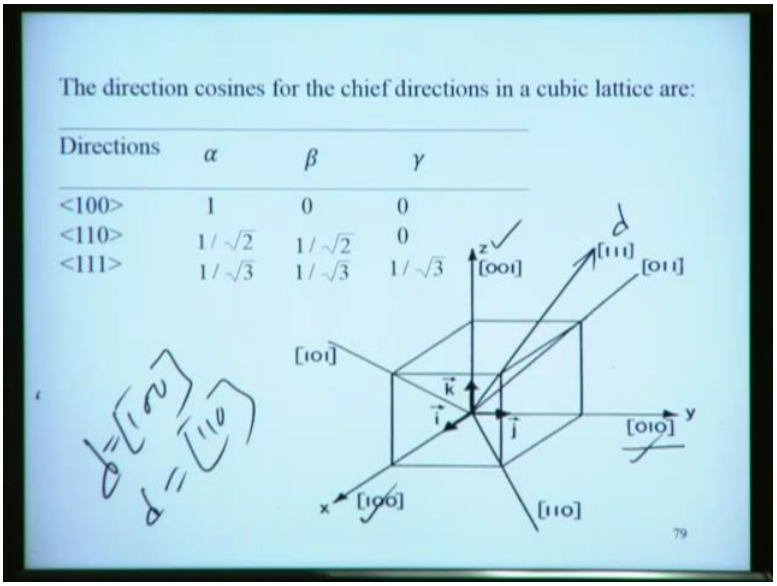

Now we use that standard formula, for that how can find out the Young's modulus 1 specific direction E, 1 by E 111 in terms of S 11, S 12 and S 44, okay? And if we put all these values and here the term, alpha, beta, gamma; that means direction cosines if we put it and we can find out 1 by E 11 equal to 0 point 37 and finally E 11 equal to 270 Giga Pascal. Similarly for iron we can find out along the direction 100 it is 125 Giga Pascal.

So if we see the Young's modulus along 111, or along 100 direction, in this case there is huge difference in the values of the Young's modulus. So, specifically that, it is very obvious that, that (ino) anisotropic behaviour that means that elastic modulus are not same along 2 different directions. So, it is, the elastic modulus is very much orientation dependence in case of structural iron.

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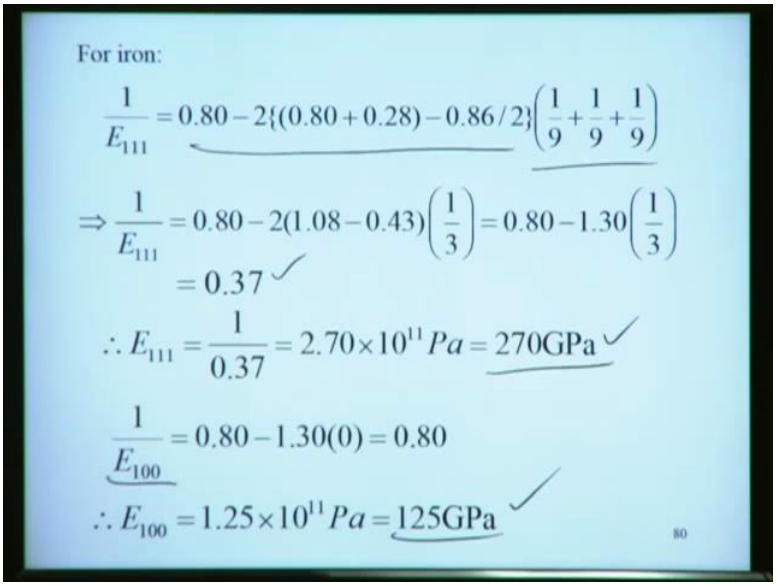

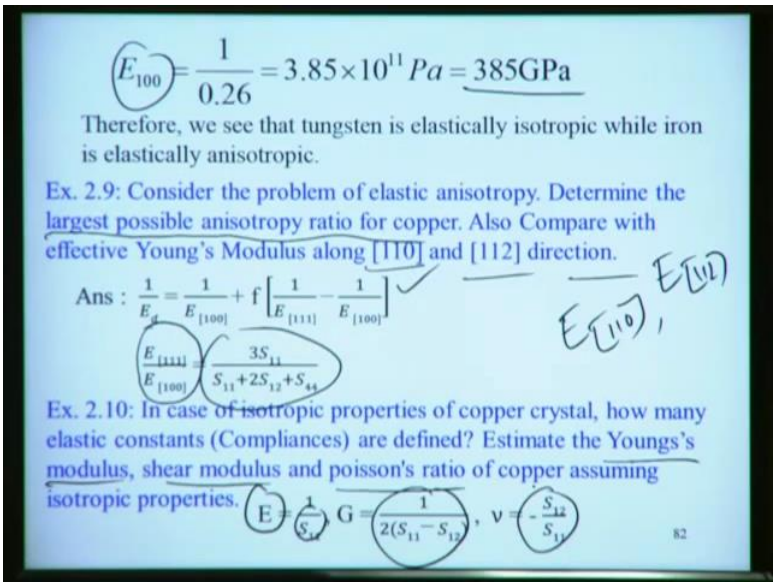

But if we look into the investigate for (tun) tungsten in the similar way we can we can find out E 11 equal to 385 Giga Pascal and E 100 equal to 385 Giga Pascal even. So, if we observe that tungsten E, along the direction 111 or along the direction 100 it is almost similar. So then we can conclude that tungsten is basically (elastic) elastically isotropic while iron is elastically anisotropic. So we should be very careful to whether we need to consider the isotropic or anisotropic behaviour for the different types of materials. So, this is the proof or this is the example of the anisotropic or isotropic behaviour in case of tungsten and the iron at different orientation.

Now we look into thats next problem. So, now consider the problem of the elastic anisotropic, determine the largest possible anisotropic ratio for copper. So, largest possible anisotropic ratio in case of cubic structure, copper, copper is having probably the FCC structure. So in case of cubic structure, largest possible ratio can be defined that ratio of the E along the direction 111 and with respect to along the direction 100. So that stand-up formulation theory we have already derived.

We use this formula and we can find out the largest anisotropic anisotropic ratio for copper can also be obtained. Similarly also compare with the effective Young's modulus along 110,112 direction so along 110 and along 112 direction we need to find out the Young's modulus along 110 direction and along 112 direction by the similar approach and we can make some comments on the, whether there is a anisotropy exist at the different directions.

Next problem we will try to find out in case of isotropic properties of copper crystal. How many elastic constants or compliances are defined? Estimate the Young's modulus, shear modulus and Poisson's ratio of copper assuming the elastic properties. As we discussed that when you try to estimate the Young's modulus, shear modulus and Poisson's ratio for a specific material we can directly evaluate the E, (we) looking into the (re) relation for the isotropic properties of the materials in terms of the elastic compliances or stiffness term, then we can directly use this relation and we can find out the, all this (el) elastic properties in case of specific materials.

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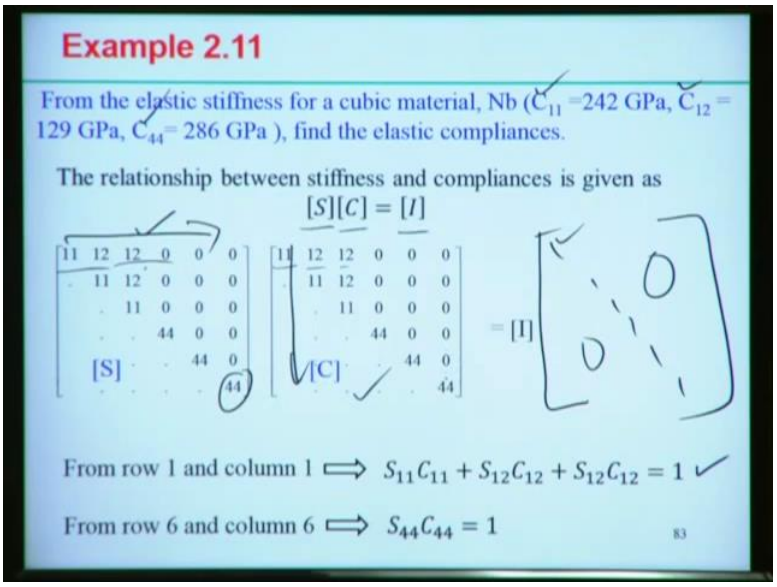

Now 1 very specific problem; for example from the elastic stiffness the, for a cubic material, then find the elastic compliances. So, in this case we have the data (or) for the (elas) elastic stiffness but we need to find out the elastic compliance system. So, in this case actually there is, elastic compliances and elastic stiffness they actually hold good for a specific relation that is the, there are 2 matrix actually multiply these things, they actually hold good the whether (un) unit matrix.

So, in a unit matrix, or maybe identity matrix in this case the diagonal elements are 1 and other elements are simply 0. So, for the cubic material this is the expression of the matrix S and for the cubic material this is the expression of the matrix C; that means stiffness matrix. So, when we multiply these things we brings the (uni) identity matrix, so in this case now we have the, there, we have expressed all these components, 11, 12, 12 considering the cubic material, cubic symmetry and here also 11, 12.

So position of all this S and C term is the same, okay? So now first if we look into that, so there is, we need to evaluate because cubic symmetry in case of cubic symmetry the, the constant terms is required, only there is a 3 constant terms, C 11, C 12 and C 44. So we, still if we try to find out the 3 equation at least, then we can solve it and we can find out all these things. But tactfully we need to consider this relation like that, so here I is actually basically 1 1 1 1 1 1 and other terms 0.

So from here if we see try to solve this equations, then first look into the row 1 and row, column 1. So, for, from row 1, to column 1, if we find out S 11 and C 11 plus S 12 C 12 plus S 12 C 12 we are getting this equation equal to 1, here is the 1 term. Similarly row 6 and column 6 if we look S 44 C 44 equal to 1, so we get 2 equations.

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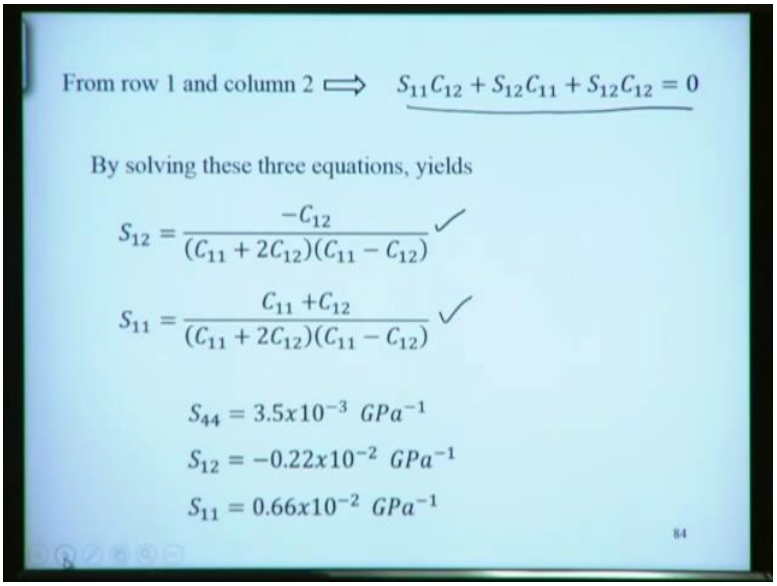

Next equation from row 1 and column 2, we can find out this equation equal to 0. Now by solving this 3 equations straightforward we can find out the relation between S 12 and S 11 in terms of C. And once we (if) evaluate S 12 and S 4 and S4 can also be directly correlated that S 44 equal to 1 by C 44. So from this expression we can find out this relation between S, S terms and the C terms and putting all these values we can find out this value. So this can, values can also be compared with the tabulated values are available in the standard literature. So, these are the way to find out the, either S or C when one is given or not, others are not available.

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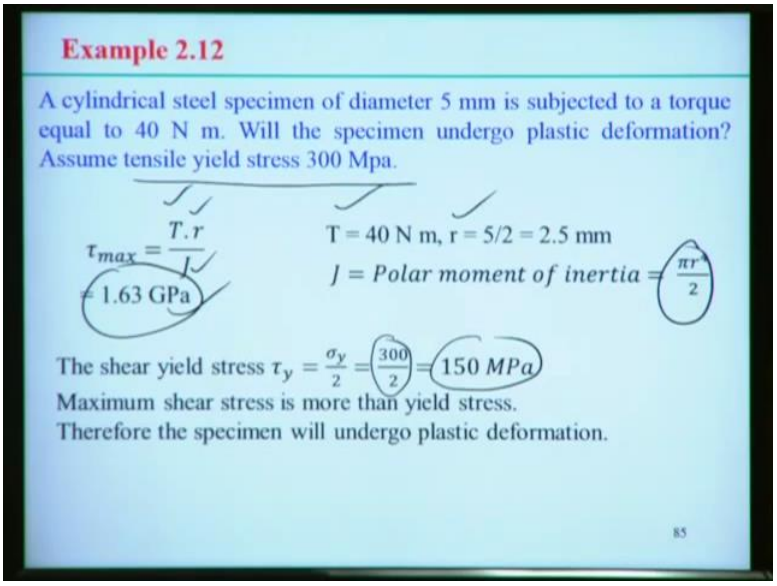

So these are the simple (pro) elasticity problem, so maybe it is necessary to mention that, a cylindrical steel specimen of diameter 5 millimetre is subject to a torque equal to 40 newton metre, when the specimen undergo plastic deformation? Assume tensile yield stress is 300. So here tensile yield stress is given but we need to (s) specify whether is there any plastic deformation or not.

So, we know that when it is subjected to a torque then maximum shear stress can also be obtained, torque and the radial distance or the optimum outer surface distance and J equal to polar moment of inertia which is equal to buy pi r to the power 4 by 2, so r is actually the radial, radius. So, T is given r is given and J is also multiply and then we can find out the, in this case, the maximum shear stress is obtained as 1 point 63 Giga Pascal, putting the numerical values.

But shear yield stress can also be said that it, in the multiplying factor 2 here so that shear yield stress the half of the what was the tensile yield stress. So tensile yield stress is given, from there we can find out the, what is the shear yield stress value. Now we can compare with respect to the maximum shear stress. So, maximum shear stress 1 point 63 Giga Pascal is much more than the 150 mega Pascal.

So, we can say that therefore the specimen will go under plastic deformation. So, here the message is that we need to know when we are comparing the (plast) plastic deformation, if the shear yield stress is given or the tensile yield stress is given accordingly we have to compare with the results and then we can take the decision whether there is any plastic deformation or not.

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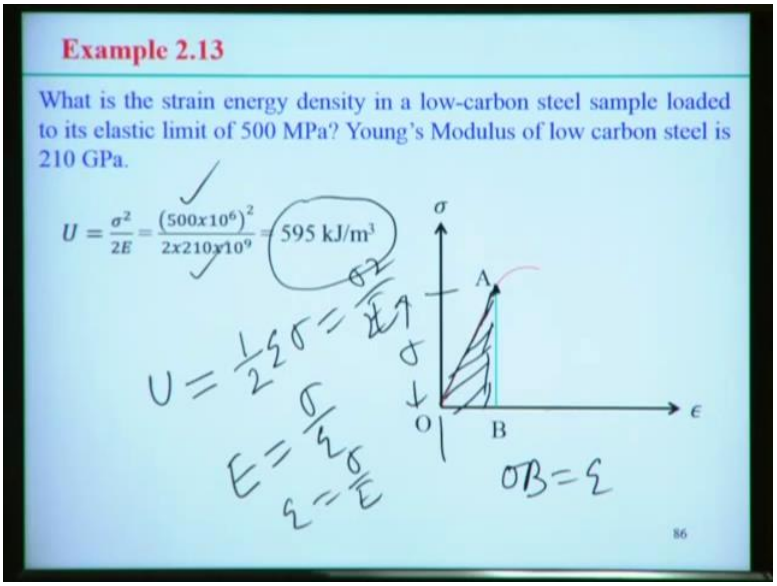

Now we will look into another problem; so in this case we need to find out the strain energy in a low-carbon steel sample loaded to its elastic limit of 500 mega Pascal; Young's modulus of low carbon steel is (2 hun) 210 Giga Pascal. So here we need to find out the strain energy when the limit is half to 500 mega Pascal. So within the elastic limit, normally in this case the (wu) (of) a low carbon steel within the elastic limit up to O to A, this is the suppose A is the elastic limit here and O to A follow the linear relation.

And in this case, that strain energy density, so if we physically estimate so (up) upto this, this is the OB is actually amount of the elastic strain, so that area actually represents the energy per unit volume. So that area physically represent energy per unit volume if we use (at it) so that actually this triangle, so area of the triangle suppose OB equal to Epsilon and OA equal to shear stress. So, half of strain into Sigma, that is the amount of energy per unit volume.

Now, Young's modulus within the elastic limit, Young's modulus equal to stress by strain, so in terms of strain equal to Sigma by E. So if we put it here, Sigma by E, so it becomes Sigma Square by twice E. Now, the Sigma Square is the elastic limit 500, so elastic limit at this point, stress value at this point is given, Sigma, 500 mega Pascal and the Young's modulus is also given then we can find out the kilojoule per metre cube; this is the amount of the energy within the elastic limit.

So thanks for your kind attention. So hopefully you have understand able to understand the theory as well as practical application of this theory through solving of the numerical and the

different kind of problems, and if some; hopefully (you) you will be able to get some idea from the analysis of the theory as well as the some numerical problems. Okay, thank you, thank you very much.