Introduction to Crystal Elasticity and Crystal Plasticity
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Week-02
Lecture-03
Elasticity

Good morning everybody. Let us start today's topic on elasticity crystal (struc differ) different crystal structures. So, so far we have discussed the different type of structure of materials, for example we explained in terms of BCC, FCC, HCP or different, and very basic units of the materials.

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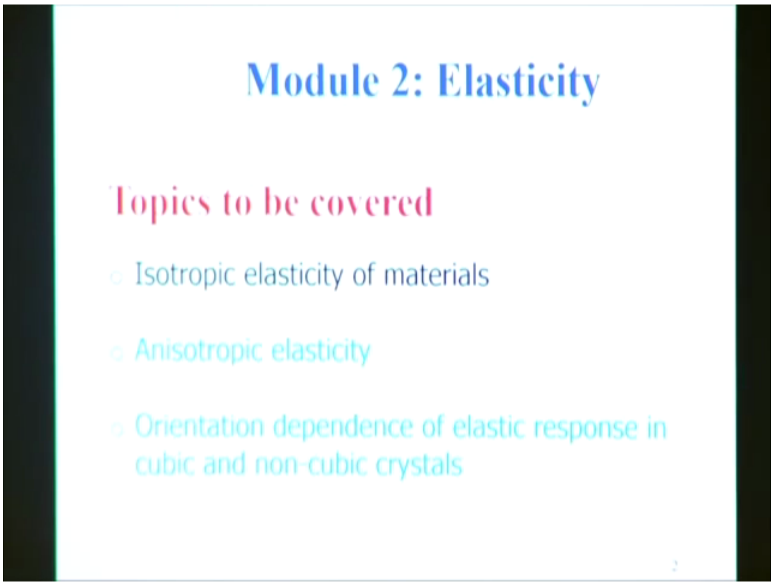

So, now today we will try to discuss this next topic that is called elasticity part of different crystals. So, first I will try to focus on isotropic elasticity of materials and then gradually will shift to anisotropic elasticity of materials and how the orientation dependence of elastic response is generally observed in case of cubic and non-cubic crystals.

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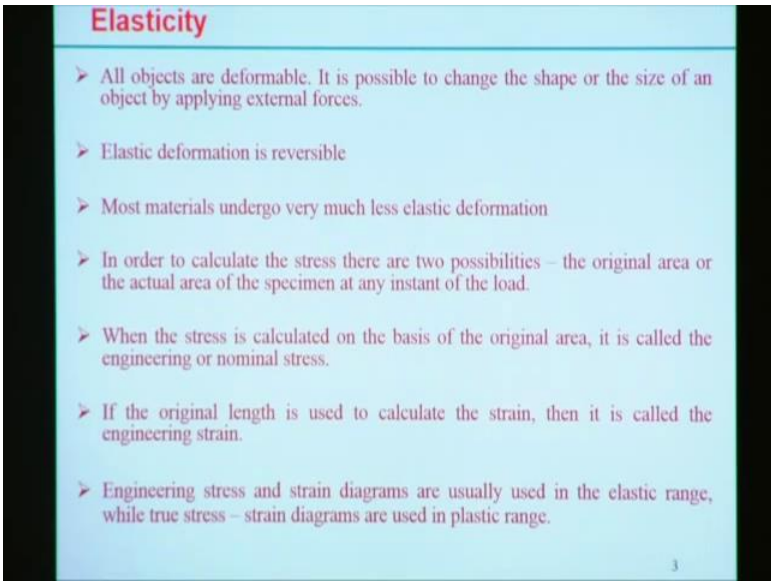

So, from the basic part start with the elasticity. So, it is known that all the objects are a deformable when it is subjected to some kind of load but at the same time it is possible to change their shape or size due to the application of the external load but how we can change and (size) size and try to explain in the mathematical form, then we need to define several parameters to analyse the deformation mechanisms of solids.

So first in terms of elasticity will try to explain the deformation behaviour, so elasticity is known that it is a reversible process that means if we apply the load and subsequently released the load it will come back to its initial position without any permanent deformation. So, most of the materials practically undergoes very small amount of the elastic deformation but (el) can go up to large amount of permanent deformation with the application of the mechanical load here.

So there are 2 possibilities to analyse the elasticity, first if we analyse the load with respect to the original area or the actual area at the instant time of load. But when the stress is calculated on the basis of original area it is called engineering stress or nominal stress or (())(2:56) similarly. When it is based on the original length to calculate the strain then it can be called as engineering strain. Of course engineering stress and strain diagrams is specifically useful within the elastic range but the true stress strain diagram is specifically significant or important in plastic range. Although there is a (())(3:23) for small deformation case engineering stress strain and or true stress strain, there is a very small difference is there within the elastic limit.

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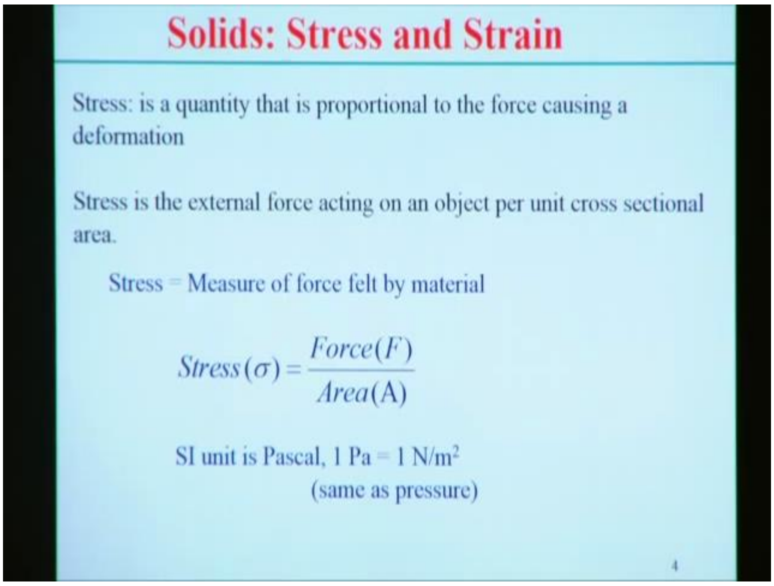

First we define the stress and strain over a solid body. Stress actually quantify, which is the proportional to the force causing deformation to that specific solid. Now stress is actually the external force acting of the object per unit cross-sectional area. So we can measure the stress like that; it is force divided by area that means force per unit area. So unit of stress is in SI it is Pascal or Newton per meter square which is same as the unit of pressure.

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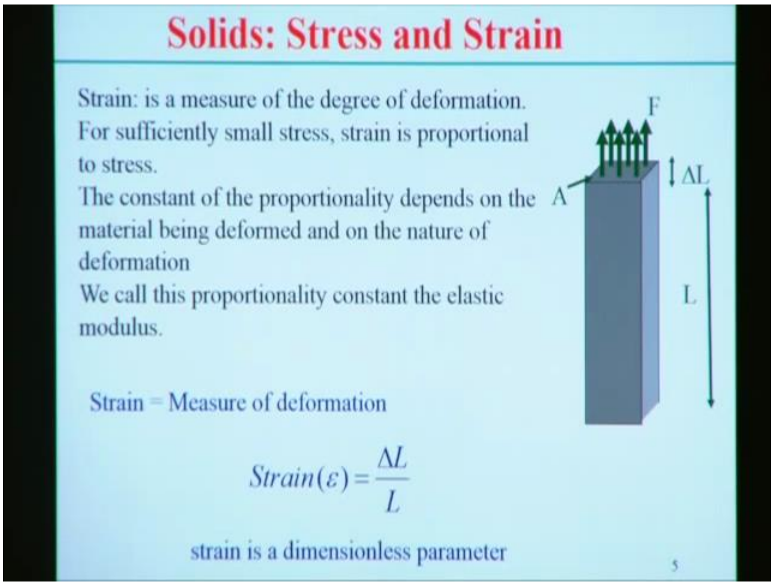

Similarly strain is a measure of degree of deformation with the application of the load. But for sufficiently small amount of the stress, strain is proportional to the stress. (The) the constant term of the proportionality actually depends on the materials being deformed and the nature of deformation. We call this proportionality constants as elastic modulus. If we look into the right-hand side figure we see the L is the original length, and with the application of the load F, it deformed to final length L plus delta L. So, specifically increment of the length with the specific direction, that means parallel to the force vector here, is the delta L.

Now here, stress is the can be defined the load over the area, so that is a stress and strain can be defined here, what is the increment of the length as compared to the original length. So of course in this case we can define stress as a engineering stress or strain as a engineering strain. The dimension of the strain, it is same that having no dimension of strain.

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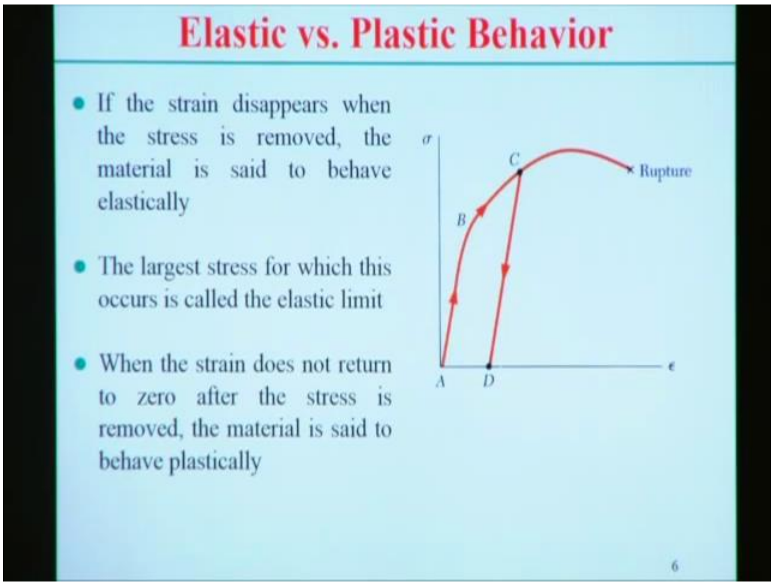

Now, if there is a application of the load in 1 specific direction that is uniaxial direction, then the figure shows the typical stress strain diagram where the initial state from A to B the chord actually follow a linear path but beyond B the diagram becomes non-linear and upto certain point it breaks, that point is called the rupture or the fracture happens at (the) at that point.

Now, strains will disappear if we remove the load that point B and it will come back to the initial position A. But at the same time if deformation happens up to the point C and then afterwards if we remove the load it comes back to the position D. So, there exists some permanent deformation AD. That (s) that is called the material undergoes plastic deformation so that deformation is not recoverable.

So in this case, that is (por up) deformation at point C is called as plastic deformation but deformation at, up to point B can be considered as a elastic deformation. So, the largest stress for which this occurs is called as the elastic limit but when the strain does not return to the 0 after the stress is removed, the material is set to behave plastically. So there is a clear distinction between elastic part as well as plastic deformation.

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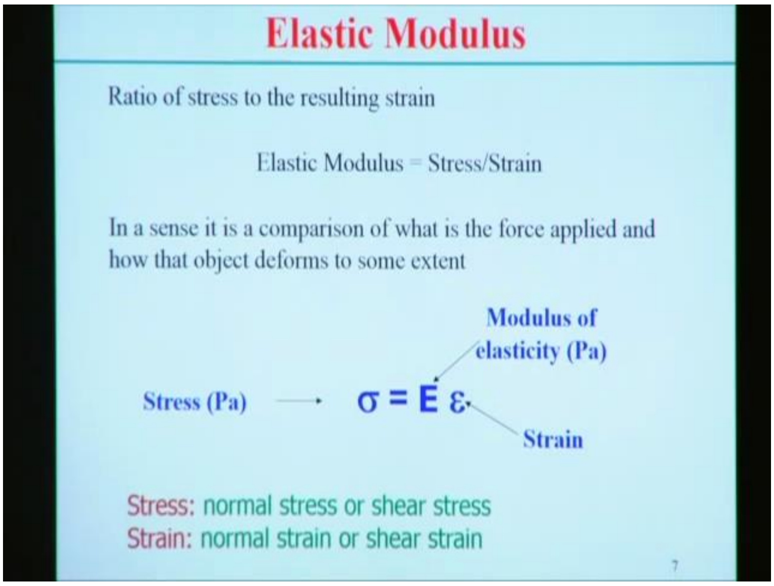

So, within the elastic limit the ratio of the stress and strain is equal to the constant term, that constant term is called as elastic modulus. In the sense that it is a comparison of what force is applied and how the object deforms upto certain extent. So if we look into that expression, the stress is equal to elastic modulus and multiplied by strain. So, since strain is dimensionless, so equation of stress is equal to the here equation of unit of modulus of elasticity.

So both the modulus of elasticity and stress, having the same unit but when we produce the stress, this stress is whether it is normal stress or shear stress and the strain, whether it is normal strain or shear strain accordingly we can define different elastic modulus and that is limited to, within the elastic limit.

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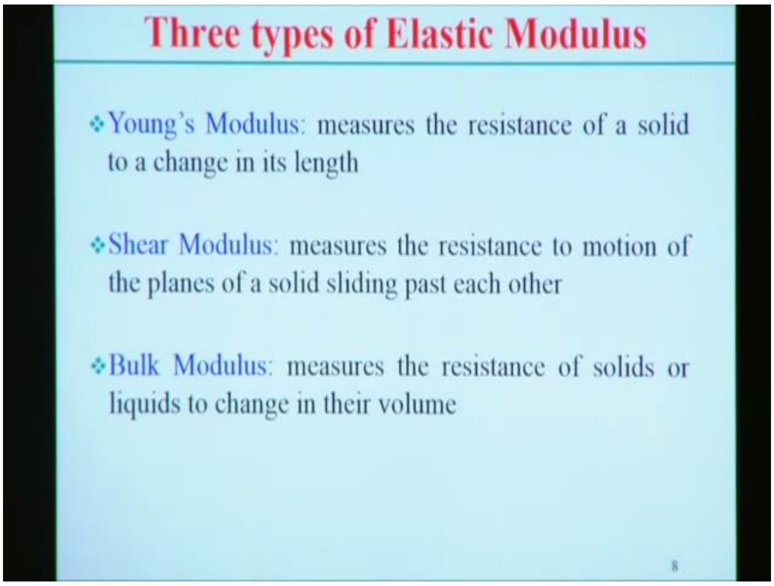

So, 3 different types of elastic modulus is generally we observe in the solid mechanics approach when a material is deformed with the application of some external load. So, 1 is the Young's modulus. Actually it measures resistance of a solid to a change in its length. So, that application of the load here may be either tensile or compressive but in this case the load acts normal to the cross-sectional area.

Shear modulus its measures resistance to motion of the plane of a solid past over (s up) solid sliding past each other. So that will graphically, I will try to explain what is shear modulus. So in this case the load actually applies which is parallel to the cross sectional area. Third 1 is the bulk modulus; this actually measures the resistance of solids or liquids to change in their volume.

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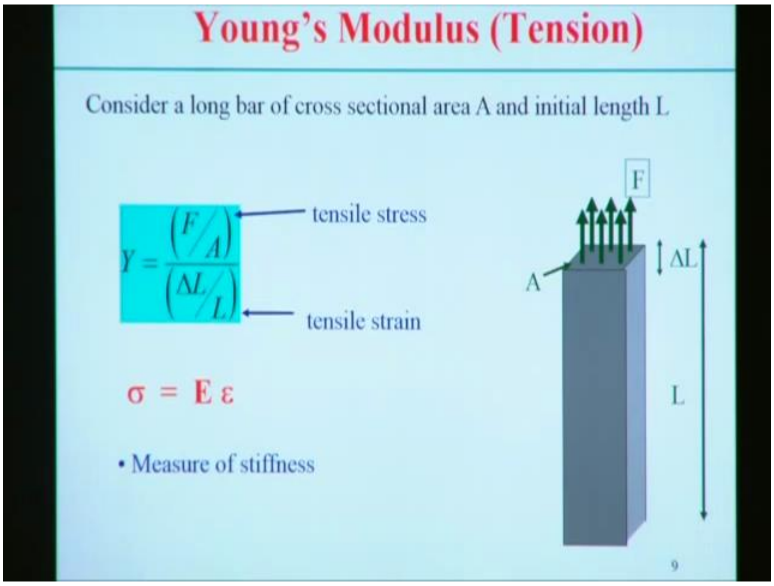

So, these 3 elastic modulus, we can explain further that if we look into the figure, the (el) application of the force at cross-sectional area, that ratio actually define the stress and if tensile loading is acting here so it can be considered as a tensile stress. But, the area of cross-section is normal to the force direction. At the same time there is a deformation along the direction of the force that is delta L and that increment of the deformation with respect to the initial length can be considered as a tensile strain.

So this tensile stress and tensile strain ratio is considered as a Young's modulus. Stress can also act in such a way that it may be the compressive load also. So in this case the Sigma equal to E into Epsilon and E specifically considered as a Young's modulus and this actually indicates the measure of the stiffness of a solid body.

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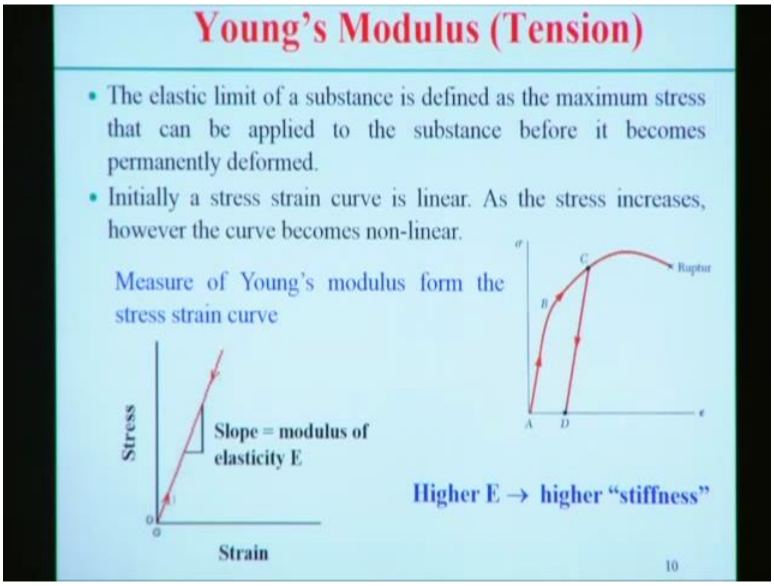

Now, the, if we look into that diagram, that elastic limit of the substance actually defined the maximum stress can be applied to the substance before it permanently deformed, that we have already explained. And if we take as a reference the right-hand side figure, here we can see the stress strain diagram of a specific material but in this in this case how we can measure the Young's modulus and the stress strain curve. Since we are explained that stress is proportional to the strain to the elastic limit and that constant of proportionality we affiliate with the Young's modulus. So the Young's modulus is actually represented by the term of ratio stress by strain but this (ratio) this relation of stress strain initial period is normally follow the linear relation.

So, up that line AB is a straight line here and the slope of the (straight line) straight line is physically represents the Young's modulus. We can find out in that way that, if we consider the angle theta between the (line) straight line and the strain axis, so tan theta represents the slope, and the tan theta equal to stress by strain but that definitely within elastic limit and that ratio actually represents the Young's modulus here. So higher (E) Young's modulus (meet) means there exist higher stiffness for a specific structure of a solid.

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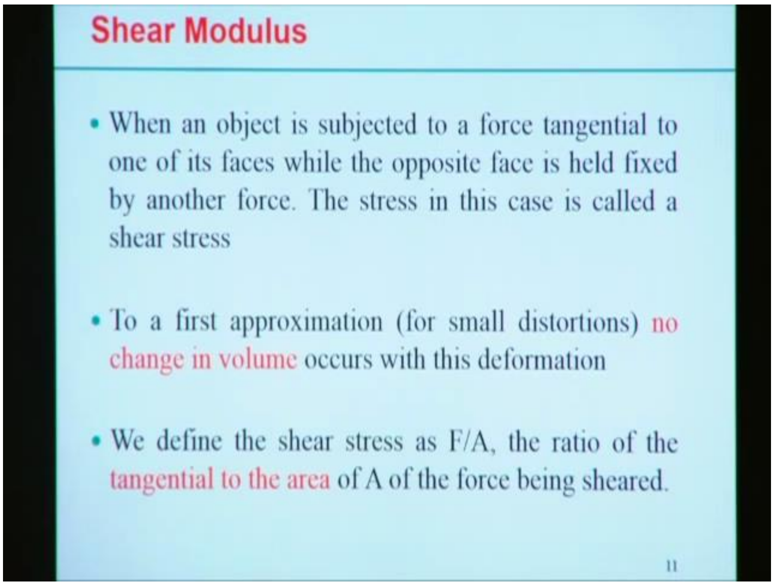

Now, next is the shear modulus. Shear modulus we can define in such a way that when an object is subjected to a force tangential to 1 of its faces while we keep (co) as a constant in the opposite face, and then the stress in this case is called a shear stress. Definitely in this case application of the load is parallel to the cross-sectional area here. And to a first approximation and specifically for small (di) distortion or small deformation in case no change in volume occurs with this deformation. We define the shear stress as a ratio similar to normal stress area sorry force divided by area, but this area is the tangential to the area of A of the force being acting.

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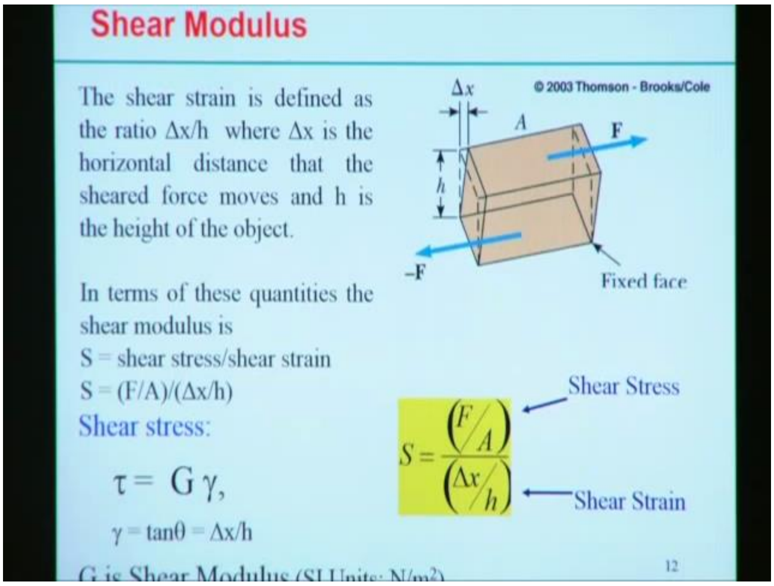

Now, if we look into right-hand side figure, if we try to find out that there is application of the force on the top of the element and bottom face is (keeped) kept as fixed. Now with the application of the load A on the top surface its deformation longer direction of the load is delta x and that deformation happens over the reference of height h. Now in this case the shear strain on be defined as delta x divided by h where delta x is the horizontal distance, that is the shear force moves and h is the height of the object.

Therefore when we try to find out the share modulus its is basically the ratio of the shear stress and shear strain. So, here the shear stress is defined as the force F divided by a cross-sectional area A but it is noteworthy that this cross-sectional area is actually acting parallel to the application of the force.

So in terms of stress we can find out that S is F by A that is shear stress and the ratio of shear strain that is delta x by h. So shear stress is generally represented by Tao that is equal to G into gamma. So here, gamma equal to (t) tan theta which is equal to delta x by h here and that is the measure of the shear strain in this case, and G is considered as a shear modulus in this case having the unit of newton per meter squared or Pascal similar to pressure.

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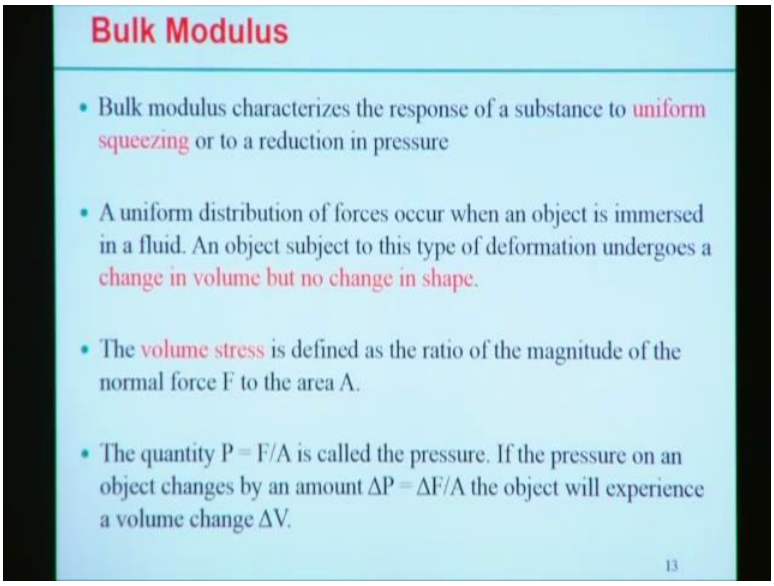

Now, the third modulus that is called bulk modulus which is characterised with the response of a substance when it is subjected to uniform squeezing or is there any reduction in the application of pressure and that actually acts over the volume of an element. So a uniform distribution of forces occurs (c) specifically when the object is immersed in a fluid. And the object is subjected to this type of deformation that is undergoes by the change of volume without any change of shape.

So this volumetric (str) stress which is defined as the magnitude of the normal force to that area A. And that quantity P, F by A, here it is termed as pressure. And if the pressure changes by an amount of delta P which is equal to the change of force by the area the object will experience a change of volume delta V.

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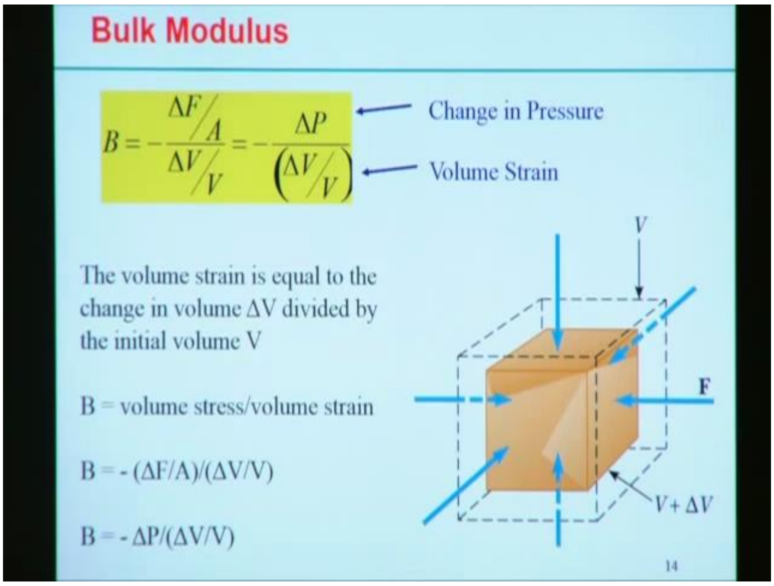

So that delta V with respect to the original V that is called volume strain or volumetric strain and here the bulk modulus is defined by the ratio of change of pressure and the volumetric strain. So the difference from the other 2 modulus with respect to the bulk modulus is that in case of bulk modulus, we representing the stress and strain term acting over the volume.

So B, if we generally define the bulk modulus and that is specific to the solid, and that is the ratio of the volume stress and the volume strain. But this volume stress actually comes from the change in pressure and volumetric strains, actually change of volume with the application of this force F without any change of the shape of the object.

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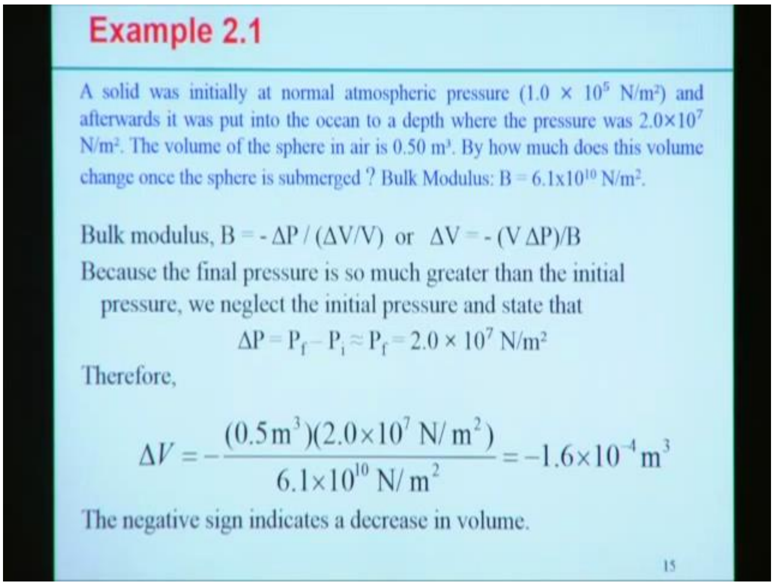

Let us look into 1 example to understand the different modulus or specifically the bulk modulus in this case. A solid was initially at normal atmospheric pressure that is 1 into 10 to the power 5 Newton per meter squared and afterwards it was put into the ocean to a depth but the pressure was 2 into 10 to the power 7 Newton per meter squared. The volume of the sphere in air is point 5 metre cube but now, we need to estimate the how much does the volume change once the sphere is submerged into the water from the atmosphere. So, bulk modulus also defined here so it is noteworthy that all the units of the modulus which is the units (is) equal to the stress, units of stress.

Now bulk modulus, if we directly apply is (del) change of pressure and delta V by V, so here the sign convention is considered as a negative depending upon the application of the pressure and whether it is expanding or whether it is squeezing. So, change of volume can be represented as the original volume into change of pressure divided by the bulk modulus. So, in this case change of pressure can also be calculate if the gauge pressure is known at a specific depth where the pressure was 2 into 10 to the power 7 Newton per meter squared.

So, in this case the difference between the final and the initial pressure is actually the 2 into 10 to the power 7 Newton per meter squared. So the change of volume, if we put the numerical values of all the parameters and we found out change of volume is calculated as 1 point 6 into 10 to the power minus 4 metre cube. this negative sign actually indicates there is a decrease in volume with the application of the pressure.

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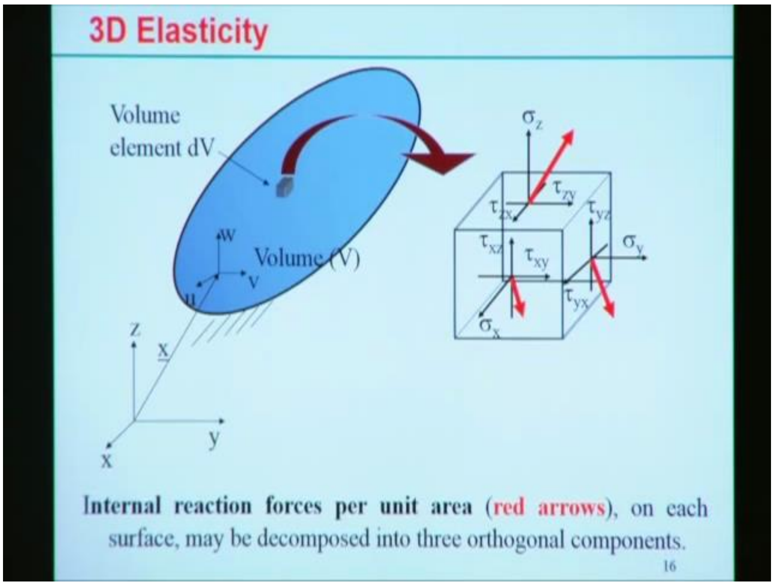

Now we come to that 3-dimensional state of the stress and of course all these cases the analysis is limited to within the elastic limit. So, so far we have discussed the 1-dimensional or 1 directional set of the stress, now we try to implement the concept of 1 directional state of the stress for 3-dimensional case. Now if we look into that figure, left-hand side figure (yeah) material volume if we consider and on that material volume if we try to find out what the element of stresses can be act on this volume of the material.

Let us look into that, there is an arbitrary forces basically acting on an element but this component of the forces can also be decomposed into the 3 orthogonal component and on this 3 orthogonal component or maybe we can say the in the Cartesian coordinate system we can define the general state of the stress. Let us look into that elemental volume here and what are the different stresses are acting on this case.

First they are the stresses, normal stress is Sigma Y, probably it is acting along the Y direction, Sigma X is the acting on X direction and Sigma Z. These are the normal stresses which acting. Now on, we see that there are several shear stress components as well and we, we if we see there are the shear stress components are Tao YX or Tao XY, Tao YZ or Tao ZY, Tao XZ or Tao ZX. I mean there are 6 shear stress components; all are (ac) acting over the surface of this elemental volume. But any state of the force condition can also be represented within the elemental volume in terms of this 9 components of the stress that means 6 are the normal stress components and sorry, 3 are the normal stress components and 6 are the shear stress components.

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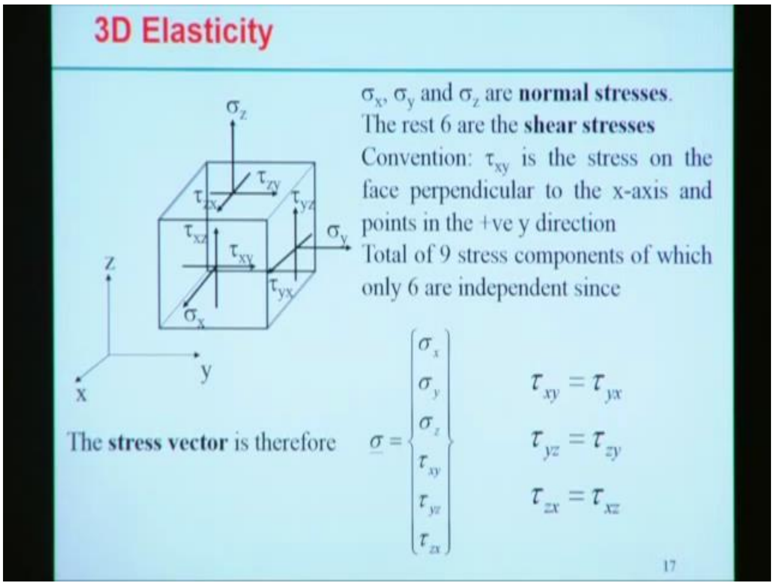

Here see that Sigma X Sigma Y and Sigma Z are the normal stresses, but the rest of the 6 are the shear stresses. It is a convention is like that; Tao XY is the stress on the face perpendicular to the axis at points in the positive Y direction. So with this Convention we can define the different shear stress components. But there are 9 stress components out of which only 6 are independent since the shear stress component Tao XY is equal to Tao YX, Tao YZ is equal to Tao ZY, and Tao ZX equal to Tao XZ, so therefore the stress vectors can be represented at the 6 component that is the Sigma X, Sigma Y, Sigma Z and 3 shear stress components, Tao XY, Tao YZ and Tao ZX.

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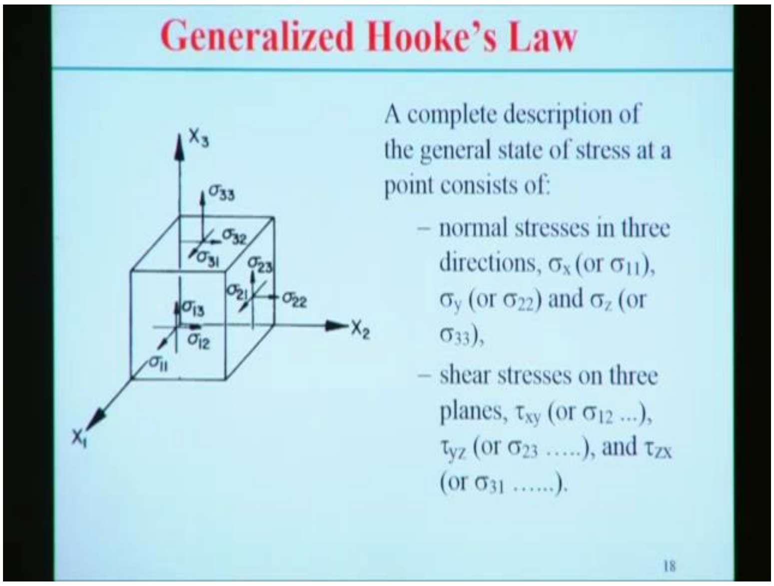

This is the another way to represents the 3-dimensional state of the stress and sometimes we define the axis as X1, X2, X3 and the sheer or normal stress components can be represented like that and here is Sigma X which is equal to Sigma 11, Sigma Y which may be able to Sigma 22, and Sigma Z which is equal to Sigma 33. Similarly the shear stress component Tao XY can be represented as Sigma 12, Tao YZ, Sigma 23 and Tao ZX (or if) which is equal to Sigma 31. So throughout our analysis we can use both of the notation, either Sigma X, or either Sigma 1 in that way, we can use to, for that analysis of 3 dimensional state of the stress.

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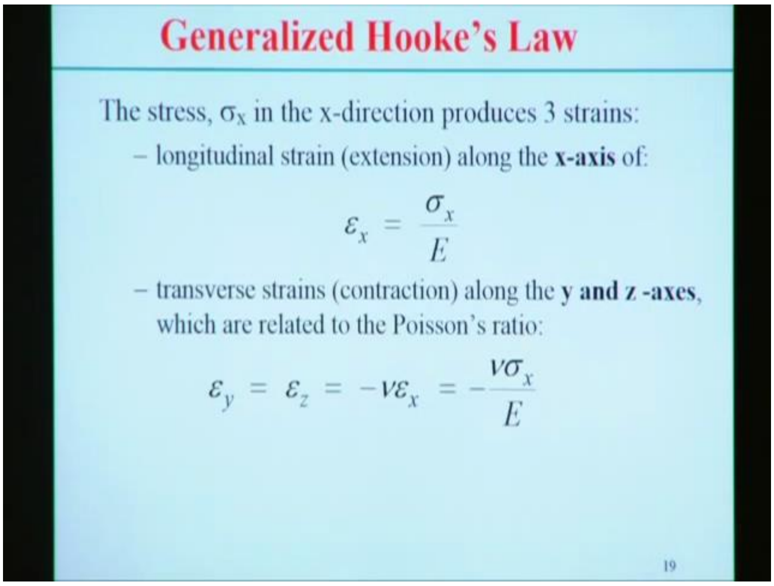

Now, the Hooke's Law states that stress is proportional to the strain, but how we can apply in case of 3-dimensional stress state, this Hooke's Law. The stress, suppose, the stress Sigma X is acting on X direction and that actually produces 3 strains; 1 is the longitudinal strain or maybe you can say that extension along the X axis, that can be defined as Epsilon X equal to Sigma X by E. But at the same time it will produce the transverse strains or maybe contraction along the Y and Z axis which are related to the Poisson's ratio.

Now in case of 3-dimensional stress state, if 1 stress is acting 1 specific direction, (let) and if there is a extension along this direction, so there must be contraction in other 2 directions to make the volume consistency over the deformation. That is why the Poisson's ratio actually comes into the picture to consider the latter contraction in there is a application of the load in 1 specific direction.

So here is (Sigma Y) Epsilon Y and Epsilon Z can be represented as the negative of the Poisson's ratio and (mult) multiply with the strain in X direction so that is is equal to that Poisson's ratio into Sigma X by E. So the negative sign comes because of the contraction in other direction as compared to the extension along in X direction.

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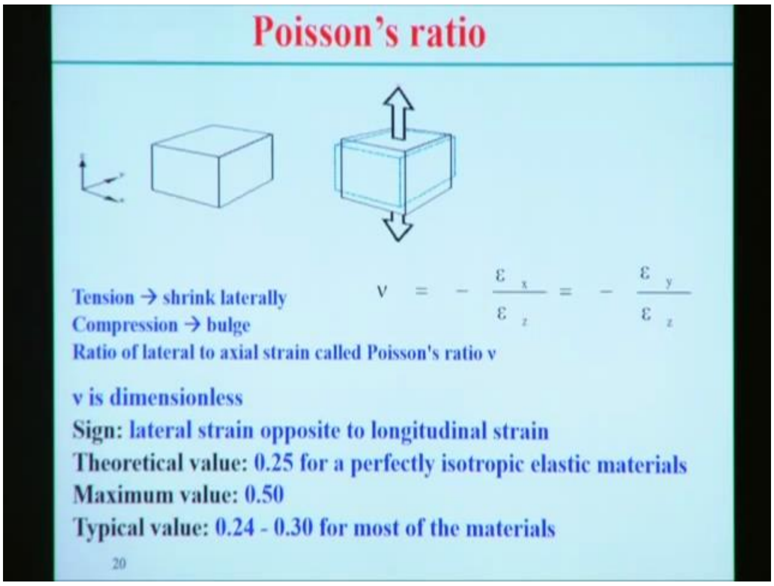

So here, how we can define the Poisson's ratio in this case? The first (())(30:47) indicates the undeformed state but if we apply the load then it will try to deform in 1 direction or maybe extension in 1 direction but other 2 directions probably it can go through the contraction. So, the Poisson's ratio is defined by the lateral strain to the axial strain and that is negative of Epsilon X by Epsilon Z or negative of Epsilon Y and Epsilon Z.

So, it is obvious that the Poisson's ratio is dimensionless at the lateral (ss) sign of this Poisson's ratio is that lateral strain opposite to the longitudinal strain. But theoretical value of Poisson's ratio is point 25 or perfectly isotropic elastic materials, but maximum limit is half or point 5 but typical values for most of the metals is observed between point 24 to point 3.

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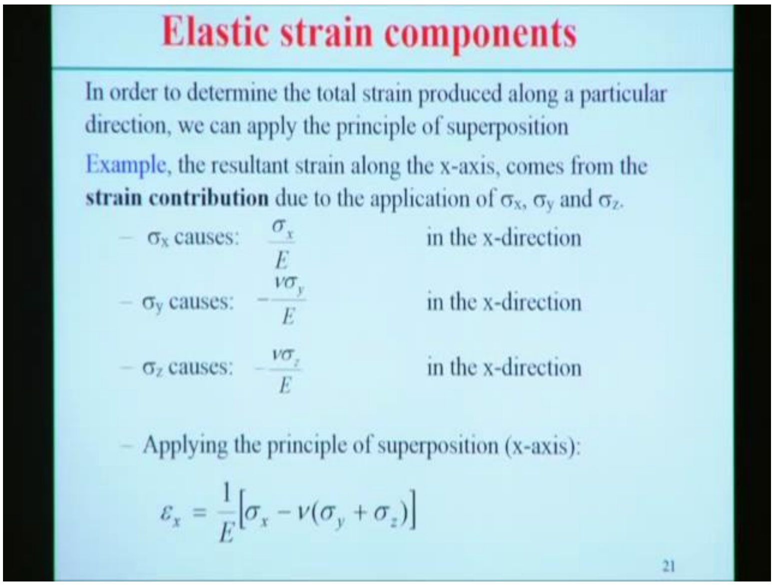

So by looking into the Poisson's ratio we will try to explain the component of the stress or strain at different directions. So in order to determine the total strain, 1 specific direction we can apply the principle of superposition. Let us look into that (ss) strain along the X axis and the (con), it comes from the contribution with application of the Sigma X, Sigma Y and Sigma Z; that means other 2 directions. So Sigma X causes actually Sigma X by E in the X direction but Sigma Y causes component of the strain in X direction and that is multiplied by the Poisson's ratio into Sigma Y by E.

Similarly the strain component along X direction due to the application of the stress Sigma Z is considered as minus Nu Sigma Z by E. So by applying the principle of superposition along the X axis, the total strain component is calculated as, 1 by E Sigma X minus Nu into Sigma Y plus Sigma Z. In this case there is a positive deformation (or pos) happens due to the application of the Sigma X while negative deformation happens due to the application of Sigma Y and Sigma Z. So effective deformation is the Epsilon X, that is the, consist of both the longitudinal strain and the other 2 lateral strain and that lateral strain actually represented (in terms) with the effect of the Poisson's ratio.

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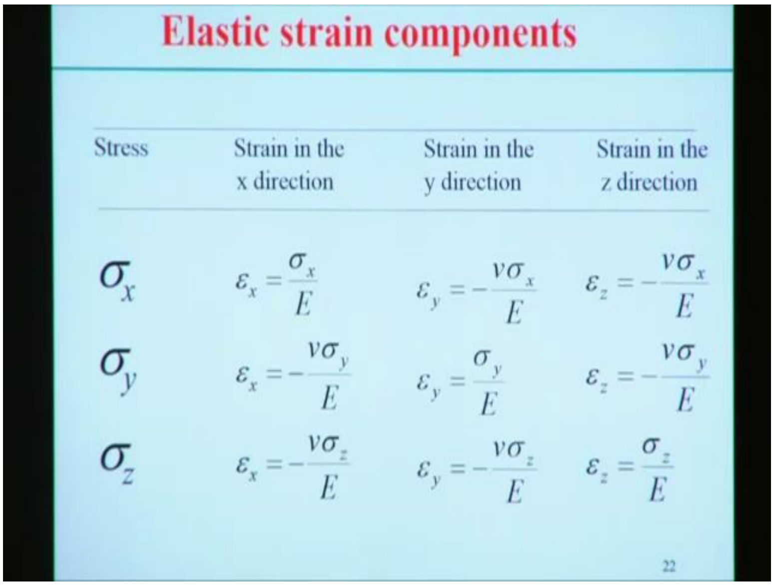

Now, if we look into that tabulated form, the stress is acting, Sigma X, Sigma Y and Sigma Z but what are the component of the strain is acting X direction, what are the component of the strain acting in Y direction and component of the strain in Z direction. So if we see that component of the strain in X direction, Epsilon X is equal to Sigma X by E, Epsilon X due to the Sigma Y, that is the multiply by the Poisson's ratio with the (st) strain to Sigma Y and similar effect can also be observed into the Sigma Z. So these are the way out to find out the individual strain component acting in different direction.

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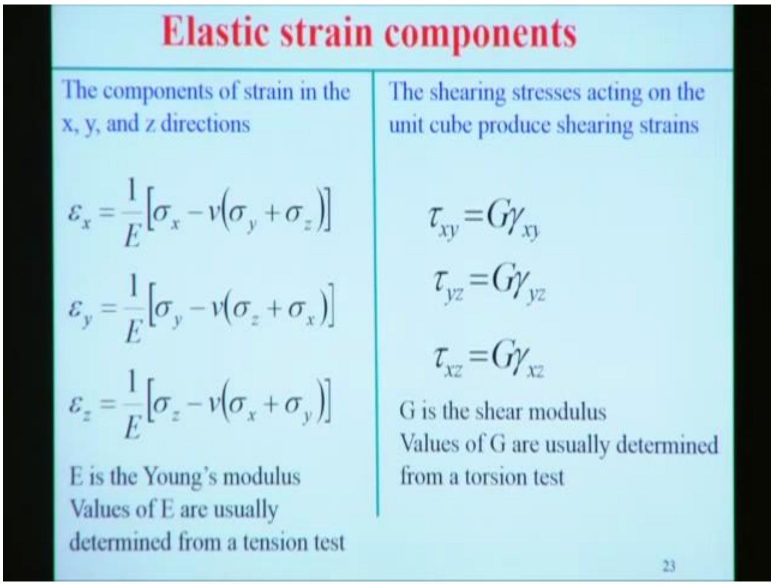

So when you apply the principle of superposition (of) of the effect of individual component we can find out the component of the strain in X,Y and Z direction are represented like that; Epsilon X equal to 1 by E Sigma X minus Nu into Sigma Y plus Sigma Z; similarly Epsilon Y and (s) Epsilon Z. But if we look into all this expression the positive deformation or positive amount of strain actually comes from their individual stress component along that specific direction. So we are talking about the positive in the sense that Sigma X, Sigma Y or Sigma Z all are positive.

But at the same time the negative contribution actually comes from the other 2 components of the stress when we focus on 1 specific axial direction. So E here is the Young's modulus but values of the Young's modulus can also be obtained from the uniaxial tension test and that Young's modulus actually represents the slope of the very initial curve or maybe we can say the initial linear part of the stress strain curve is a measure of the Young's modulus.

The shear stress also acting on the unit cube and that is, shear stress is proportional to the shear strain and that proportionality is represented in terms of the shear modulus here. So similarly all the shear strain component (can be) can also be represented individually with respect to the (s) shear strain along the specific plane. So here G is the shear modulus and values of the G can also be obtained from a torsion test as compared to E in case of tensile test.

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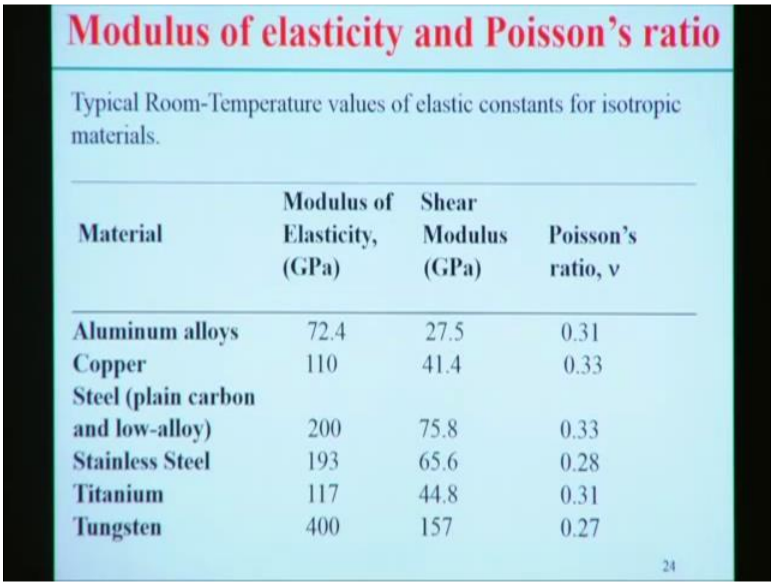
Modulus of elasticity and Poisson's ratio

Typical Room-Temperature values of elastic constants for isotropic materials.

Material	Modulus of Elasticity, (GPa)	Shear Modulus (GPa)	Poisson's ratio, ν
Aluminum alloys	72.4	27.5	0.31
Copper	110	41.4	0.33
Steel (plain carbon and low-alloy)	200	75.8	0.33
Stainless Steel	193	65.6	0.28
Titanium	117	44.8	0.31
Tungsten	400	157	0.27

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These are the some basic idea about the specific values of the elastic constants for isotropic materials. And if we see that modulus of elasticity, shear modulus and Poisson's ratio are defined for the different materials and out of these materials, tungsten is having very high modulus of elasticity. That means the stiffness is very high in case of tungsten, other way we can say the slope is very high on the stress strain diagram in case of tungsten as compared to the other materials.

And similarly if we see, roughly observe the Poisson's ratio is the maximum point 33 and minimum is point 27 for this materials. And of course the aluminium alloy is having low amount of the modulus of elasticity (())(38:34). Yet stiffness is specifically less as compared to the other materials.

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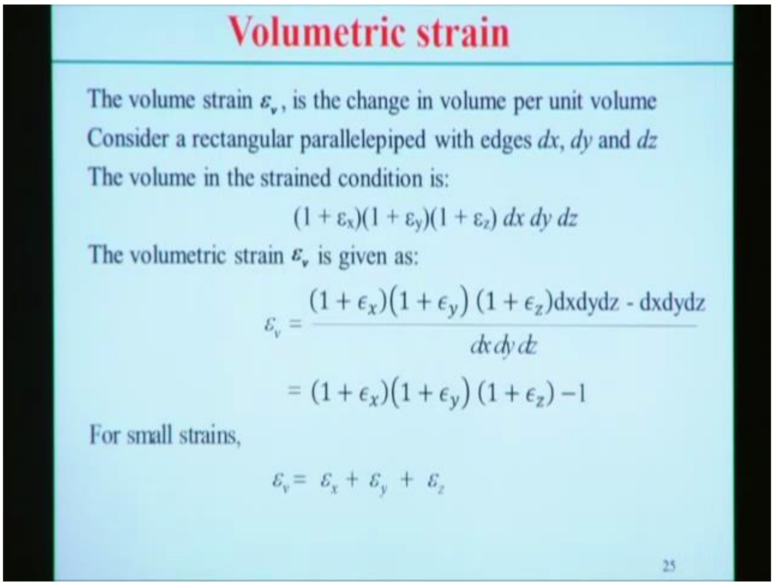

Now if we try to estimate the volumetric strain which is the changing of the volume or unit volume and that can be calculated as simply considering 1 rectangular parallelepiped with edges dx, dy and dz. Therefore if we consider the engineering strain here and that is the engineering strain is a difference between the final length specifically I am talking about when we focused on 1 specific direction so that is what was the final length minus what was the initial length divided by the original length or initial length; that actually measured the engineering strain.

So, that total volume can be calculated as 1 plus Epsilon X into 1 plus Epsilon Y into 1 plus Epsilon Z multiplied by the initial volume dx, dy, and dz. And now overall volumetric strain due to the change of the volume can also be calculate like that, that (f) ev equal to first is the final volume minus initial volume divided by the initial volume or original volume and it can be calculated is like that; 1 plus Epsilon X into 1 plus Epsilon Y into 1 plus Epsilon Z minus 1.

So if we consider that, neglect the higher-order term specifically (ep) multiply Epsilon X into Epsilon Y and into Epsilon Z. So that quantity is very small (sp) in case of small strain so we can approximate that volumetric strain is the linear sum of all the individual strain components; that means ev equal to Epsilon X plus Epsilon Y plus Epsilon Z. So this volumetric strain is actually varied when there exists small amount of the strains.

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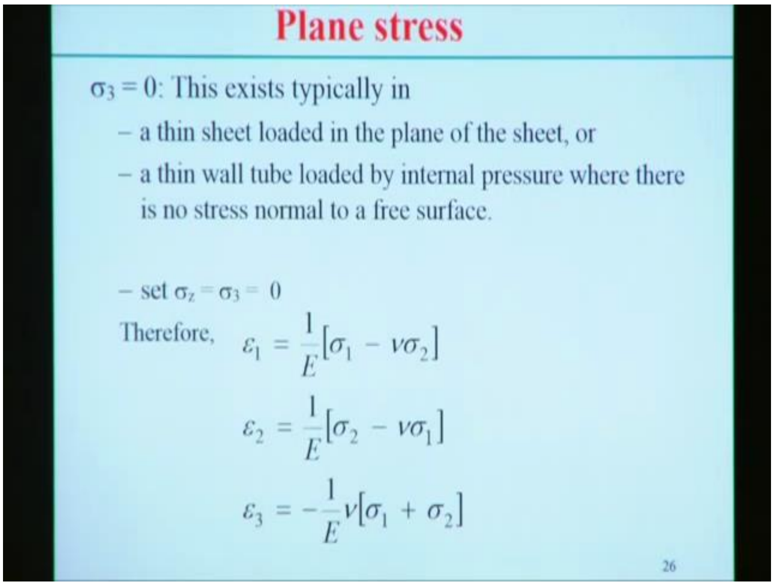

Now if we look into very specific cases; the 3-dimensional state of the stress 1 specific case is the plane stress condition. In this case typically exists when Sigma 3 equal to 0 and practically this happens in case of the things she and when it is loaded in a plane of the sheet or a thin wall tube which is loaded by internal pressure there is no stress on normal to the free surface.

So these are the typical conditions of the plane stress where Sigma 3 equal to 0 that means third directional stress is 0 here. And if we set either Sigma Z or Sigma 3 equal to 0, then we can find out the 3 component of the strain; 1 by E into Sigma 1 minus Nu into Sigma 2, here the Sigma 3 equal to 0. So we find out that expression of the 3 component of the strain but there exist 2 component of the stress in case of plane stress condition.

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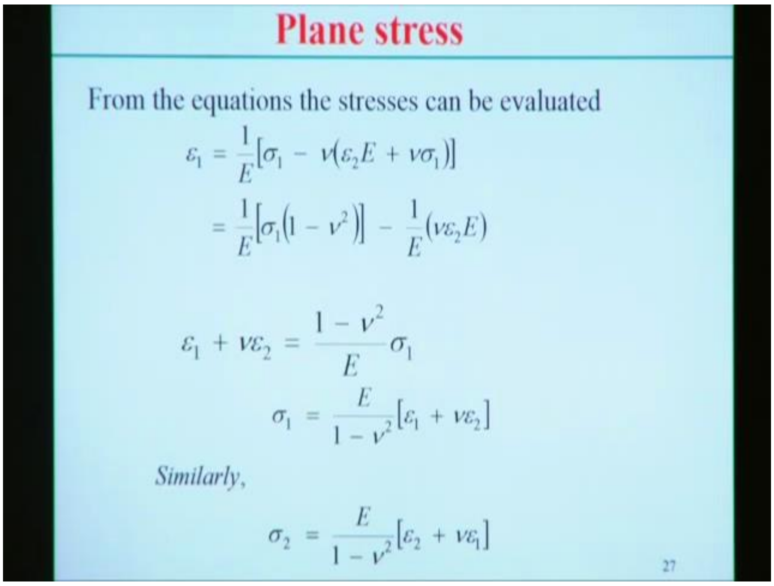

Now, it is possible further to rearrange this equation and it is possible to find out the value of the 2 stress is here in terms of the strain component. So, here we see that Sigma 1 and Sigma 2 represented in terms of the material properties that means Young's modulus and Poisson's ratio as well as strain components. However third strain component can also be represents by adding the other 2 stresses.

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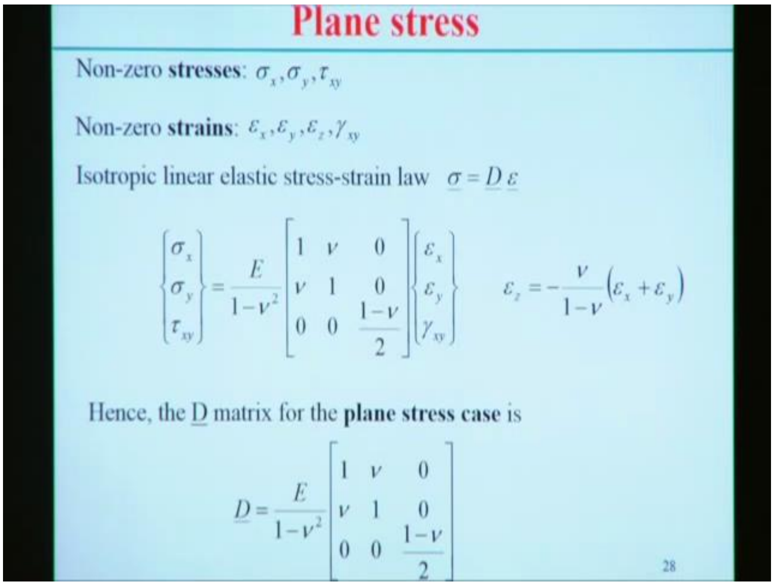

So in plane stress condition, in say we can say that non-0 stresses exist Sigma X Sigma Y and shear stress is only Tao XY and non-0 strains, Epsilon X, Epsilon Y, Epsilon Z and gamma XY. So if we see there are 3 component of the non-0 stresses and there are 4 component of the non-0 strain in case of plane stress conditions. So looking into that expression or relation in terms of the elastic modulus and we rearrange this equations we can find out in the matrix from like this Sigma equal to D into Epsilon; so stress equal to D into Epsilon.

So D actually related to the properties of the materials, that means D actually is a function of the material properties like Young's modulus and Poisson's ratio here. If we see the (())(44:28) vector Sigma, Sigma X, Sigma Y and Tao XY, stress component, the D matrix and then finally, Epsilon X, Epsilon Y and gamma XY. So this is the relation between the stress and strain but individually the Epsilon Z can also be calculated in terms of Epsilon X and Epsilon Y. So in this case, D matrix for the plane stress case is defined like this.

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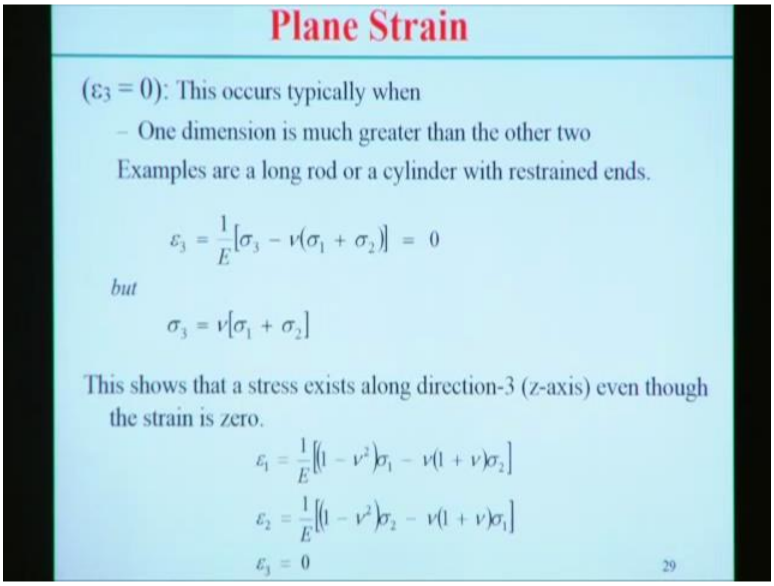

Similarly another special case that is called plane strain condition and this prevails when strain is Epsilon 3 equal to 0 this case. This actually occurs in case of 1-dimension is much greater than the other 2 dimension. 1 example are the long rod or a cylinder with some restrained ends. So if we put the strain in third direction, that is when Epsilon 3 equal to 0 here we can correlate the Sigma 3 that means, in terms of Sigma 1 and Sigma 2. Actually it shows there exists a component of the stress but 1, 2 component of the strain.

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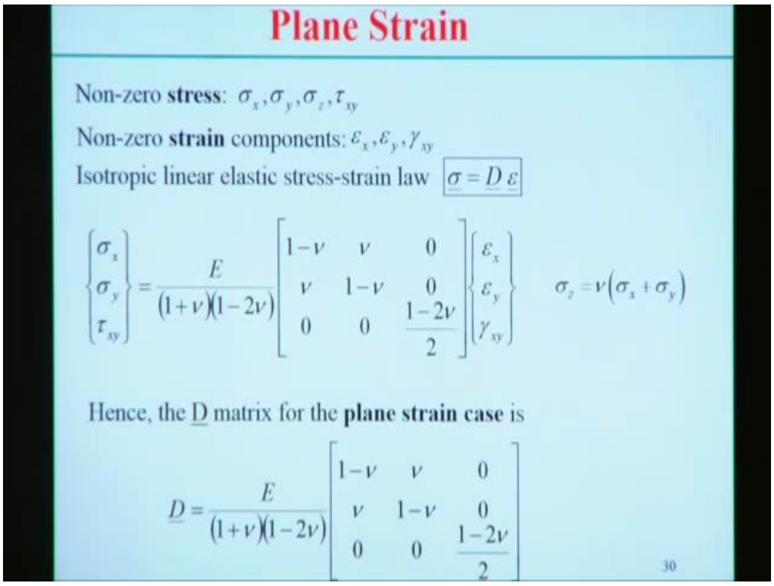

So in general we represents the plane strain condition like this, non-0 stresses are Sigma X, Sigma Y, Sigma Z and Tao XY whereas non-0 strain components are Epsilon X, Epsilon Y and gamma XY. So for isotropic linear (elasto) elastic stress strain law of can be represented, Sigma equal to D into Epsilon; again D actually represents that (po) properties of the materials, but in terms of Young’s modulus and the Poisson’s ratio.

So in this case the expression of D is different from that of plane stress conditions but it actually links between the stress and strain and here we can link the 3 component of the stress in terms of 3 component of the strain, but Sigma Z can also be calculated in terms Sigma X and Sigma Y. So looking into that D matrix, different plane stress and the plane stress condition we can correlate between the stress and strain when the condition exists in terms of plane stress or condition exist in terms of plane strain condition.

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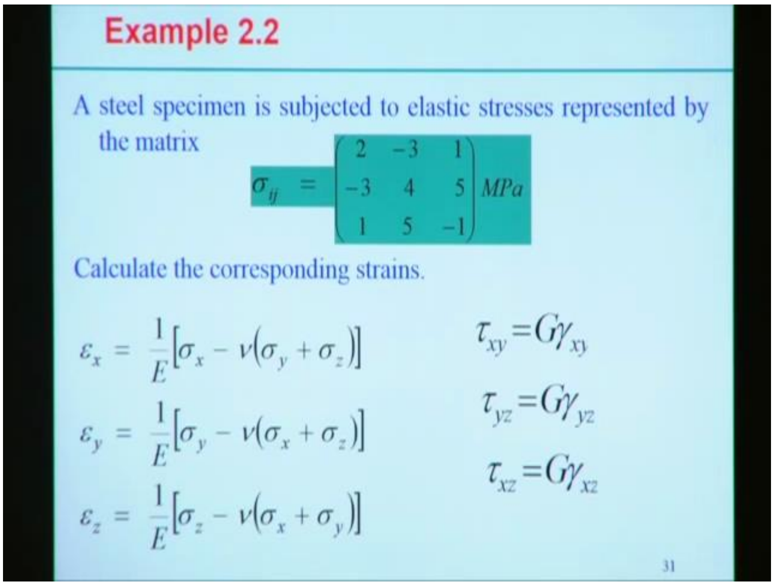

Now if we look into that 1 specific example then, we will be able to identify different component of the stress and when it is subjected to some elastic deformation. So, a steel specimen is subjected to elastic stresses represented by the matrix, if we see the different component of the matrix and here we need to find out the corresponding strain for example, the 3 component of the strain Epsilon X, Epsilon Y and Epsilon Z. Since stress are given so we can find out exactly the stresses from the given matrix, but how to find out the different stresses from this matrix, let us see.

The first component of the matrix, 11 component that is, is equal to Sigma X the numerical value is 2 here. But if we look into the Tao XY or if we see the 12, Tao 12 or Sigma 12, that is actually minus 3. If we look into that there are 2 minus 3 here, so that means it is a, produce the symmetric matrix, that means here physically Tao XY equal to Tao YZ is following here. Similarly Tao Y Z equal to Tao (ZX) ZY or Tao XZ equal to Tao ZX are following here. So basically here this matrix, there are 6 components and if we pick up that diagonal component (si) as Sigma X, Sigma Y and Sigma Z, if we put the numerical values we can easily estimate the corresponding strain in this case.

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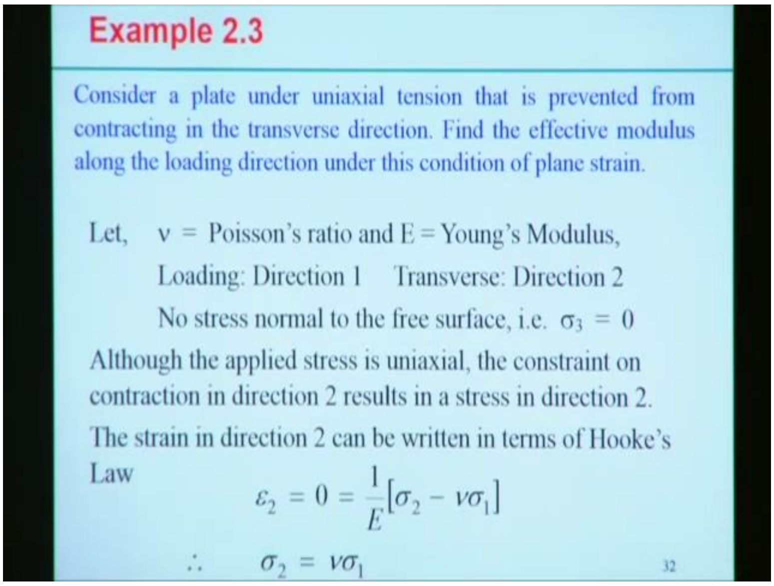

I will try to explain the another example here that; consider a plate under uniaxial tension that is prevented from the contracting in the transverse direction, and need to find out the effective modulus along the loading direction under the condition of plane strain. So, it is clearly mentioned that the plane strain condition prevails in this specific problem. Now, it is a basically uniaxial tension testing but 1 direction it is restricted to deformation, so if there is existence of some restrictions on specific directions that will try to produce some amount of the stress, but not the strain.

So that condition can be represented like that; Epsilon 2, that means second direction, if we consider the loading direction is direction 1 and transverse direction (is) as direction 2, so second direction the deformation is restricted. So in this case there may not be the change of length, so that means strain component will be 0 but there must be some of a stress component, that means it will try to create the, some amount of stress along direction 2.

But, if we investigate overall the problem no stress actually acting normal to the free surface, maybe in the third direction, there does not acting any stress that means the Sigma 3 equal to 0. Now, this condition if we apply the Hooke's law can find out that restriction of the strain along direction 2 equal to 0 from that condition we can find out Sigma 2 equal to Nu into Sigma 1, so relation between the Sigma 2 and Sigma 1.

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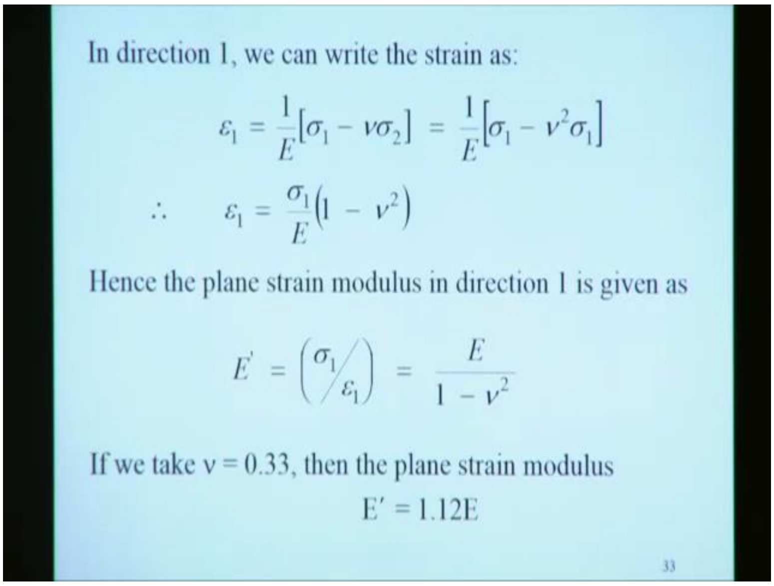

Then (effective) effective strain direction 1 is the represent in terms of Sigma 1 and Sigma 2 and report here the values of the relation between Sigma 1, Sigma 2 in the second part of this equation, then we can find out Epsilon 1 in terms of Sigma 1 and other material parameters; (here I am) that means E and Nu. So therefore, and plane strain (condition) modulus in the direction one can also be represented by stress by strain in the specific direction.

So that means Sigma 1 by Epsilon 1 which can be represent E by 1 minus Nu square and this effective value can also be very precisely evaluated if we know the numerical value of the Nu if we consider Nu as point 33 then the plastic strain modulus is calculated as 1 point 12 E. So this case the effective modulus along on specific direction maybe along direction 1 is 12 percent more as compared to the Young's modulus which (is) which was measured in case of uniaxial tensile testing.

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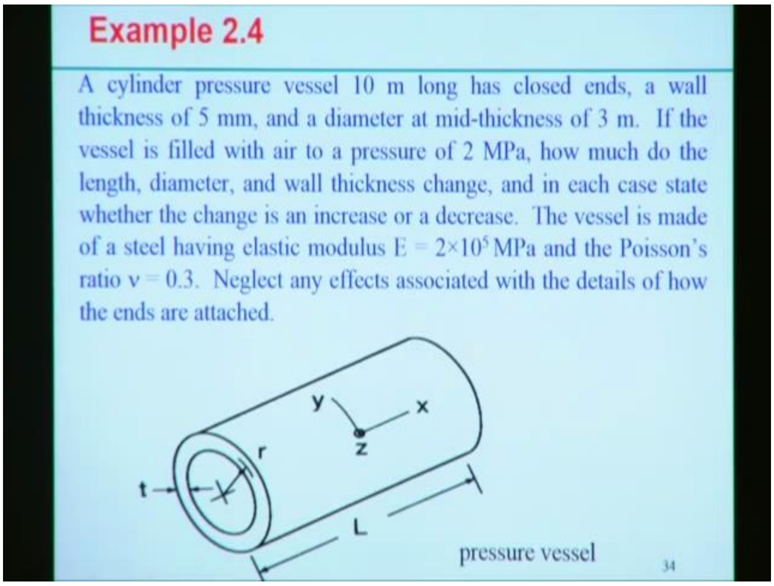

We look into another example problem like that; a cylinder pressure vessel 10 meter long has closed ends and all other parameters, for example thickness, diameter and the internal pressure also given, Young's modulus also given, the materials of this vessel is made of steel and Poisson's ratio are also given. Then, if we neglect any effects associated with the details of the how the ends are attached, we can find out at different amount of the strain or stresses and what are the change of the length along different directions.

If we look into that what are the stresses or strain acting in this case, if we see the stress is acting along X direction due to the internal pressure of this thick cylinder. Also about the circumference there is acting of the stress Sigma Y but Z direction, since this is a thin sheet so there may not be any amount of the stress acting on the Z direction or variation of the stress is negligible due to the thin sheet pressure vessel assumptions.

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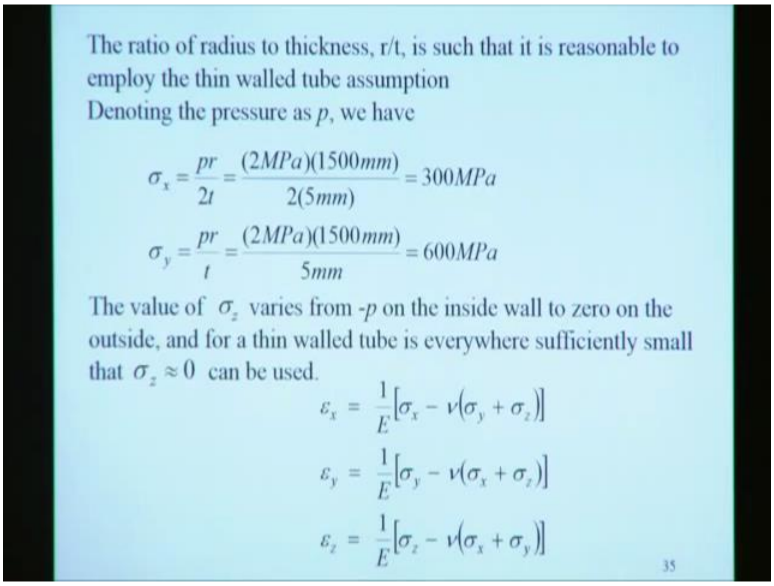

So of course it is not that the ratio of the thickness, radius to the thickness is very small here so that it can be considered as a thin-walled (b) pressure vessel and we can estimate that stress acting direction X is pr by 2t where p is the internal pressure, r is the radius and t is the thickness of the vessel and we can find out Sigma X as 300 mega Pascal. Similarly we can estimate the Sigma Y pr by t where 600 mega Pascal.

But as already I mentioned, that Sigma Z varies actually minus p, which is the internal pressure and outside it is 0, so we can assume that this is sufficiently small that can produce any significant variation over the thickness of the wall, so Sigma Z can be considered as 0 in this case. So, when Sigma Z equal to 0, so this can be considered as a plain stress problem here. Now we can found out the different component of the strain along X, Y and Z direction so here we have mentioned the X, Y and Z direction.

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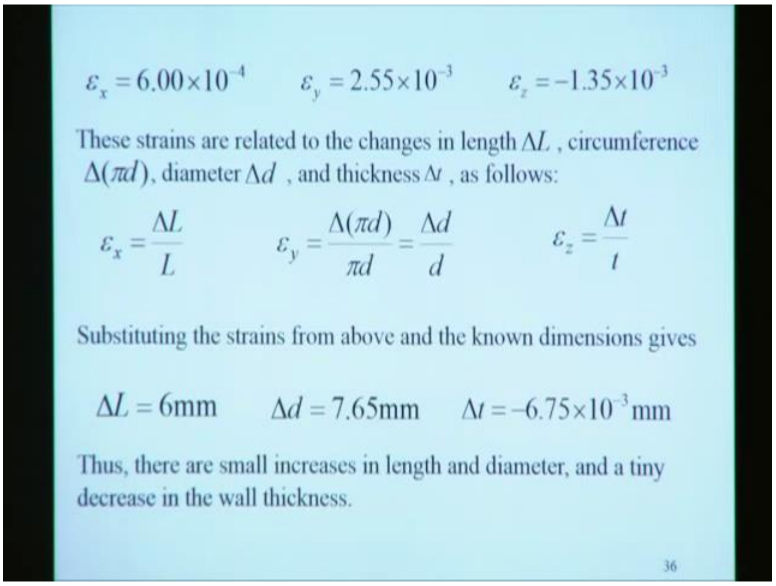

So if we put all the numerical values here we can find out the X direction, Y direction and Z direction the strain component, but this strain we can find out that strain in along X direction, that is a change of the length of the cylinder with respect to the original length, and similarly over the circumference Sigma, Epsilon Y can also be calculated over the integer of the length and that is the ratio of the change of the diameter of the original diameter and similarly Epsilon Z can also be (calculated) estimated with the change of the thickness with respect to the original or initial thickness.

So putting all these strain component values we can find out what is the change of the length, change of the diameter and change of the thickness here. And we just observe that change of the thickness is very small in this case as compared to the change of the length or change of the diameter.

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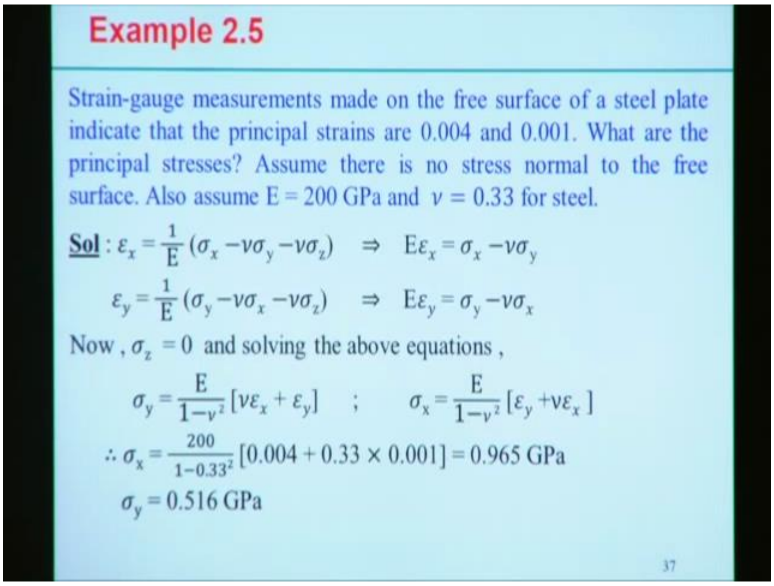

We shift to another problem we can find out that some data which is the measurement from the strain gauge made on the free surface of a steel plate indicate that the principal strain components are point 004 and point 001. What are the principal stresses? Here the assumptions is that no stress normal to the free surface and the values of the Young's modulus and Poisson's ratio for steel are also given.

So we straightforward apply the strain component along the X direction that is 1 by E in terms of Sigma Y Sigma (X Y) X and Sigma Z and (al) also in terms of Nu. So since no stress acting normal to the free surface that means Sigma Z equal to 0 here and using that we can find out (and we) we get 2 equations in terms of the stress and strain, (I) I mean Epsilon X, Epsilon 1 stress component or Sigma X and Sigma 1 these are the stress components. So if we solve it we can find out Sigma Y and Sigma X in terms of Epsilon X (Eps) Epsilon Y and Young's modulus as well as Nu; that means Poisson's ratio.

So, if we put all the numerical values, we can find out that Sigma X point 965 Giga Pascal and Sigma Y equal to point 516 Giga Pascal. So this example problems actually give some practical idea, how we can apply 3D elasticity theory to solve very different kind of problems so it is a, this was a very basic problems, but we, I think we can understand how this applications or expression of the different strain or stress components is applied to solve any practical problems.

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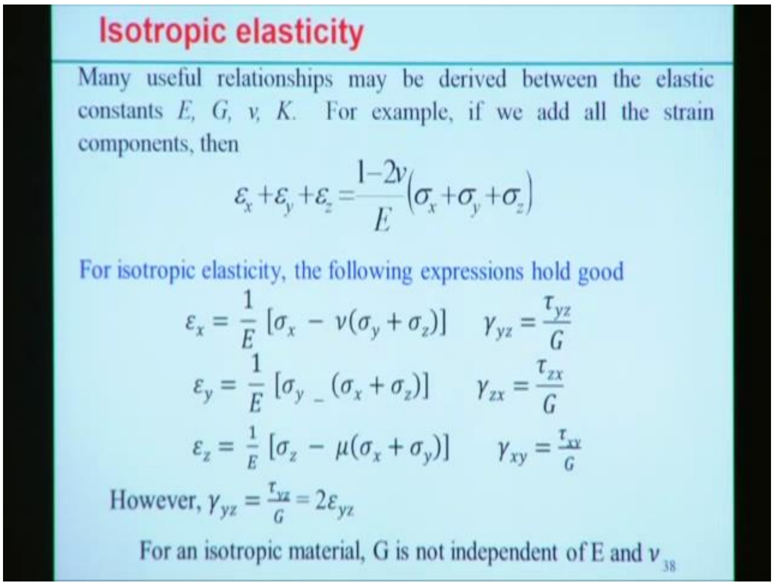

Now, we have discussed there exists 3 different types of (module) modulus as well there are existence of the Poisson's ratio, but using all this elastic modulus or elastic components or we can say elastic constants can find out different correlations among the different parameters. Let us investigate how all these parameters, for example E, Young's modulus, shear modulus, Poisson's ratio, K or D, it is a bulk modulus can also be correlated.

So it is a very first thing if we look into that expression of Epsilon X, Epsilon Y and Epsilon Z, if we add it step forward and find out this relations. But for isotropic elasticity, the following expression also (())(60:11) that we already discussed these things but here point to be noted that shear strain, for example shear component YZ is the shear stress Y shear modulus which is equivalent to the 2 times of the (s s s) normal strain component, that means Epsilon Y Z. So for an isotropic material it can be says that G, shear modulus is not independent of Young's modulus and Nu, it should depends on that.

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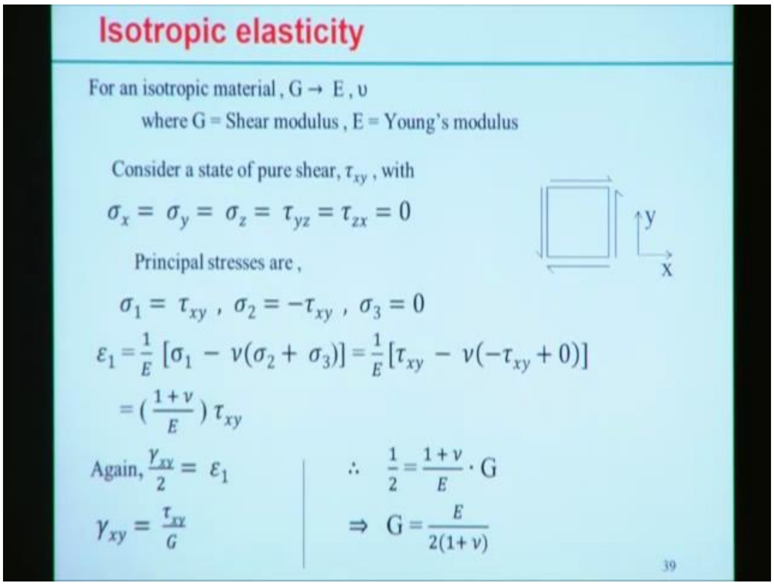

Now let us look into the relation between G, E and Nu, that means between the shear modulus, Young's modulus and Nu; this 3 constant term. First we will try to focus on that, the state of a pure shear condition. So, state of pure shear condition, there exist only one shear stress that is Tao XY, of course with respect to the other components of the other 5, 3 normal stress and 2 shear components as 0.

So with the state of the pure shear and if we analyse the (())(61:26) also we can find out the principal stress components in this case, Sigma 1 is equal to Tao XY. Second principal stress component is the minus Tao XY and Sigma 3 is definitely 0. So this is the state of the 3 principal stresses. Now when this principal stresses for pure shear condition is known to us and we can find out the Epsilon 1; that means shear strain along direction one can be represented in terms of the stress components and if we straightforward put the stress component, we can find out the Epsilon 1 equal to 1 plus Nu by E into Tao XY.

But we know that Epsilon 1 at the (safe) same time the normal strain component is the half of the shear strain component and shear stress component is related to the shear stress and the shear modulus. So, from this relation we can find out this half of the, equal to, half is equal to 1 plus Nu by E into G. So, from here we can find out the relation between G in terms of Young's modulus and Nu. So, this is the straightforward relation between the shear modulus, Young's modulus and Poisson's ratio.

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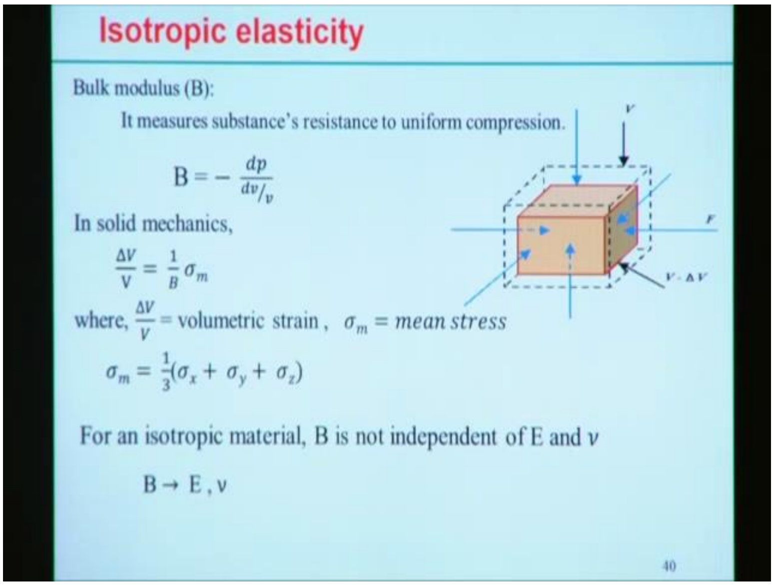

Similarly we can find out, for in case of isotropic elasticity the bulk modulus and with respect to the Young's modulus or Poisson's ratio. So it is already discussed that bulk modulus can be defined minus of dp, that means change of pressure over the volumetric strain. If the negative sign actually indicates, depending upon the direction of the stress as well as is there is a contraction of the volume with the application of this type of loading condition or this type of pressure for this type of stress condition.

So in solid mechanics, that Delta V by V is the volumetric strain is equal to 1 by B into Sigma M, Sigma M can be considered as a mean stress in this case or Delta V by V is the volumetric strain, but this mean stress actually equal to the 1 third of sation of the stresses along X, Y and Z direction. So for an isotropic material it can be says that he is not independent of E and Nu, so how we can represents the bulk modulus in terms of E and Nu from this basic concept of the relation between the mean stress and the volumetric strain.

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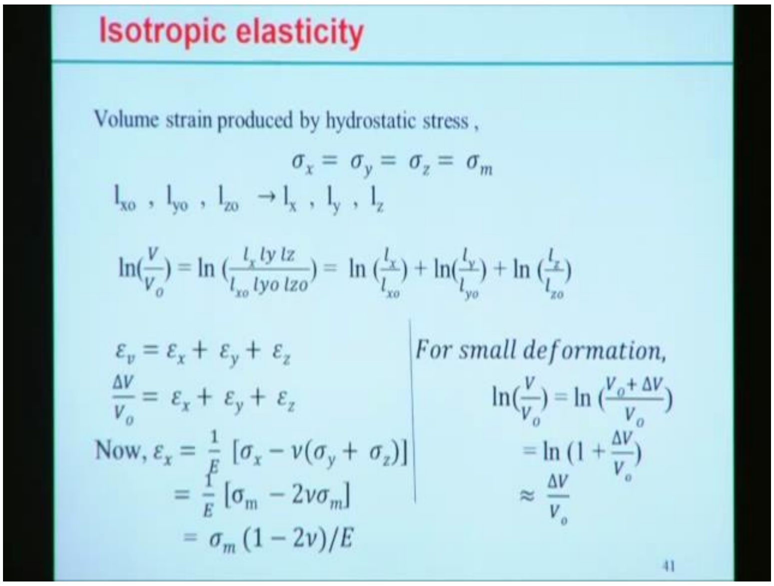

Now suppose volumetric strain actually produced by the hydrostatic stress and in the hydrostatic stress component, the mean stress component is actually called the Sigma X, Sigma Y and Sigma Z. So when there is application of the hydrostatic state of the stress that means (if we) if we consider is as a in terms of pressure, so irrespective of the X, Y and Z, this 3 Cartesian coordinate system all are equal in this case.

So, but if we try to estimate the volumetric strain here if we see that lxo, lyo, lzo was initial length dimension of an object and final dimension was lx, ly and lz. So therefore, V by V0, that means final length by initial length can also be (re) represented in terms of the logarithm, and individual component. But if we know this volumetric (s) strain actually represented in terms of the Epsilon X, Epsilon Y and Epsilon Z; but here all the terms are (log) true strain.

So it is obvious that in case of the true strain and if the deformation is very small the value of the volumetric strain or individual strain component, the true strain is actually equal to the engineering strain component. So here we find out the right-hand side expression, that we can find out the logarithm V by V0 is actually the volumetric strain and that is, approximation is Delta V by V0; that is actually considered as a engineering, in the perspective of engineering strain.

Now, Epsilon X can also be represented in terms of the Sigma X, Sigma Y, Sigma Z and Nu and if we find out the (represent) replace Sigma X, Sigma Y and Sigma Z as against stress value we can find out the Epsilon X in terms of mean stress and in terms of Poisson's ratio and Young's modulus.

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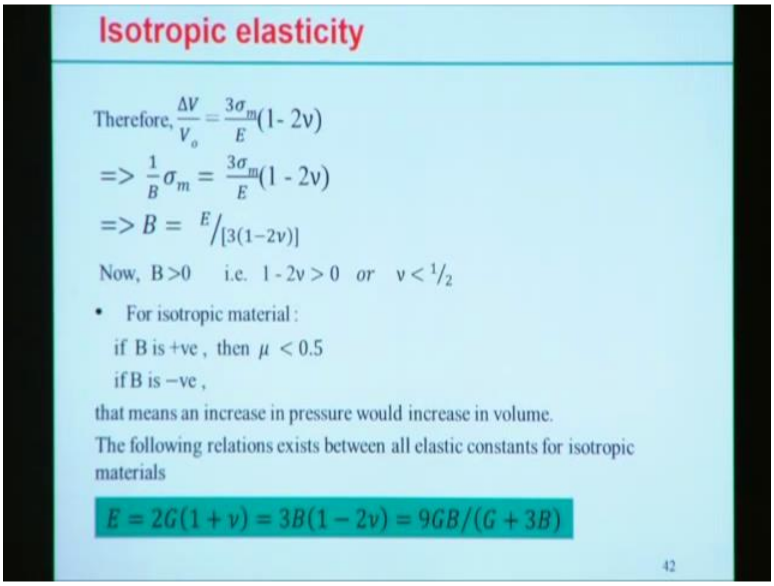

So from this relation we can find out that the definition of the bulk modulus, Delta V by (v) V0 which is equal to 1 by V into Sigma M and right-hand side also there is a Sigma M. So, here we can find out that bulk modulus is a ratio of the E divided by the 3 into 1 minus 2Nu. So these are the expression of the bulk modulus in terms of Young's modulus and Poisson's ratio. Now, if B greater than 0 that means 1 minus 2Nu should be greater than 0 which indicates that Nu should be less than half.

So for isotropic, if bulk modulus is B is positive , then we can say Nu should be less point 5. But if B is negative, then that actually indicates that (at) an increase in pressure should increase in volume. But we have already mentioned that practical values of the bulk modulus is (most) for most of the materials (is) sorry (Nu) Nu; that means Nu here the Nu; Mu is the Poisson's ratio (here), that is actually limit is point 5 in in case of solid materials, but practical for all the materials it lies between around point 24 to point 3 in between.

So we can derive so many relations between all these 4 parameters but here is the sary E in terms of shear modulus and Nu, that means Poisson's ratio and E can be represented in terms of bulk modulus and Poisson's ratio and similarly E can be represent in terms of shear modulus and bulk modulus. So, so many correlations can also be possible among all these 4 parameters in case of isotropic elasticity.

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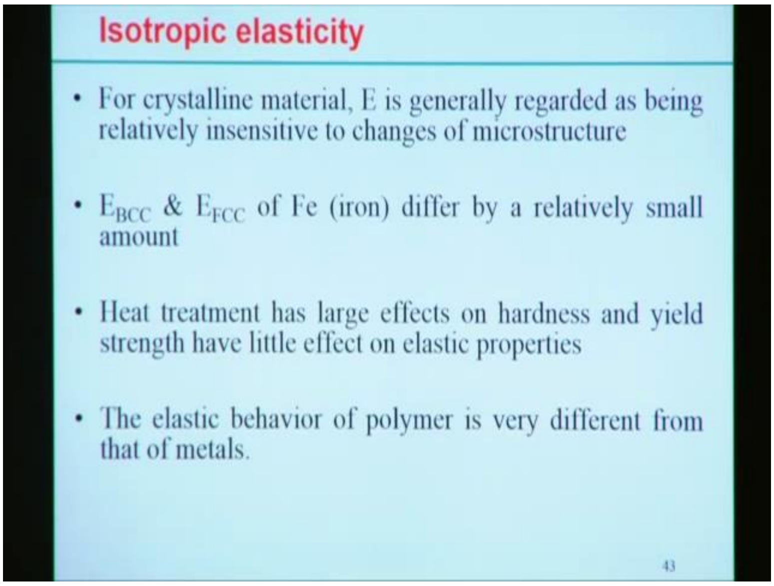

So, there is a few comments on this isotropic elasticity that for crystalline material, E is generally regarded as being relatively insensitive to change of the microstructure, but for BCC and FCC, Young's modulus in case of iron differ by a relatively very small amount. Heat treatment (at) practically having the large effect on the hardness and the yield strength but very little effect on the elastic properties.

The elastic behaviour of the polymer is actually very different from that of the metals. So, this isotropic elasticity, or all the parameters so far we have discussed, it is a basically applicable for the solid materials and most of the engineering materials we can used all this relations and main thing is that we need to remember that all this correlation we have derived assuming there is a exist some isotropic elasticity.

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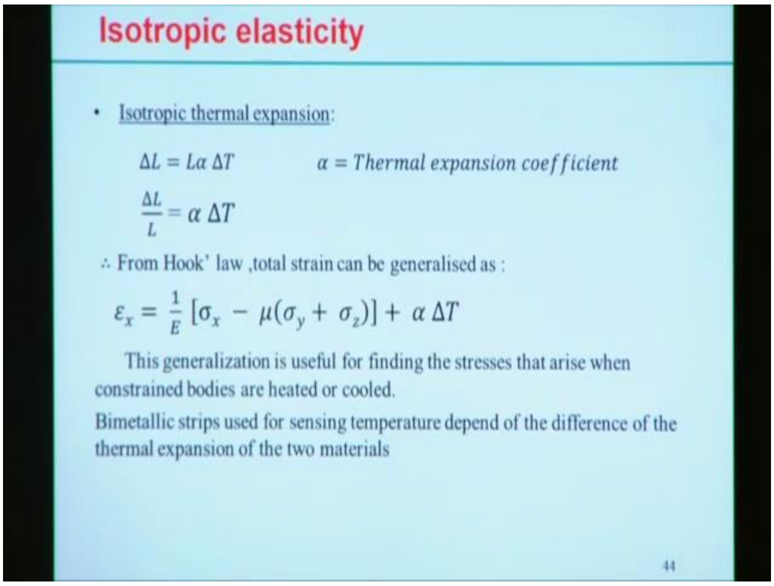

Now, further extension of the isotropic elasticity when we consider there is a thermal expansion; that actually produces some amount of the thermal strength. So, thermal, so actual if there is a application of the temperature (())(70:23) difference within the body itself then increment of the length or maybe linear strength can also be represented by the difference or in terms of thermal expansion coefficient alpha and what is the temperature difference.

So, practically in solid materials if that existence of the thermal load, thermal load in the sense of this (difference) difference of the temperature, it actually produce some amount of the strain that is called the thermal strain. And when you (super) ; this thermal strength can be directly added to the mechanical strain component we can find out or we can modify the linear strain component along X direction which is the first part indicates in the, due to the mechanical load and second part indicates due to the thermal load.

So this generalisation actually useful for finding the stresses that actually observed in case of constrained bodies when they are heated or (they are) they are cooled. 1 typical examples of the bimetallic strip used for sensing temperature depends on the difference of the thermal expansion of the 2 materials.

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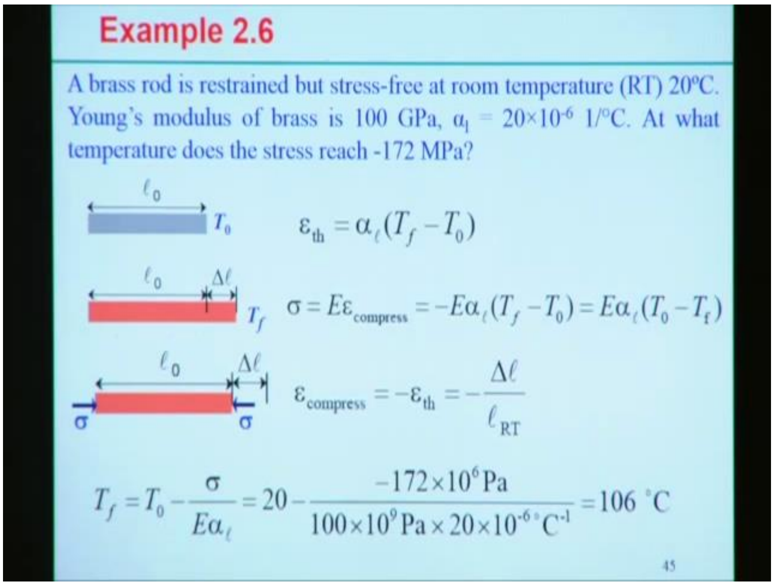

Let us look into 1 example to explain the effect of the thermal expansion. So first, a brass rod is restrained but stress free at the room temperature which is 20 degree, Young's modulus is given, the coefficient of thermal expansion also mentioned, (or) at what temperature does the stress reach to minus 172 mega Pascal. So here in this case in absence of any mechanical load only the strain will be produced due to the thermal load. So that thermal strain can be defined like that (ex) thermal expansion coefficient multiplied by the difference in temperature.

So, first figure actually indicates that original length which is having (at) the temperature, room temperature T0 at 20 degrees centigrade. Now, if there is a difference of temperature, then it will try to expand (if) freely if there is no constraint or no obstacle on this deformation . So, second figure actually indicates the length or (s) of the material when it is in final temperature, but in this case, no constant is applied to the material.

If we look into the third figure, if it is constant, if it is resist to move, or if it is resist to deformation due to the application of the temperature change then definitely it will create some amount of the stress here. So third figure actually indicates some amount of the compressive stress will be generated if we try to restrict the elongation or deformation (mo on) on specific direction. So that thermal strain is useful to estimate the amount of the stress here.

So, Sigma can be considered as the Young's modulus into the thermal strain here and that is the Young's modulus into coefficient of the thermal and the temperature difference and that thermal strain can also be calculate from the available data and from there we can find out what is the final temperature if we will try to produce, there is a compressive stress of 172 mega Pascal here. So, it is observed that final temperature here to produce the 172 mega Pascal compressive stress, the temperature should raise to 106 degrees centigrade.

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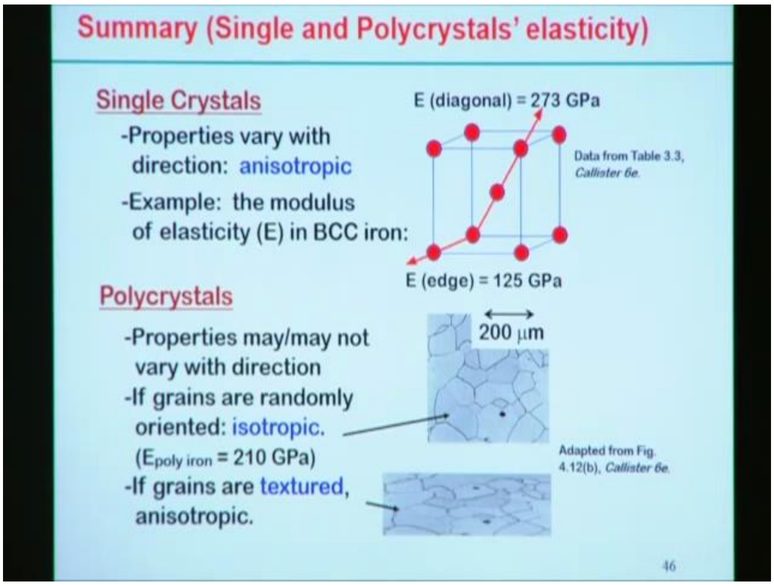

So, in say we can say that for single crystal the properties actually vary with the direction and we know that atomic arrangement of the single crystal is different in different directions, so there is a difference of the Young's modulus at different direction if we look into that picture. So at the diagonal direction, for BCC iron, it is a, the Young's modulus is 273 Giga Pascal, but along the edge the Young's modulus is 125 Giga Pascal.

So, if we, here we find out the difference, the Young's modulus at 2 different directions. That means that (si) perfect single crystal actually follow some anisotropic behaviour. And second thing is that polycrystal, that properties may or may not vary with the direction. If grains are randomly oriented that we can consider as a isotropic properties. But if grains are textured then 1 properties are, maybe, mechanical properties may be strong, 1 specific direction as compared to the other direction.

So, in sary the, depending upon the application whether it is need to use elastic (properties) elastic properties in case of anisotropic or plastic properties in case of isotropic, that actually depends on the different structure. So, in sary we can say that single (crys) crystal structure follow in different direction, the different properties; that means anisotropic properties hold good in this case.

But in case of polycrystal, in general if there is no textured structure then we can follow the isotropic properties and we can apply the theory of the isotropic elasticity in this case to evaluate or to analyse the different properties or to correlate the amount of the stress, strain using the material properties. So thank you very much for your kind attention. Thank you.