Introduction to crystal elasticity and crystal plasticity

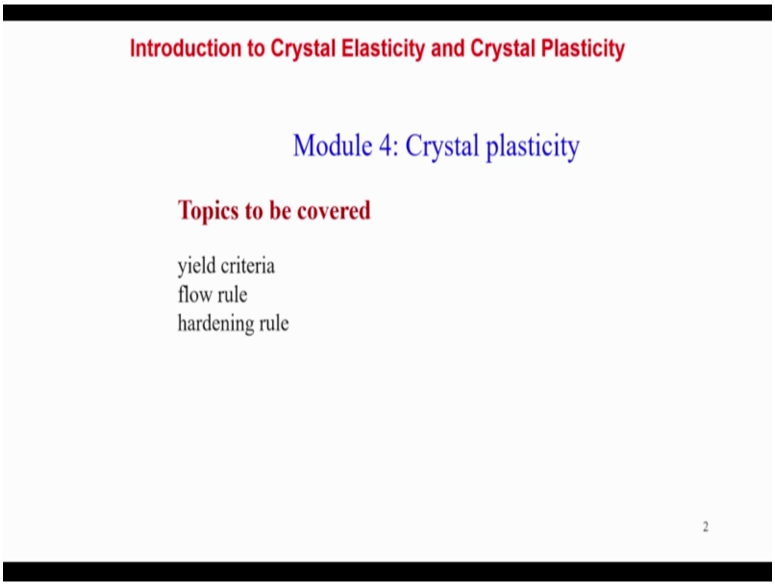
Prof Swarup Bag
Mechanical Engineering Department
Indian Institute of Technology Guwahati

Week-03

Lecture-06

Hello everybody! Let us start today's lecture of crystal plasticity. So few parts I have covered the last lecture. Now we will try to wind up the last part of the crystal plasticity.

(Refer Slide Time: 1:55)



Introduction to Crystal Elasticity and Crystal Plasticity

Module 4: Crystal plasticity

Topics to be covered

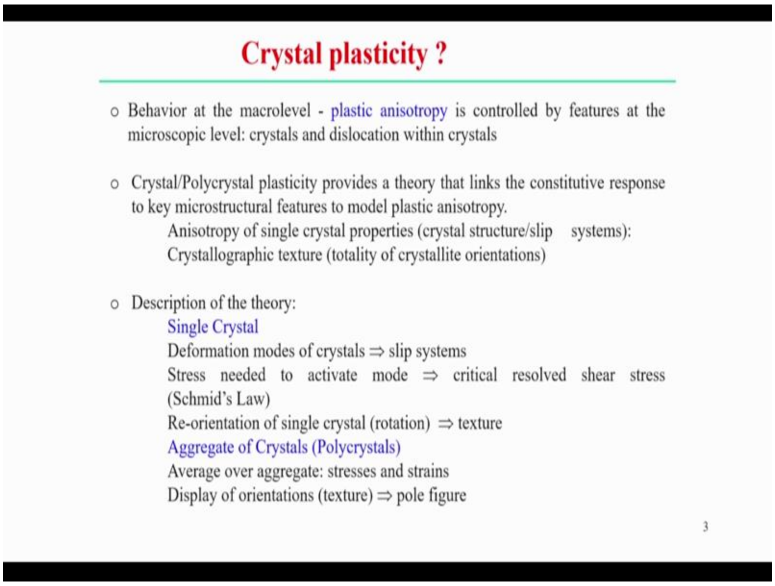
yield criteria
flow rule
hardening rule

So here we will focus on few things, like first is the yield criteria, flow rule and finally the hardening rule. Basically we have seen that continuum plasticity, we try to define that what maybe the yield criteria we can follow and based on that we have develop the Von Mises yield function of form for a specific material and then we the different flow rule to predict the plastic behavior of the material and also we use the different hardening rule also, to explain the phenomena of the implement of the strain level after the first yield point.

So there are different constrictive relation exist and we will try to using that relationship and we explain the different continuum plasticity model, but in crystal plasticity it may not be very straight forward to defining the yield criteria, flow rule and the hardening rule because in this case at a microscopic level the behavior of the plastic deformation for a single crystal is a more complicated.

So there maybe the several theory exist but we will try to overview on that and we will try to get the basic understanding on the topic of the yield criteria flow rule and the hardening rule in case of crystal plasticity or specifically when we try to focus on the single crystal plastic deformation behavior. Finally we will try to find out in case of polycrystal form how we can use this theory in some aggregate form or we will try to explain the different theory behind the polycrystalline plastic deformation.

(Refer Slide Time: 3:02)



Crystal plasticity ?

- Behavior at the macrolevel - plastic anisotropy is controlled by features at the microscopic level: crystals and dislocation within crystals

- Crystal/Polycrystal plasticity provides a theory that links the constitutive response to key microstructural features to model plastic anisotropy.
  Anisotropy of single crystal properties (crystal structure/slip systems):
  Crystallographic texture (totality of crystallite orientations)

- Description of the theory:
  Single Crystal
  Deformation modes of crystals ⇒ slip systems
  Stress needed to activate mode ⇒ critical resolved shear stress (Schmid's Law)
  Re-orientation of single crystal (rotation) ⇒ texture
  Aggregate of Crystals (Polycrystals)
  Average over aggregate: stresses and strains
  Display of orientations (texture) ⇒ pole figure

So to do that first we will try to understand the crystal plasticity, why is it necessary and what is the bottom line theory exist to understand the plastic behavior of single crystal structure. So one thing is that behavior at the macro level is basically plastic anisotropy which control by feature at the macroscopic level and in this case crystal and their dislocation behavior or dislocation exist within the crystal structure and you need to analyze that. But of course here we can see that the macro level even her also the anisotropic plastic behavior exist.

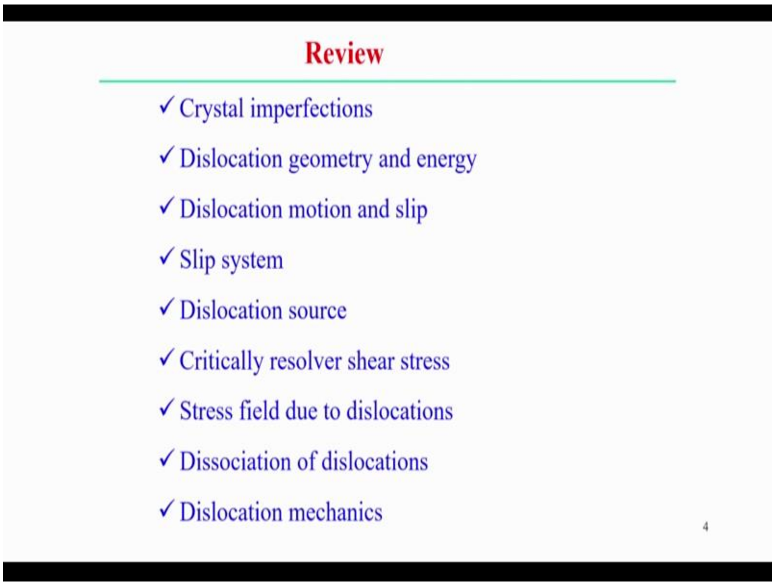
Second point is that crystal plasticity or maybe polycrystal provides a theory that actually links the constructive response to the key microstructural feature to model the plastic, specifically the plastic anisotropy. So anisotropy of single crystal properties actually it is related to the crystal structure and the slip system and at the same time we need to consider the crystallographic structure in account to explain the anisotropy plastic behavior when you try to focus on the single crystal deformation behavior.

So theoretical description on this plasticity model, first we will try to focus on the single crystal. So here the deformation mode, in terms of the slip we have explained and the slip phenomena is also explained in cased of single crystal structure by defining the single slip plane and the several slip direction so one such critical amount of the shear stress that is required to initiate the yielding that is called critical resolved shear stress, which we have already explained that how we can estimate the critical resolved shear for a specific crystal structure.

But another point is that so when we try to explain the plastic deformation by the slip system or by the mechanism of the slip in a single crystal structure so it is actually associated with the reorientation of the single crystal. So there maybe some rotation of the crystal and that can be after rotation they can orient one specific direction and that is specifically structure, so structure is having very important phenomena to analyze the crystal plasticity and that action brings the complexity in this plasticity theory which is different from the continuum plasticity theory in this case.

So once we done for the single theory or application or analysis of the crystal plasticity for single crystal then we will try to estimate to we will try to link with the aggregate of the crystals, for example in case of polycrystals. So average aggregate over the stress and strain can be done in case of the polycrystals and that polycrystalline structure the display of the orientation of the structure actually better link or better explain with the pole figures but we will not do into much details on pole figure in this case, we will try to understand the basic theory behind the crystal plasticity.

(Refer Slide Time: 7:52)



Review

- ✓ Crystal imperfections
- ✓ Dislocation geometry and energy
- ✓ Dislocation motion and slip
- ✓ Slip system
- ✓ Dislocation source
- ✓ Critically resolver shear stress
- ✓ Stress field due to dislocations
- ✓ Dissociation of dislocations
- ✓ Dislocation mechanics

So to do that or to enter into the actual topic probably we can look into the review work, so far we have done related to crystal plasticity, first we started with the crystal imperfection or crystal defects, we will try to identify the broad category of the crystal defect, point defects, line defects and the surphase defects, so here we have explained the crystal (())(7:08) in most of the cases in terms of the line defects that means in terms of the dislocation.

Then we have tried to estimate the dislocation of geometry two basic element, true dislocation and edge dislocation and what is the associated amount of energy with screw and edge dislocation that we have estimated. Next we have discussed on the dislocation motion and the slip system for a specific crystal structure, simple crystal structure for example, we consider the FCC, BCC and HCP crystal structure, what are the slip system exist all this simple materials.

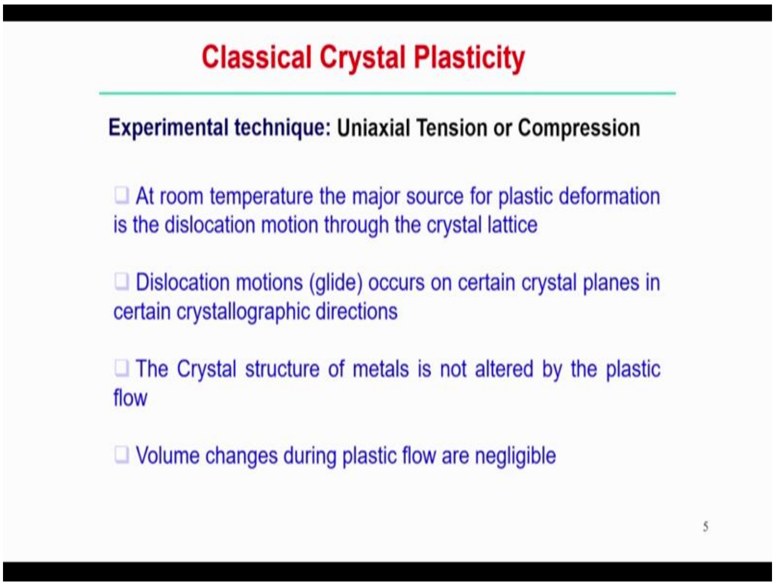
Then we have define the slip system that means nit constitute on defining the slip plane as well as the direction, slip direction on the specified slip plane that uhh constitute actually the slip system of a specific crystal structure. Then we discussed the dislocation source by using the Frank – Reed source of dislocation generation, with that mechanism we have explained the further straining or plastic deformation of the crystals there is a generation of the dislocation and that actually increases the dislocation density of a specific sample, that we have explained.

Next we have tried to define the resolved shear stress and from the resolved shear stress the critical value of the resolved shear stress that actually starts the initiation of the yielding that is called critically resolved shear stress that we have evaluated for the simple crystal structure. And then we have derived the stress field associated to the dislocation individually we consider the stress field associated with the screw dislocation and edge dislocation and finally how the this stress field are interacting in the presence of the another dislocation of opposite sign that we have explained.

And then we also explained the dissociation of the dislocation maybe full dislocation, how it can be dissociated into two partials or maybe dislocation reactions in terms of the two partial dislocation can form in the another dislocation. By looking into that vector summation of the partial or looking into the or by analyzing the energy level changes during the reaction, we have discussed the reaction of the dislocation.

And finally we discussed dislocation mechanics, different mechanics associated with the dislocation for example dislocation pile up, dislocation intersection, cross slip, all this phenomena we try to explain in terms of the dislocation and overall if we see that all discussion has been done with a two basic reference points one is the in terms of either edge dislocation or in terms of screw dislocation, but practically this is the very basic elemental thing, the edge and screw dislocation. And impractical there maybe the combination of the both the components must be there.

(Refer Slide Time: 10:48)



Classical Crystal Plasticity

Experimental technique: Uniaxial Tension or Compression

☐ At room temperature the major source for plastic deformation is the dislocation motion through the crystal lattice

☐ Dislocation motions (glide) occurs on certain crystal planes in certain crystallographic directions

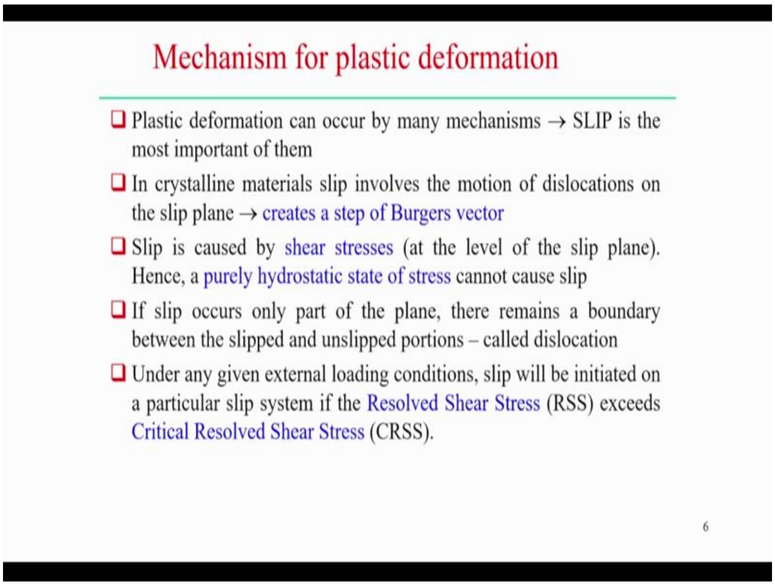
☐ The Crystal structure of metals is not altered by the plastic flow

☐ Volume changes during plastic flow are negligible

Now we come to the actual party of this lecture that is the classical plasticity, how we can explain this thing. Actually the most feasible experimental technique we generally consider that is either Uniaxial tension or compression and from that we get the data itself and we compare with the theoretical prediction of the single crystal structure or maybe polycrystalline structure. Now few facts to analyze the classical plasticity theory or associated with that one point is that a room temperature, a major source of plastic deformation is the dislocation motion through the crystal lattice.

So we need to understand the dislocation motion to understand the plastic deformation in the single crystal structure. Second point is that dislocation motion or maybe sometimes we can call dislocation glide basically occurs specifically on certain crystal plane and in certain crystallographic direction. Third point is that crystal structure of metals actually not alter by the plastic flow or maybe we can say by the dislocation motion. What are the volume changes associated with the plastic flow are negligible in this case so this is a typical key fact to remember when you try to enter into the analyzing of the basic crystal plasticity theory.

(Refer Slide Time: 12:49)



Mechanism for plastic deformation

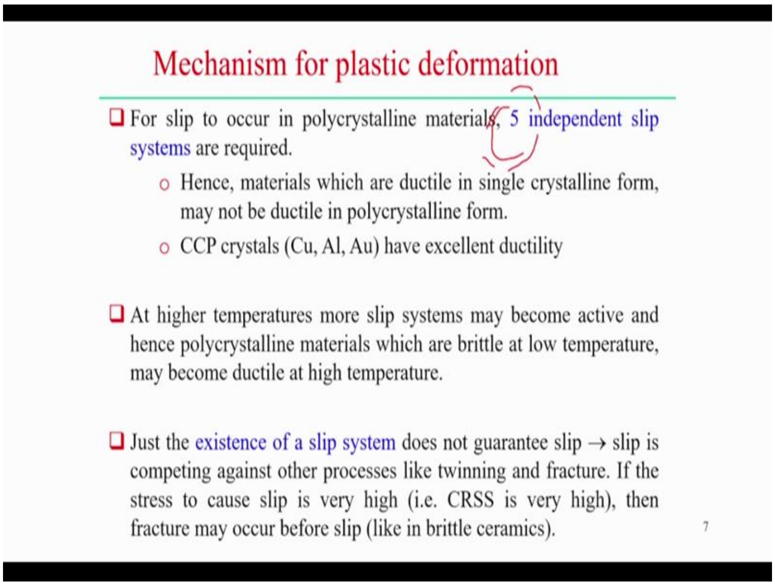
- Plastic deformation can occur by many mechanisms → SLIP is the most important of them
- In crystalline materials slip involves the motion of dislocations on the slip plane → creates a step of Burgers vector
- Slip is caused by shear stresses (at the level of the slip plane). Hence, a purely hydrostatic state of stress cannot cause slip
- If slip occurs only part of the plane, there remains a boundary between the slipped and unslipped portions – called dislocation
- Under any given external loading conditions, slip will be initiated on a particular slip system if the Resolved Shear Stress (RSS) exceeds Critical Resolved Shear Stress (CRSS).

Now what are the mechanism of the plastic deformation, so first is the plastic deformation basically several times we have discussed that plastic deformation can occur by many mechanism but one of these is the slip mechanism which is the most important here and we are trying to analyze all these phenomena based on these slip mechanism. In the crystalline materials slip involves the motion of the dislocation on a specific slip plane and we have already define that creates one single step that actually associated with the Burgers vector or we represent this thing the motion of maybe slip in terms of the Burgers vector.

Third point is the slip is caused by the shear stress, so at the level of the slip plane but since purely hydrostatic state of the stress cannot cause the slip. Since slip occurs only by the shear stresses, this is very important phenomena to consider that purely hydrostatic state of the stress does not influence the slip mechanism in this case. Next point is that if slip occurs only part of the plane, definitely that actually creates a boundary between the slipped and unslipped portion and that actually physically called the dislocation.

So under any given external given conditions slip will be initiated on a particular slip plane the amount of the stress or shear stress exist certain value that is called the critical resolved shear stress when it is reached then only slip will be initiated. So this is the key fact we have already discussed, I am just reviewing on this fact to analyze the mechanism of the crystal plasticity.

(Refer Slide Time: 14:37)



Mechanism for plastic deformation

- For slip to occur in polycrystalline materials, 5 independent slip systems are required.
    - Hence, materials which are ductile in single crystalline form, may not be ductile in polycrystalline form.
    - CCP crystals (Cu, Al, Au) have excellent ductility
- At higher temperatures more slip systems may become active and hence polycrystalline materials which are brittle at low temperature, may become ductile at high temperature.
- Just the existence of a slip system does not guarantee slip → slip is competing against other processes like twinning and fracture. If the stress to cause slip is very high (i.e. CRSS is very high), then fracture may occur before slip (like in brittle ceramics).

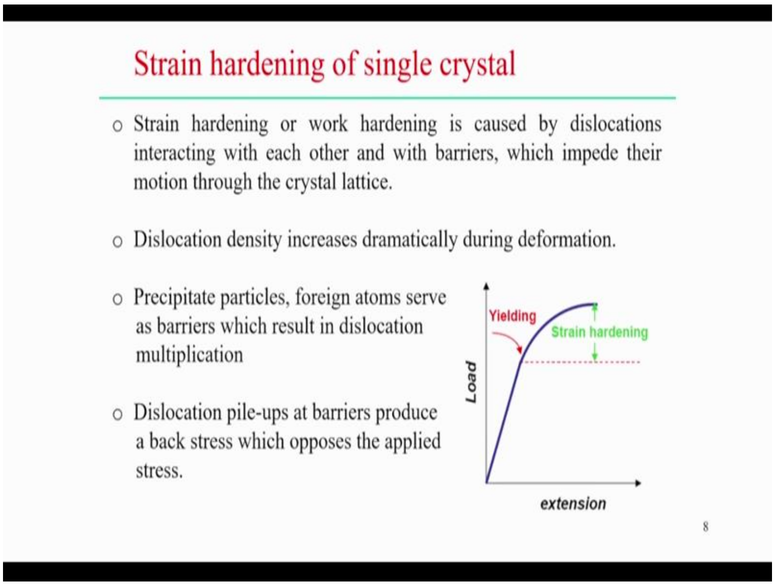
So but, in this one point is that for a slip in a polycrystalline material there require a five independent slip system so here the significant terms is the 5 here that we can say for a polycrystalline material actually we need to define 5 independent slip system, so why this number is important because so far we have discussed the slip occurs only one specified slip plane at in one direction but we have not discussed what is the consequence or whether is there at any time whether several slip system is active or not, that we have not discussed.

But polycrystalline material it is required at least the 5 independent slip system is required, we will try to explain or we will try to mathematically prove that why 5 independent slip is system is required to initiate the slip in case of the polycrystalline materials. Then that means uhh at one time a number of slip system is important here to look into that aspect if you see that material who is a ductile in single crystalline form, they may not be the ductile in polycrystalline form because in polycrystalline form there may be at the same time the initiation of the several slip plane or initiation of the several independent slip system.

CCP crystal that means Cubic Closed Packed system has explained ductility because of the less number of slip system probably active in this case. But at high temperature more slip system may become active and hence polycrystalline materials actually which is brittle at the low temperature probably they may ductile at the high temperature. And just the existence of a slip system does not guarantee that slip will occur because at the same time slip is the competing agianst other processes like twinning and fracture.

If the stress to cause the slip is very high probably critically resolved shear stress is very high then fracture may occur before slip it is like the brittle ceramic, for example even in continuum mechanics also we have observed that brittle facture actually occurs without the much plastic deformation, for example in glass can be considered as the brittle material so we apply the load, almost no deformation it simply breaks, so directly with the application of the load it leads to the fracture point. So similar phenomena can also observed in this case when you try to explain the crystal slip system in case of the single crystal structure.

(Refer Slide Time: 17:51)



Strain hardening of single crystal

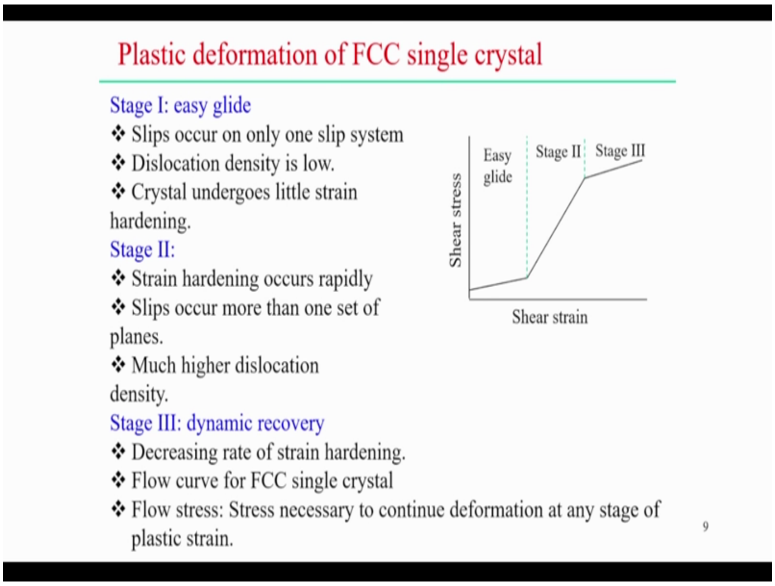
- Strain hardening or work hardening is caused by dislocations interacting with each other and with barriers, which impede their motion through the crystal lattice.
- Dislocation density increases dramatically during deformation.
- Precipitate particles, foreign atoms serve as barriers which result in dislocation multiplication
- Dislocation pile-ups at barriers produce a back stress which opposes the applied stress.

Now like continuum mechanics also if we see how the sip occurs that strain hardening or work hardening actually the this mechanism also active but the dislocation interacting with each other when there exist some barrier which impede their motions through the crystal lattice. So in uhh strain hardening for the single crystal structure when there is a resistance to the dislocation motion that actually raise the further stress level so that is the because of the strain hardening mechanism exist what is particular materials.

So definitely dislocation density increases dramatically during the deformation, so once it starts the yielding point with a further straining or further plastic straining of the material there is a generation of a multiple dislocation, so practically in a specific sample there is a increment of the dislocation density. But presence of the precipitated particles for an atoms actually acts as a barrier which results in the dislocation multiplications or that actually that barriers resist the dislocation motions as well.

Finally the dislocation pile up if there some exist some barrier for at a specific barrier that actually produce a back stress which actually improves the level of the stress with a further straining of the material. So basically in a single crystal also the strain hardening mechanism exist and that actually explains by the resistant with the dislocation motions in presence of certain barriers. This is the basic phenomena of the strain hardening in case of the single crystal structure.

(Refer Slide Time: 19:52)



Plastic deformation of FCC single crystal

Stage I: easy glide

- ❖ Slips occur on only one slip system
- ❖ Dislocation density is low.
- ❖ Crystal undergoes little strain hardening.

Stage II:

- ❖ Strain hardening occurs rapidly
- ❖ Slips occur more than one set of planes.
- ❖ Much higher dislocation density.

Stage III: dynamic recovery

- ❖ Decreasing rate of strain hardening.
- ❖ Flow curve for FCC single crystal
- ❖ Flow stress: Stress necessary to continue deformation at any stage of plastic strain.

Now if we look into the plastic deformation of single crystal structure we can see graphically that the it is graph of the shear stress and shear strain and we can define the deformation zone in stage one and stage 2 and stage 3. If we see that in stage 1 actually that is called the easy gliding, so basically easy movement of the dislocation probably. So in this case starts with a slip occurs only one slip system at a very initial stage. So at this stage basically the dislocation density is very low so probably the interaction of the dislocation and chances here are also very low, so in this case of course with the further deformation of the plastic deformation of the materials there may not be much increment of the stress level due to the strain hardening mechanism.

So basically crystal alter undergoes very little amount of the strain hardening in a very initial stage that means stage 1. Stage 2 if we see that strain hardening actually occurs very rapidly that means there is a increment of the stress level within the small amount of the strain which physically indicates that the effect of the strain hardening is very high in this case, because in this case actually slip occurs more than one set of planes not is very confined to the single slip system, so therefore multiple slip system is active on the stage 2 and that actually raise the level of the stress due to the effect of the strain hardening.

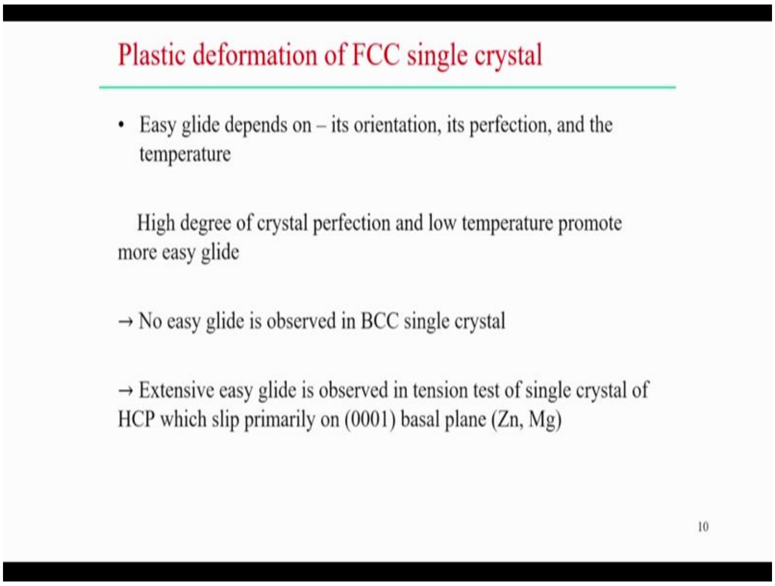
At the consequently same time, so if there is a strain hardening effect is very in this case so at the same time the dislocation density also improves or also increases at the second stage. In hot stage in this case the plastic actually there is a decreasing rate of the strain hardening rate is very low at the third stage, and the third stage it can be say that the shear stress is reached to the flow stress or flow curve for the FCC single crystal structure. Now flow stress, what we mean by flow stress, actually the stress necessary to continue the deformation at any stage of the plastic strain.

So probably at the stage 3 the amount of the stress can considered as a flow stress, it remains almost nearly constant and where the strain hardening rate is very low, and if we see that the overall structure, shear stress versus shear strain of FCC single crystal structure if we look into that there is this is the approximation of the actual curve and the if we see the mechanism involved in the stage 1, basically we have divided into the three stages, stage 1, stage 2 and stage 3, basically in this stage 1 there is a small increment of the stress level, stage 2 high increment of the stress level and stage 3 less increment of the stress level.

But try to reach some specific steady state value that can be considered as a flow stress value in this case and we explain the plastic deformation of the single crystal structure by considering the strain mechanism and which case the strain hardening mechanism is less or more. At the same the the very initial stage the active slip system may be less and if we go on further straining then there maybe the increment of the active slip system, the more number of slip system becomes active in the latter stage.

Similarly at the same consequence we can say that the dislocation density maybe at the very initial stage is very low, and gradually the dislocation density actually increases and probably at the certain point the dislocation density reaches the maximum point.

(Refer Slide Time: 25:42)



Plastic deformation of FCC single crystal

- Easy glide depends on – its orientation, its perfection, and the temperature

High degree of crystal perfection and low temperature promote more easy glide

→ No easy glide is observed in BCC single crystal

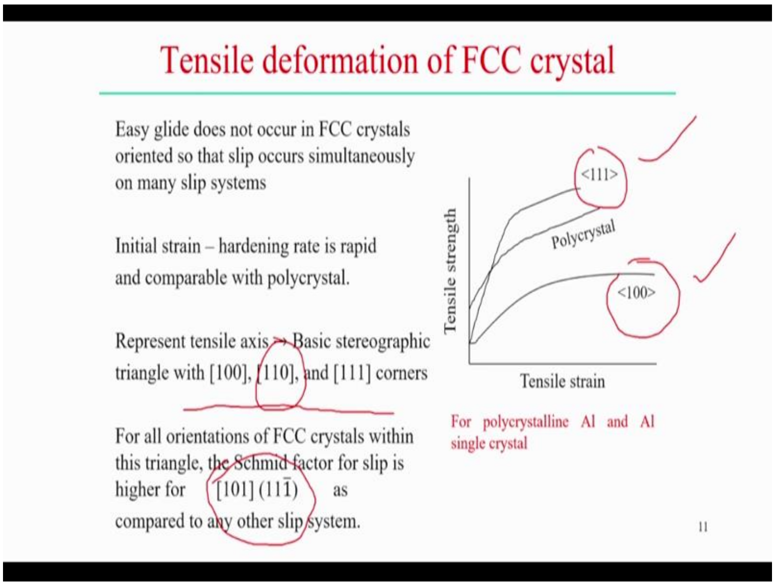
→ Extensive easy glide is observed in tension test of single crystal of HCP which slip primarily on (0001) basal plane (Zn, Mg)

Now what maybe the effect of the easily glide on this specifically the plastic deformation of the FCC single crystal structure. So if we see that easy glide actually depends on its orientation, its perfection and the temperature. So all this parameters actually insignificant on the easy glide but high degree of the plastic perfection, a crystal perfection and a low temperature actually promote more easy glide. So probably if presence of the dislocation is very less that means it tends to the very prefect crystal and very low temperature probably the number of active slip system is very less so that it is, it easy gliding is possible in that condition or that condition actually promotes the easy glide.

But if you see no easy glide is observed in BCC single crystal structure. If we see or if we have analyze that what is the total number of slip system, theoretically it is having 48 number of slip system as compared to the FCC structure because FCC structure is only 12 number of slip system. So since the presence of the several slip system in BCC crystal structure, so in this case there may not be the possible of the easy gliding of the dislocation.

Third point is that extensive easy glide is observed in tension test in case of the single crystal like HCP crystal structure and which slip primarily on tipple 0 1 plane that is your basically basal plane. So the most feasible reason in this case the easy glide in case of HCP crystal structure because in a HCP it is having very limited number of the slip system so that is the reason for the easy gliding phenomena in case of HCP crystal structure.

(Refer Slide Time: 30:21)



Tensile deformation of FCC crystal

Easy glide does not occur in FCC crystals oriented so that slip occurs simultaneously on many slip systems

Initial strain – hardening rate is rapid and comparable with polycrystal.

Represent tensile axis → Basic stereographic triangle with [100], [110], and [111] corners

For all orientations of FCC crystals within this triangle, the Schmid factor for slip is higher for [101] (11$\bar{1}$) as compared to any other slip system.

For polycrystalline Al and Al single crystal

Now if you see the tensile deformation of FCC crystal structure, right hand side if you look into the graphical form the tensile strength versus tensile strain, here see that actually represents the stress versus strain curve and if we see that there are two different direction so 111 that is basically the body diagonal in that direction, so in that direction if we apply the tensile load the curve represents like that, in the figure. I think this is the curve that represents the stress versus strain along 111 direction and if we see the stress versus strain along 100 direction that means along one of the axis x, y, z axis if we apply the tensile load we are getting the stress versus strain diagram.

If we see there is a huge difference between the tensile behavior of a single crystal structure so it depends actually which direction the load is acting, so there is a variation from the maximum possible variation to the minimum probably in this case the huge variation observe, so but if we consider that polycrystalline material so basically if we consider randomly oriented grains having polycrystals and that if you test for that polycrystal with application of a load then we are getting somehow in between that two direction so some aggregate in between this two specific direction of the tensile axis.

So this is the typical for aluminum single crystal structure or aluminum polycrystalline structure, we observed this type of tensile behavior. Now if we try to explain or if we analyze this phenomena so in point wise let us see the first thing is that easy glide does not occur in FCC crystal oriented so that slip occurs simultaneously on many slip system. So here if we see for this specific crystals we don’t find the very easy glide at very initial stage because probably in this case the very initial stage the large number of slip or more than one slip system is active, so that strength level is high.

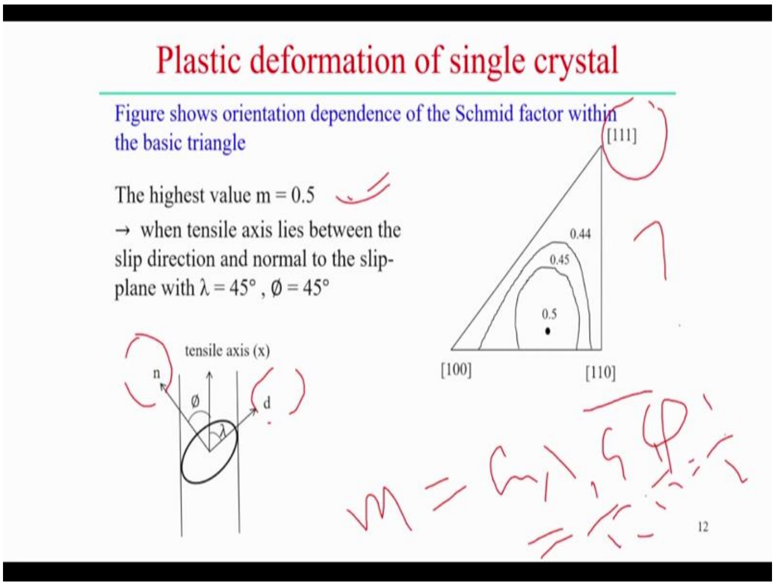
Initial strain hardening rate is rapid and comparing with the polycrystal. So if we consider individually the direction, it is initial at the very initial part it is comparable with the polycrystal form. But there is a variation and how we can explain the variation that can be better represented by the, if you represent the tensile axis basic stereographic triangle with 100, 110 and 111 corners, what does it mean? Because in one single crystals the tensile stress can also be done at a several orientation or several direction, for example here if you see the two cases, one is the suppose 100 direction, we apply the tensile axis along 100 direction.

That here if you see 111 direction, so it is variation and if we do probably 100, 111 and another is second one 110 direction, that is 110 direction is the phase diagonal. So here if we construct the basic stereographic triangle representing that three direction at the three corner of the triangle. For example, one first corner probably 100 that represents the direction of one axis. 110 that is the direction of the phase diagonal of a unit cell there is the another corner of a triangle and third one is the 111 that is the body diagonal direction of a cubic cell that represents the third corner of the triangle.

So within this triangle if we consider there, there may be the variable direction if we consider the tensile stress probably the stress strain diagram will be different but how to capture or how to consider all the, all this uhh values of the specifically the tensile stress strain diagram that is oriented within that three axis, within that three corner of the triangle. So that we can represent in the contour plot of the this thing with the variable, with variation and probably we can use that data to represent, to correlate that stress strain diagram of a single crystal structure to the polycrystalline structure.

We will try to see how it can vary, but basic philosophy is that first we need to fix the three corners of the triangle so that we can capture minimum to the maximum variable of the tensile strain that exist between the three direction that is 100, 110 direction and 111 direction. But it observed that the for all orientation of FCC crystals within this triangle the Schmid factor for the slip higher for the specific one 101, 11 minus 1 is the specific slip system. We will try to explain that how this factor can be estimated theoretically and to that we can look into that how this triangle can be formed.

(Refer Slide Time: 34:23)



Plastic deformation of single crystal

Figure shows orientation dependence of the Schmid factor within the basic triangle

The highest value m = 0.5

→ when tensile axis lies between the slip direction and normal to the slip-plane with λ = 45° , Ø = 45°

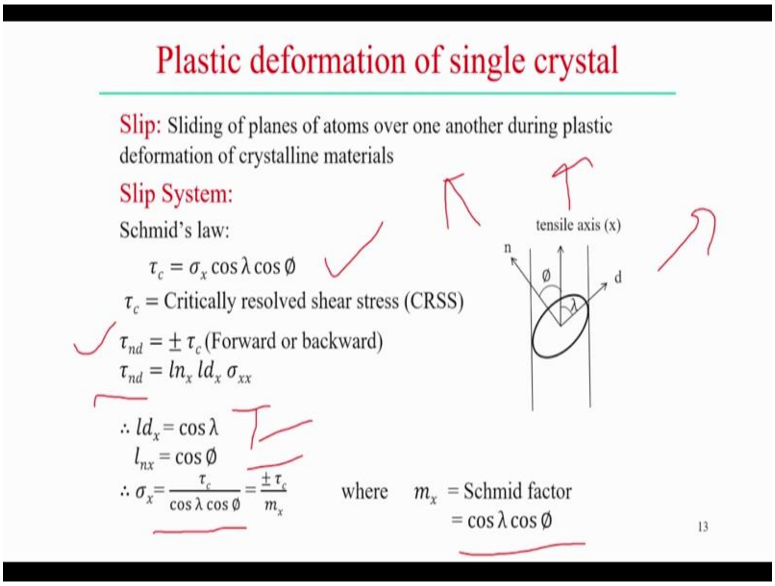
See right hand side figure, if you see that one corner point represents the direction of the one axis of a unit cell. Second point is that, that direction actually represents the direction of the phase diagonal of a cubic unit cell and third point represents the direction along the body diagonal of the cubic unit cell. So we assuming that within this direction there are several orientations and that orientation can be represented in case of uhh in case of contour plot we can find out, so iso-contour we can plot so one contour probably represents the same value so that is called isocontour in this case.

So here in case of FCC crystal structure we found that highest value of m, m is the Schmid factor so it is 0.5, so other is the, if you see that other variable if you see in this case that isocontour is a 0.45, 0.44 that kind of variation exist. So in between this variation several orientation can be better represented by this basic triangle form. The highest value in this case if we see that is m equal to 0.5, so m equal to 0.5 corresponds to that suppose we consider uhh resolved shear stress and we define one slip plane that normal to the slip plane is represented by the direction n and d is the slip direction and suppose along the x axis there is a tensile axis applying the load.

With this configuration say we have defined lambda n phi so we have already defined how we can link, how we can find out the resolved shear stress or maybe how to find out the shear stress acting on the direction t to initiate the, or to initiate the slip of the crystals. So in this case that lambda equal to 45 and phi equal to 45 probably it is may be the optimum condition, for this and optimum condition here m equal to actually phi, so m is actually here defined cos lambda and cos phi will later on how we can estimate this m and corresponding to 1 by root 2 into 1 by root 2, that means half or this is point phi.

So this is the maximum possible value of the m with the several orientation between the three axis of the triangle, so we can use the information and can find out the loading condition or stress strain diagram probably by looking into the m value, so let us see how we can use this m value to do the further analysis on that.

(Refer Slide Time: 41:30)



Plastic deformation of single crystal

Slip: Sliding of planes of atoms over one another during plastic deformation of crystalline materials

Slip System:

Schmid's law:

$$\tau_c = \sigma_x \cos\lambda \cos\emptyset$$

τ_c = Critically resolved shear stress (CRSS)

$\tau_{nd} = \pm\tau_c$ (Forward or backward)

$\tau_{nd} = l n_x \, l d_x \, \sigma_{xx}$

$\therefore l d_x = \cos\lambda$

$l_{nx} = \cos\emptyset$

$\therefore \sigma_x = \frac{\tau_c}{\cos\lambda\cos\emptyset} = \frac{\pm\tau_c}{m_x}$ where m_x = Schmid factor $= \cos\lambda\cos\emptyset$

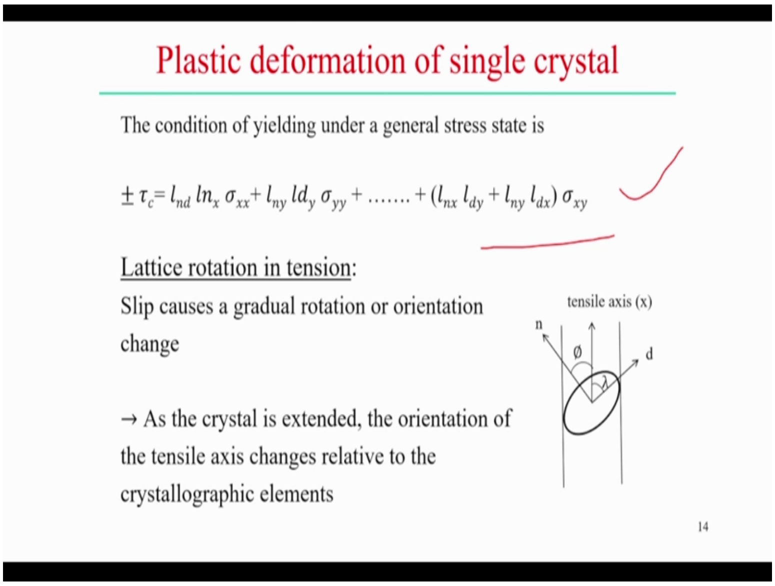
Now we start with the slip process but we have already discussed this thing but we try to represent the different to make the understand the plastic deformation in case of the single crystal structure. So slip we understand the sliding of the planes of the atoms over one another during the deformation of the crystalline materials. Now when you try to do that for the slip system we define this Schmid's law is this one, tau c probably that tau c is the critically or resolved shear stress is linked with the tensile stress, sigma x cos lambda cos phi, that slip direction normal to the slip plane and this is the application of the load or tensile axis along which the direction the tensile stress is acting.

So with this configuration this is the Schmid's law that we have already derived this law and we can find out the actual value of the, or critical value or resolved shear stress value on the to cause the slip of the atoms along the specified direction, here it is suppose the direction d. So when this tau c is the critically resolved shear stress value then it starts the yielding of the material. Now we can find out in other way that see here we can see the tau nd plus minus tau c we have considered here.

So tau nd we find out that in the stress system this is the shear stress value and another is the, tau nd the shear stress value and sigma xx is the suppose normal stress value so that we link in terms of the direction cosine lnx or ldx, so nx or dx that link with the axis d and axis x or between the axis n nor axis x. So that with a definition of the direction cosine we just simply linking the shear stress and the normal stress value. So here it is well defined that ldx or lnx, ldx probably we can cos lambda and lnx is basically cos phi.

So once we do that we can find out the sigma x equal to tau x by cos lambda cos phi and basically plus minus tau c by mx so plus minus we can consider the slip may occur when you consider the critically resolved shear stress maybe forward or backward direction it may move depending upon the loading condition so that is when we consider wither tensile or compression accordingly we can find out that plus and minus we have consider. Now if we see that sigma x the normal stress value equal to tau c by some factor cos lambda cos phi. So that factor we represents at mx that is called the Schmid factor and we represent this as a cos lambda cos phi that is the Schmid factor in this case.

(Refer Slide Time: 41:34)



Plastic deformation of single crystal

The condition of yielding under a general stress state is

$$\pm\, \tau_c = l_{nd}\, ln_x\, \sigma_{xx} + l_{ny}\, ld_y\, \sigma_{yy} + \ldots\ldots + (l_{nx}\, l_{dy} + l_{ny}\, l_{dx})\, \sigma_{xy}$$

Lattice rotation in tension:

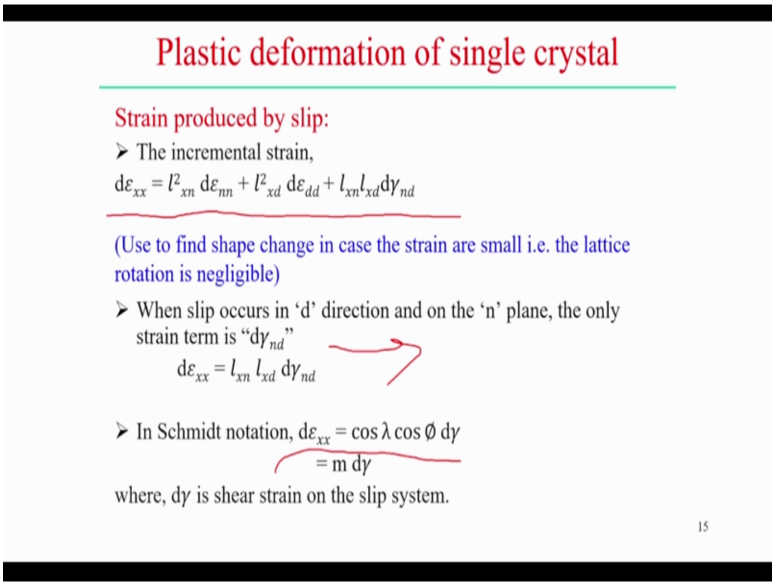
Slip causes a gradual rotation or orientation change

→ As the crystal is extended, the orientation of the tensile axis changes relative to the crystallographic elements

So with this calculation if we see the condition of the yielding under a general stress state that can be written like that, the plus minus tau c the critically resolved shear stress is in terms of the state of the general state of the stress so here we see the normal stress component and the shear stress component and we find out all this component in terms of the direction cosine and this is the general law of that. Now when you try to predict the yielding under a general state of the case, in this case probably we are not considering the lattice rotation during the tension, that is the one significant phenomena associated with the slip, because slip causes a gradual rotation or there maybe the orientation change.

So as the crystal is extended the orientation of the tensile axis actually changes related to the crystallographic elements. So that lattice rotation actually with this theory we are neglecting the effect of the lattice rotation but we will try to explain the linking of the lattice rotation with a uhh plastic deformation with a slip system with a slip mechanism later on by just simply looking into the deformation different, plastic deformation theory. So here the significant point is that although we are predicting this simple yielding behavior in terms of the strain by linking into that general state of the stress but lattice rotation we are not considering here.

(Refer Slide Time: 46:58)



Plastic deformation of single crystal

Strain produced by slip:

- The incremental strain,

$d\varepsilon_{xx} = l^2_{xn}\, d\varepsilon_{nn} + l^2_{xd}\, d\varepsilon_{dd} + l_{xn} l_{xd} d\gamma_{nd}$

(Use to find shape change in case the strain are small i.e. the lattice rotation is negligible)

- When slip occurs in ‘d’ direction and on the ‘n’ plane, the only strain term is “$d\gamma_{nd}$”

$$d\varepsilon_{xx} = l_{xn}\, l_{xd}\, d\gamma_{nd}$$

- In Schmidt notation, $d\varepsilon_{xx} = \cos\lambda \cos \emptyset\, d\gamma$

$$= m\, d\gamma$$

where, $d\gamma$ is shear strain on the slip system.

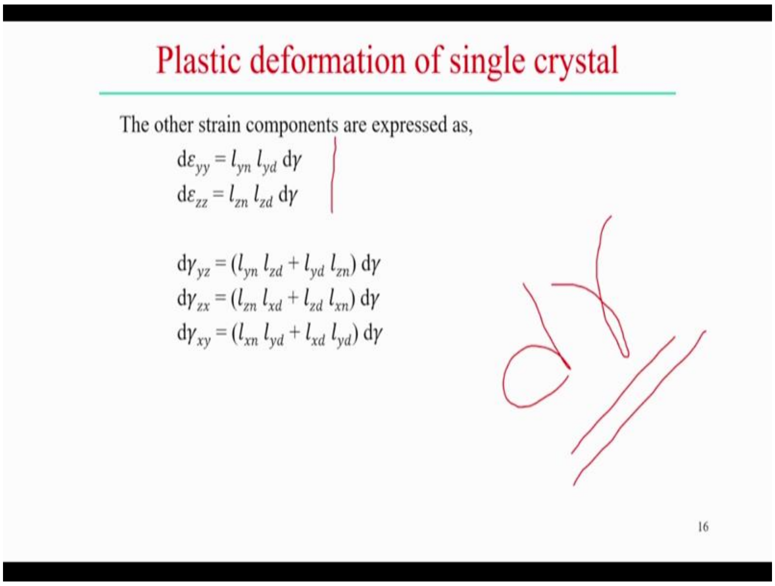
Now we can further simplified calculation on that, that what is the amount of the strain produced by slip. So previously we tried to link the amount of the stress, now we try to link with the amount of the strain produced by the slip, so in this case probably in the plastic deformation, always we try to predict the strain in terms of the incremental mode. So that incremental strain d Epsilon xx so that means along the x direction what is the normal strain component in the incremental form we can find out by looking into the simple direction cosine between the different axis and what are the component of the strength we can find out that.

So here if we see that the three component are there, the Epsilon nn, d Epsilon dd, and d gamma nd. So basically on the slip plane two normal strain components and one is the shear strain component and associated the direction cosine we can find out the incremental amount of the strain along the x axis this, d sigma, d Epsilon xx in this case. Definitely this formula or this correlation is actually used to find out the safe the change in case of specific, in case of the small strain tension where the lattice rotation is negligible. Now if the slip occurs in d direction and on the n plane the only strain terms exist d lambda nd.

That actually define internally when you try to explain the slip mechanism for a single crystal structure so that occurs due to the amount of the shear stress and corresponding the amount of the shear strain and that is the only shear strain associated with this then we can link the amount of the normal stress along xx direction is only in terms of the shear stress component d lambda nd, of course all these cases we are representing in terms of the incremental form.

Now the Schmid notation it says that Epsilon xx can also be represented like that, the factor cos lambda cos phi basically if we look into that uhh we represents the direction cosine here and we can find out the simply m into d lambda, so here d lambda is the shear strain on the slip system, so only the shear strain component on the slip system d lambda and we can represent the amount of the stress in terms of the Schmid factor m and the corresponding incremental amount of the shear stress on the specific slip system.

(Refer Slide Time: 47:01)



Plastic deformation of single crystal

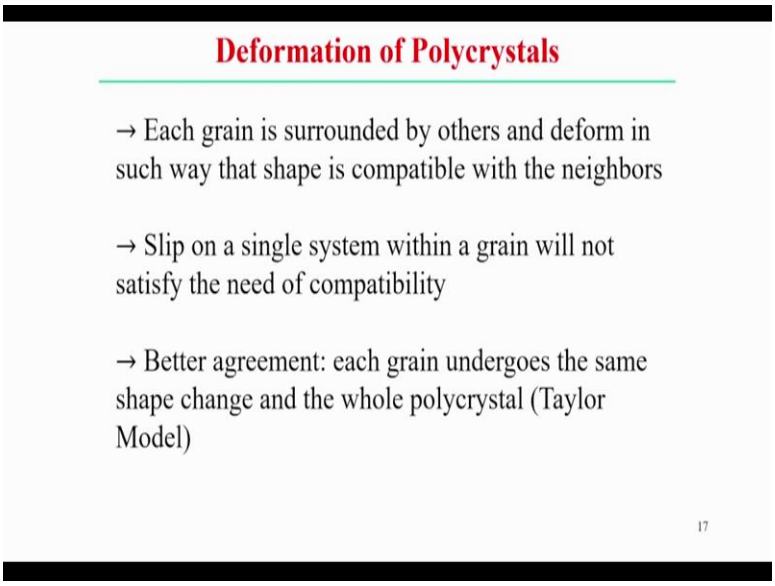
The other strain components are expressed as,

$$d\varepsilon_{yy} = l_{yn}\, l_{yd}\, d\gamma$$
$$d\varepsilon_{zz} = l_{zn}\, l_{zd}\, d\gamma$$

$$d\gamma_{yz} = (l_{yn}\, l_{zd} + l_{yd}\, l_{zn})\, d\gamma$$
$$d\gamma_{zx} = (l_{zn}\, l_{xd} + l_{zd}\, l_{xn})\, d\gamma$$
$$d\gamma_{xy} = (l_{xn}\, l_{yd} + l_{xd}\, l_{yd})\, d\gamma$$

So we can do the further expression on the further component on the strains is like that d Epsilon yy d Epsilon zz in terms all actually representing in terms of the that one constant value here that amount of the shear strain d lambda and we representing the all amount of the strain component in terms of the d lambda that is actually happening during the plastic deformation of single crystal structure and we can use this correlation to find out the component externally to other different different direction or in a x, y, z the coordinate system in terms of the internal shear strain within the crystal structure.

(Refer Slide Time: 47:59)



Deformation of Polycrystals

→ Each grain is surrounded by others and deform in such way that shape is compatible with the neighbors

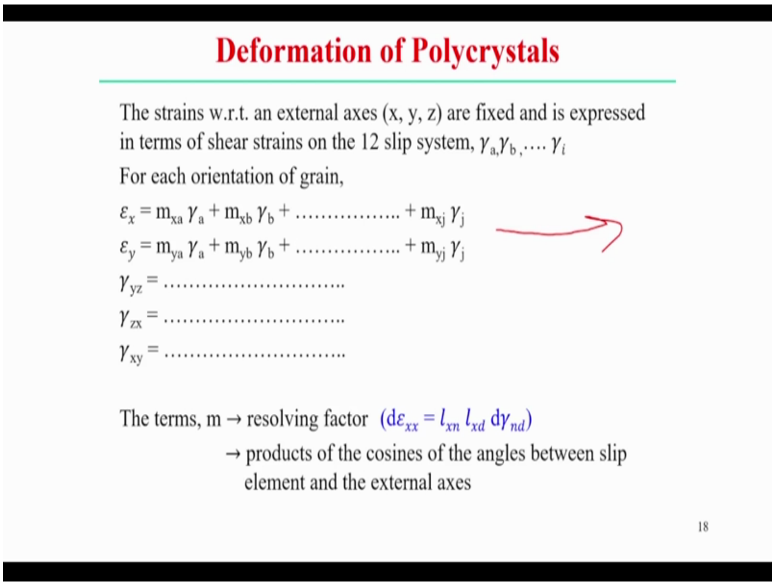
→ Slip on a single system within a grain will not satisfy the need of compatibility

→ Better agreement: each grain undergoes the same shape change and the whole polycrystal (Taylor Model)

So after looking into this single crystal deformation behavior but when we try to analyze the deformation behavior of the polycrystal, here we see that each grain is surrounded by others and deform in such way that shape is compatible with the neighbors. So of course when you try to analyze the deformation behavior in aggregate of the several grains at a time then there maybe the compatible issue with the respect to the surrounding neighbors. So you have to keep in mind this phenomena when you try to analyze the deformation behavior of the polycrystal.

Second point is that slip on a single system within a grain may not satisfy the need of the compatibility, so we have to look into that how we can satisfy the better way the compatibility issue in case of the plastic deformation of polycrystal. Third point is the better agreement can also achieve if each grain undergoes the same amount of the shape change that is the generally assumed by in case of Taylor Model. So we will try to mathematically represents this polycrystalline behavior with the application of the plastic deformation loading condition.

(Refer Slide Time: 50:10)



Deformation of Polycrystals

The strains w.r.t. an external axes (x, y, z) are fixed and is expressed in terms of shear strains on the 12 slip system, $\gamma_{a,}\gamma_{b}, \ldots \gamma_i$

For each orientation of grain,

$\varepsilon_x = m_{xa}\gamma_a + m_{xb}\gamma_b + \ldots\ldots\ldots\ldots\ldots + m_{xj}\gamma_j$

$\varepsilon_y = m_{ya}\gamma_a + m_{yb}\gamma_b + \ldots\ldots\ldots\ldots\ldots + m_{yj}\gamma_j$

$\gamma_{yz} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$\gamma_{zx} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$\gamma_{xy} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

The terms, m → resolving factor ($d\varepsilon_{xx} = l_{xn}\, l_{xd}\, d\gamma_{nd}$)

→ products of the cosines of the angles between slip element and the external axes

So the strain with respect to an external axis x, y, z are fixed and it is expressed in terms of the shear strain on the 12 slip system. So if we assume that the 12 and specific now 12 slip system we consider probably in case of FCC structure that exist the 12 slip system so assume that in case of the FCC structure we consider the 12 slip system and each slip system internally we define the amount of the shear strain is the gamma a gamma b or corresponding to the total stress components is there.

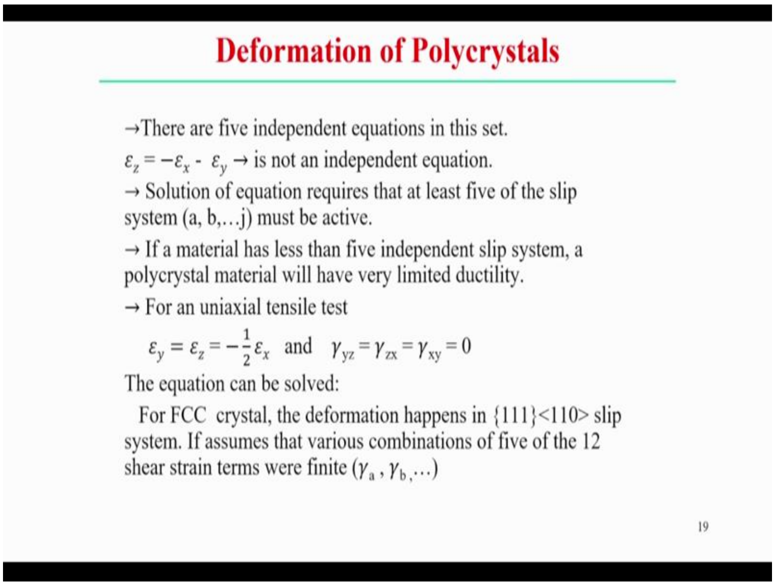
Now for each orientation of the grain, corresponding to the ach orientation of the grain we represents the amount of the normal stress with respect to that resolving factor or probably that link the direction cosine with respect to the different axis we see that state of the equation is represents is like that Epsilon x equal to some resolving factor m and that corresponding the all the different slip plane, the corresponding amount of the shear strain. So with the different orientation we can possibly form that Epsilon x Epsilon y, gamma yz, zx, xy, so two normal strain and three shear strain component we can represent the system of the equation.

But here in this case if you see there is absence of the z component, Epsilon z. Probably that will come later on but in this case if we see that m is basically represent the resolving factor and that resolving factor exactly comes from the nature of the direction cosines between the different axis system. So that resolving factor actually product of the cosines of the angles between the slip elements and the external axis, so m is can be defined.

Now with a five system of equation, there are five independent equation in the system of the equation but we have not considered the Epsilon z in that equation, if we see that Epsilon z is not considered here because Epsilon z is not an independent system, because compatibility conditions or maybe plastic incompressibility condition or if we assume there is no volumetric change probably in this case the summation of the all the strain component equal to 0, so from that condition Epsilon z actually not independent equation, it is depends on the Epsilon x and the Epsilon y. So we have not considered the independent set of the equation uhh as Epsilon z.

Now if you see there are five system of the solution of the equation required at least five of the slip system must be active. So then if the material has less than 5 independent slip system then a polycrystalline material may have very limited ductility but if you want to solve this equation at least we need to define, at least five of this slip system should be active, that’s why the very first few slides we already mentioned that 5 independent slip system is required. So this is the reason here we have tried to explain mathematically that five system to be active.

(Refer Slide Time: 53:33)



Deformation of Polycrystals

→There are five independent equations in this set.

$\varepsilon_z = -\varepsilon_x - \varepsilon_y$ → is not an independent equation.

→ Solution of equation requires that at least five of the slip system (a, b,…j) must be active.

→ If a material has less than five independent slip system, a polycrystal material will have very limited ductility.

→ For an uniaxial tensile test

$$\varepsilon_y = \varepsilon_z = -\frac{1}{2}\varepsilon_x \quad \text{and} \quad \gamma_{yz} = \gamma_{zx} = \gamma_{xy} = 0$$

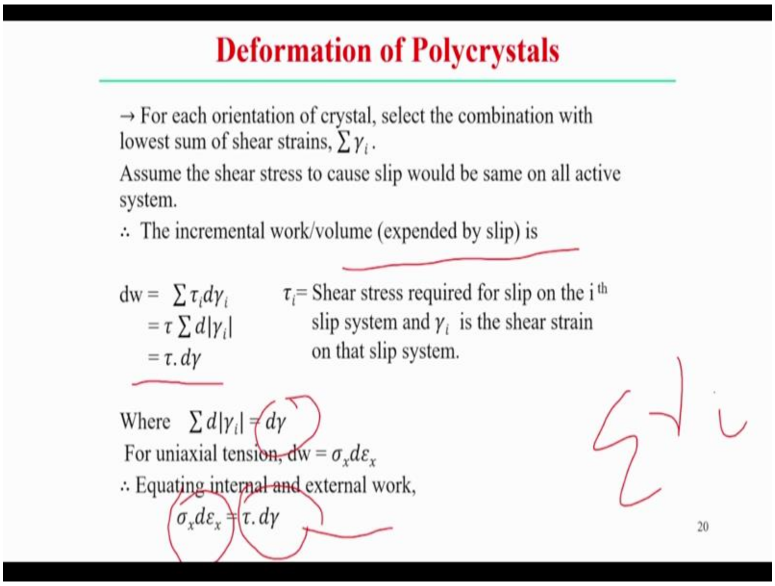
The equation can be solved:

For FCC crystal, the deformation happens in {111}<110> slip system. If assumes that various combinations of five of the 12 shear strain terms were finite ($\gamma_a, \gamma_b, \ldots$)

Now for an uniaxial tensile stress probably we can correlate the amount of the Epsilon y Epsilon z in terms of the Epsilon x and or the shear stress component are zero, so with this information we can it is possible to solve the equation. So here I am not explaining how to solve this equation for a specific system just to know the basic idea that system of equation can be solved in case of polycrystalline material when we consider the several slip system is active during the plastic deformation of that.

But the FCC crystal the deformation happens on the 111 plane and specifically 110 in that slip system. But if we assume that various combination of the five of the 12 shear strain terms is the finite term so out of the 12 slip system probably we can assume having the five combination, the finite.

(Refer Slide Time: 57:19)



Deformation of Polycrystals

→ For each orientation of crystal, select the combination with lowest sum of shear strains, $\sum \gamma_i$.

Assume the shear stress to cause slip would be same on all active system.

∴ The incremental work/volume (expended by slip) is

$$dw = \sum \tau_i d\gamma_i = \tau \sum d|\gamma_i| = \tau . d\gamma$$

τ_i = Shear stress required for slip on the i th slip system and γ_i is the shear strain on that slip system.

Where $\sum d|\gamma_i| = d\gamma$

For uniaxial tension, $dw = \sigma_x d\varepsilon_x$

∴ Equating internal and external work,

$$\sigma_x d\varepsilon_x = \tau . d\gamma$$

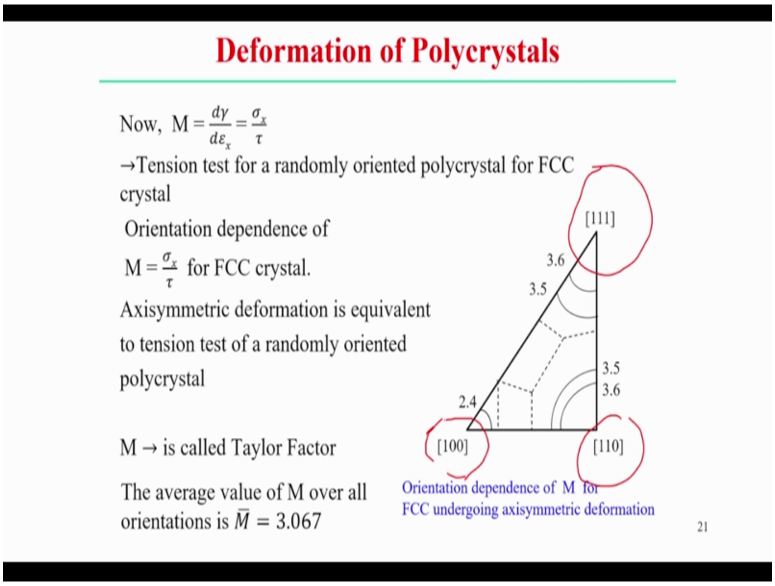
$\sum \gamma_i$

Then for each orientation of crystal, we can select the combination with the lowest sum of the shear strain. So that is the summation of the I, that is represents the lowest sum of the shear strain and out of the 12 system five system is at least active. Now assume the shear stress to cause the slip, maybe the same for all the active system, so amount of the shear stress to cause actually slip that assume which is active for, which is same for all the active system. Now definitely incremental work in terms of per unit volume, that is the expended that unit volume expended by the, so that is restricted or that is expended by the slip.

So that can be represented dw equal to simply the amount of the work done we can say the stress into strain, so that represent the amount of the work done per unit volume. So stress into strain we represent and this finally we can find out that t into d lambda so in this case shear stress actually required for the slip of the I slip system and gamma is the shear strain on the specified slip that is the representation of the tau i. Now summation of the d lambda I so basically now we try to represent the five active slip system there is a small amount of the slip and the summation of this thing all having the equal effect and we can say it is the summation of each basically d lambda.

Now from the external uniaxial tension we can find out the amount of the work done per unit volume at the same time the normal stress into the normal strain. But here the in terms of the incremental volume d Epsilon x here. Now equating the internal work done and the external work done we can find out this correlation, sigma x d sigma x and tau equal to dy. So tau dy is basically it is a theoretical prediction from the theory we can find out the amount of the incremental work per unit volume, this and left hand side that actually represents that amount of the work done, when we step forward we can do the tensile test of a specimen.

(Refer Slide Time: 59:33)



Deformation of Polycrystals

Now, $M = \frac{d\gamma}{d\varepsilon_x} = \frac{\sigma_x}{\tau}$

→Tension test for a randomly oriented polycrystal for FCC crystal

Orientation dependence of $M = \frac{\sigma_x}{\tau}$ for FCC crystal.

Axisymmetric deformation is equivalent to tension test of a randomly oriented polycrystal

M → is called Taylor Factor

The average value of M over all orientations is $\bar{M} = 3.067$

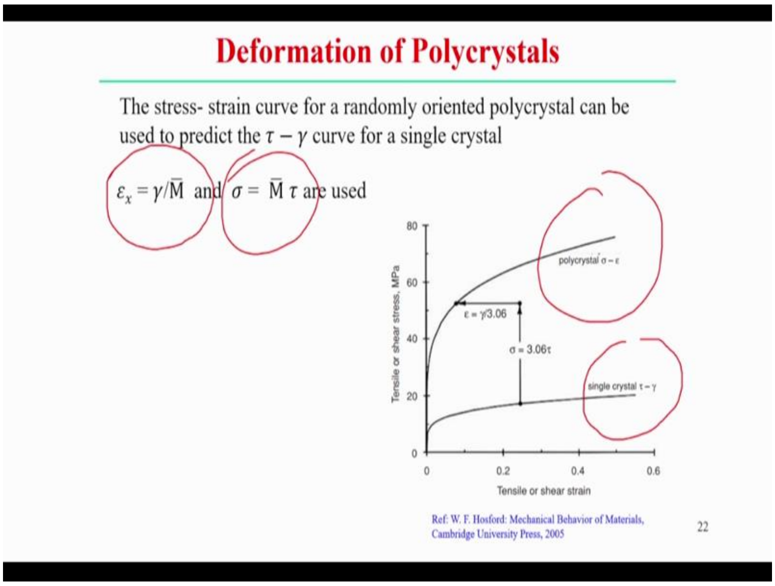
Orientation dependence of M for FCC undergoing axisymmetric deformation

So with this correlation we can find out there is a m equal to d lambda by d Epsilon x and sigma xy tau. So that is actually one factor that is called Taylor Factor, capital M. So in this case tension test can be randomly done, can be randomly oriented FCC crystals and orientation dependence of Taylor factor can be represents the amount of the sigma x by tau in case of the FCC crystals. Now Axisymmetric deformation is basically equivalent to tension test of a randomly oriented polycrystal. So if we see that similar way the orientation dependence of m that is the factor, Taylor factor for the FCC that is undergoing the Axisymmetric deformation which is equivalent to the tension test for a randomly oriented polycrystal and the three direction 100, 110 and 111, then basic of the corner of a triangle.

Now within that here it is possible to predict the distribution of the m in terms of the contour or isocontour can be seen from the figure that represent the distribution of the m and if you see you see that if you look into the distribution, specifically for FCC crystal structure, here we can find out the average value of m, m equal to 3.067 and that is estimated the average value, so looking into the several possible orientation of the polycrystalline start which direction the stress is actually applied, or plastic deformation of the polycrystal occurs.

So once we estimate the Taylor factor from the work done principal of the external uhh internal work, so we can predict that stress strain diagram or we can link the stress strain diagram of the polycrystalline form with respect to the single crystal structure.

(Refer Slide Time: 59:49)



Deformation of Polycrystals

The stress- strain curve for a randomly oriented polycrystal can be used to predict the $\tau - \gamma$ curve for a single crystal

$\varepsilon_x = \gamma/\bar{\mathrm{M}}$ and $\sigma = \bar{\mathrm{M}}\,\tau$ are used

Ref: W. F. Hosford: Mechanical Behavior of Materials, Cambridge University Press, 2005

22

Now if we look into that stress strain curve of randomly oriented polycrystal can be used to predict the shear stress or shear strain curve for a single crystal by simply looking into, by simply knowing the Taylor factor and that Taylor factor is the average. So when you compare or when you try to predict the strain diagram for a single crystal structure we can use the average value of the Taylor factor for a specific crystal structure material. So that Taylor factor use this relation between the stress and strain in terms of the Taylor factor, normal strain, sorry this is strain shear strain and normal strain and this is the normal stress versus shear stress in terms of the Taylor factor.

So from right hand side figure also shows the same thing that the polycrystalline behavior of polycrystalline stress versus strain figures is shown here and corresponding the single structure stress strain behavior and this represents the polycrystal and using this relation we can find out or we can link between the stress strain behavior of the single crystal structure or stress strain behavior of the polycrystal structure. This is the most one of the significant point if we possible to find out further crystal structure material, the Taylor factor we can predict the stress strain behavior of the single crystal structure material as compared to the polycrystalline material by simply using the average value of the Taylor factor over the specific orientation of the three different in between the three different direction of a cubic unit cell.

So with this uhh I can stop today with the analyze of the deformation of the polycrystalline behavior. Now next step we will try to focus on the constructive model for the single crystal structure or how the constructive model can be done in case of the single crystal structure or polycrystalline structure and which is almost in the similar direction what we discuss for the continuum plasticity, there we have seen that different constructive model can be formed or different plasticity model can be formed based on the theory, in the similar direction we can form in case of the single crystal structure.

Thank you very much for kind attention.