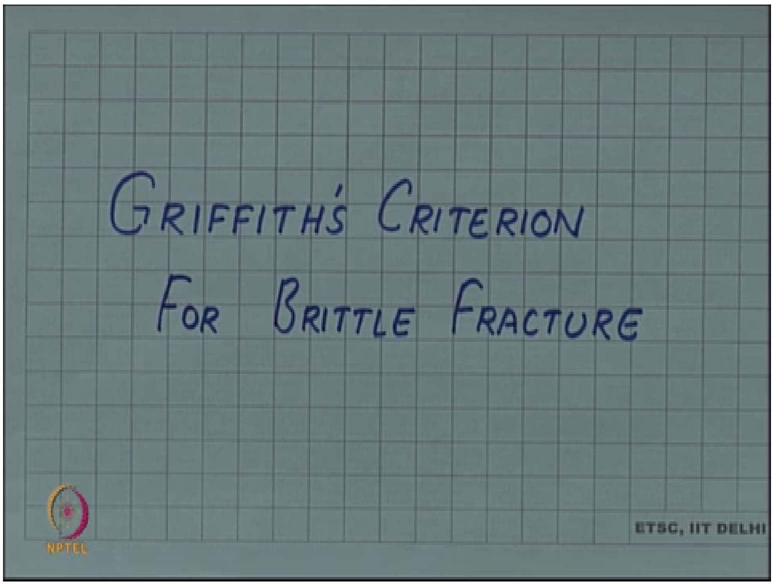
Introduction to Materials Science and Engineering
Prof. Rajesh Prasad
Department of Applied Mechanics
Indian Institute of Technology, Delhi

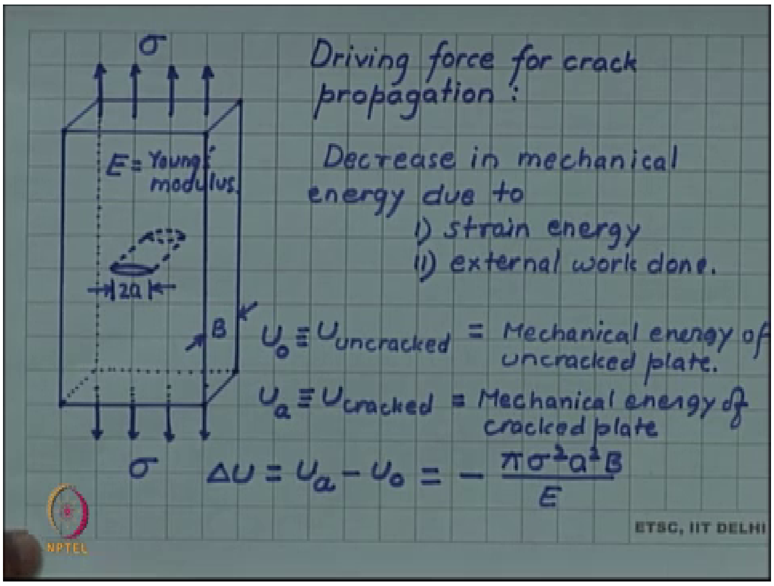
Lecture - 140
Griffith's Criterion

(Refer Slide Time: 00:06)



We saw in the last video through experiments on paper that the fracture stress depends upon the crack size. We also saw that fracture stress depends upon material property probably stiffness. The first person to establish quantitative relationship between a fracture stress and these quantities cracker size and stiffness was Griffith. And he did it for brittle fracture or brittle solids and is known as Griffith's Criterion for brittle fracture.

(Refer Slide Time: 00:43)



So, we will try to establish this criterion in this video. So, the first question is driving force for the crack propagation. Here in this figure I am showing a plate; plate of thickness let us say B, and we assume there is a very wide plate and there is a nominal stress sigma applied to the plate at its end and in the center of the plate there is a crack and I characterized the crack by its length and I call the length 2a.

So, there is a sharp crack of length 2a in the material and if I pull this sheet we have already seen that if sufficient stress is not there will be no crack propagation no fracture, fracture happens only at a critical stress. So, the question is what is driving the crack propagation? Why does the crack propagate? So, if we think in terms of energy we know that some sort of energy some sort of potential energy should decrease for any natural process and if under stress crack is propagating it is a natural process.

So, what is that what is that energy? What is that driving force for crack to propagate? What energy is decreasing? So, Griffith could see by his analysis that the energy which is decreasing in case of crack propagation is the mechanical energy of the system.

So, the driving force for crack propagation is decrease in mechanical energy. Mechanical energy has 2 components we will not go into the details of this. So, we just write decrease in mechanical energy due to strain energy, elastic strain energy and external work done.

So, since this mechanical energy reduces or decreases let us write that U uncracked is mechanical energy of uncracked plate and U cracked will mechanical energy of cracked plate, maybe I use a simpler symbol. So, for uncracked let me write U o that is crack of 0 length is there and in crack I write U a that is crack of length 2a is there.

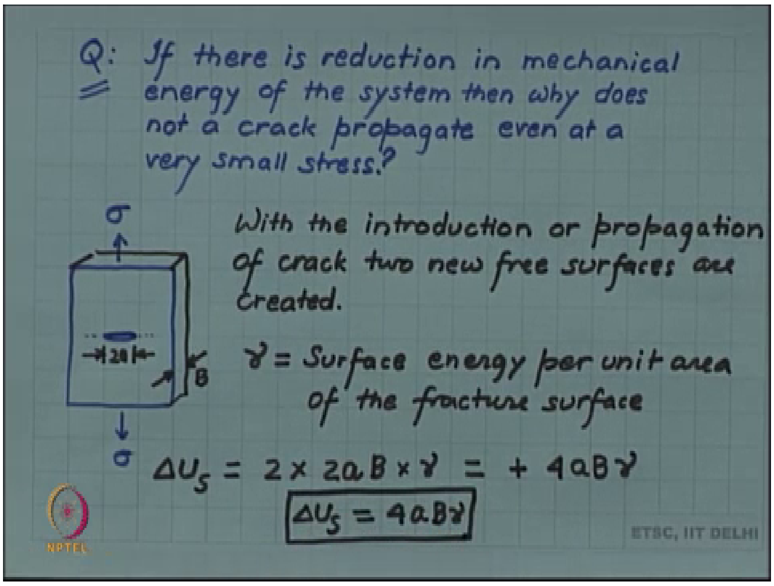
Now, that so the change in mechanical energy the change in mechanical energy due to the introduction of the crack can be written as delta U, which is U with crack minus U without crack and this can be calculated in terms of the stresses, the crack size, the plate thickness and the elastic property of the material which in this case is the Young's modulus.

So, let me write E as the Young's modulus of the plate, E is the Young's modulus. So, in terms of this parameter if I if it is calculated now the calculation is much more involved, it is a problem in solid mechanics or continuum mechanics. We will not get into that we will simply take the result from the calculation of Griffith and that is minus pi sigma square a square B by E.

So, this is the difference in the mechanical energy; negative sign shows that the mechanical energy is decreasing and mechanical energy is decreasing by this much amount that is it is pi times a square of stress times square of the crack length or half crack length times the thickness divide by the Young's modulus.

So, higher is the applied stress more will be the decrease in elastic energy, higher is the crack length more will be the decrease in elastic energy, higher thickness will also lead to greater decrease in elastic energy, but higher modulus leads to lower decrease in elastic energy.

(Refer Slide Time: 07:33)



So, this is the driving force, this is the driving force for crack propagation reduction in mechanical energy. But then the question comes if there is a reduction in mechanical energy and you can see from this formula that for a given crack size whatever is the stress; however, small the stress is they will always be a decrease in energy.

So, whatever be the crack size and however, a small stress I apply on it there will be a decrease in mechanical energy if the crack propagates, but we have just seen in the paper experiment and we know from our experience that even if crack is there the crack will not propagate unless and until the applied stress is certain critical value very small stress; very feeble stress will not led to crack propagation.

So, if there is reduction in mechanical energy of the system then why does not a crack propagate even at very small stress? So, this was an important question, this was an important question which Griffith asked and he came to a very very interesting answer.

What he realized that mechanical energy is only one of the one aspect of the energy of the system. The other aspect of the energy of the system is its surface energy. As the crack will propagate or as you introduce the crack you also introduce 2 free surfaces, 2 surfaces of the crack.

So, let us write that with the introduction of crack, with the introduction or propagation of crack two new free surfaces are created. And we know that every free surface is associated with a positive surface energy per unit area. So, let gamma be the surface energy per unit area of the fracture surface.

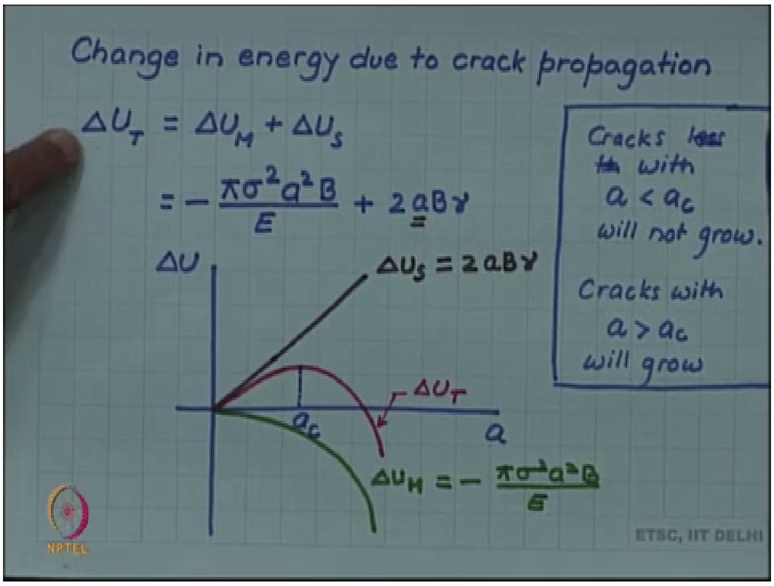
So, this is a very very interesting concept. This is a novel concept, this was novel concept at the time of Griffith, before Griffith no one had realized about this and nobody had introduced this into the analysis of fracture. So, if it is the central importance to Griffith's analysis that he introduced the concept of surface energy of the fractured surface into the analysis of crack propagation.

So, with this we can now write that there is another energy change component let us write that delta U surface. So, if we introduce a crack of length 2a in a plate of thickness B, how much new surface energy we have introduced? So we can see that we have created two surfaces.

So, let me write a factor 2 and each surface is of length 2a, and width B, width equal to the thickness of the plate. So 2aB is the surface area of the upper surface, 2aB is the surface area of the lower surface. So, 2 times 2aB is the total surface area created and since gamma its surface energy per unit area.

So, we see that there is an increase in the energy equal to 4aB gamma due to the crack. So, we call this the surface energy delta U S is equal to 4aB gamma. Look at the previous expression that was a reduction in energy due to the mechanical energy so let us call that delta U M.

(Refer Slide Time: 13:11)



So, now, I have two component of energy change in the system that is a the mechanical energy change delta U M, and the surface energy change delta U S, we saw that we did not derive but we saw that the delta U M can be written as minus pi sigma square a square B by E whereas, the surface energy can be written as 2aB gamma.

Let us try to plot let us try to plot these two energy components into as a function of the crack size. So, let me first write the mechanical energy sorry, first I am plotting the surface energy; surface energy is a easier to plot because it is just proportional to a, So, it is a straight line.

So, I am just drawing a straight line representing delta U S equal to 2aB gamma. The other term delta U M mechanical energy is proportional to a square. So, this will go a parabolically and we can draw it something like this. So, this is delta U M, the mechanical energy which is minus pi sigma square a square B by E.

The sum total delta U T will be the sum of these two. And how the graph of that will look like? You can see near the origin delta U M is very small because the slope is 0 whereas, delta U S is of constant slope. So, initially the total will have a slope equal to that of delta U S, but gradually you can see that although delta U S is increasing with constant slope, delta U M is decreasing with increasing slope.

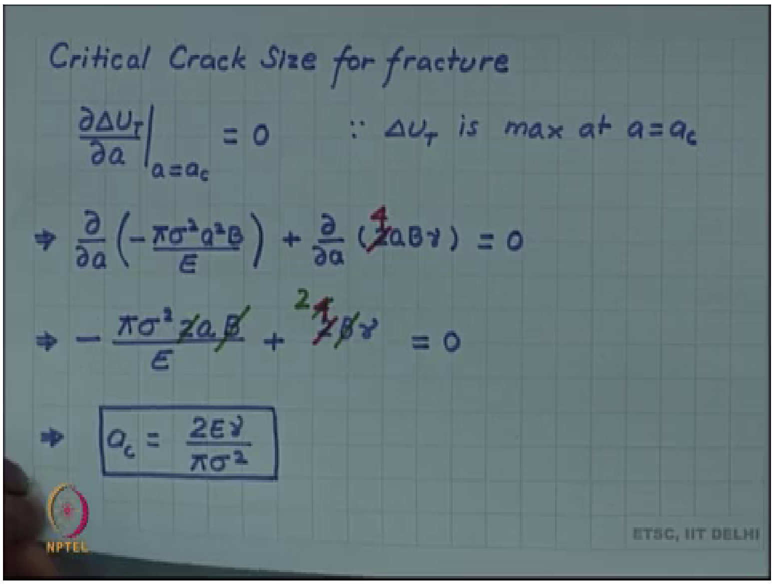
So, gradually it will start becoming more and more dominant and finally, it will pull the curve down towards itself. So, this is delta U T. You can see now that there is in this curve there is a maximum. Let me call the crack size at which this maximum happens as a c, the critical crack size.

So, if we now think of the situation if you have a crack size less than a c and if you think of it growing then the energy of the system will increase along the red curve. So, a crack size is smaller than a c should not grow because the energy of the system will increase. So, cracks less than let me write it here; cracks or other cracks with a less than a c will not grow.

Whereas, if you have a crack size larger than a c and if you allow it to grow you can see that the energy of the system will now drop along with the red curve. So, cracks with a greater than a c will grow; that is a very important conclusion from this analysis.

Now, let us try to try to get an expression for a c. Now this is easy to do that because we already have expression for delta U T and a c is the value of a, at which delta U T is maximum.

(Refer Slide Time: 18:41)

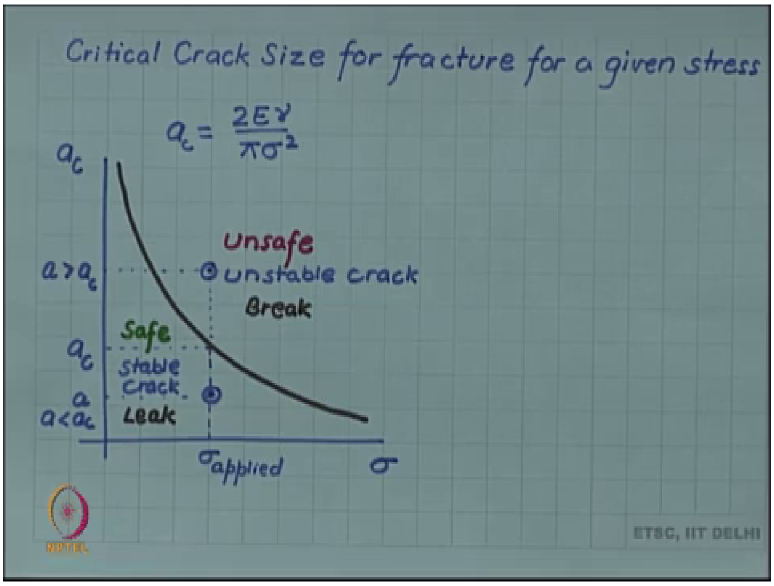


So, critical crack size is where because delta U T is maximum at a is equal to ac. So, we can easily do this differentiation, there are two simple terms; the mechanical energy term and the surface energy term and both of them are simple function one is a square and other is a.

So, we just differentiate a square differentiate such 2a. I had just made a mistake here; here also I had made a mistake. So, let me correct that let me correct that this is not 2aB gamma, this is 4aB gamma, this is also 4aB gamma, so after differentiation we get this.

And if we now solve it for so here we you can see B will cancel with B and 2 will cancel with this 2. So, you will have a c has 2 E gamma by pi sigma square. So, you have an expression for a critical crack size in terms of the properties of the material, the elastic property of modulus and the surface energy as well as the mechanical stress which you have applied sigma. So, we have got an expression for critical crack size and let us see what are the consequences of this expression.

(Refer Slide Time: 20:36)



If we plot this curve, if we plot this curve a c as a function of sigma you can see with increasing sigma it will keep decreasing. So, you have you have a curve like this. How do we interpret this curve?

Let us see this is telling us that for a given stress. So, let us let us assume that we have this stress sigma applied suppose we are applying this stress and then this is for that applied stress, this is the critical size, this is the critical size. So, if the actual crack size is let us say this, if the actual crack size is this then we can see that for this if this is the actual crack size a, so a is less than a c and we have already seen that for such cracks such cracks will not propagate for this applied stress.

So, this will be the case everywhere below the curve. If you have a point below this curve the crack of that length will not be growing or increasing in size for the corresponding applied stress. So, this region below this curve is a safe region. So, let me write this with green this is safe region; however, for the same applied stress if we have a crack size which is higher, if we have this crack size which is greater than a c then obviously, this crack according to our analysis or according to Griffith's analysis this crack will propagate and we will call fracture.

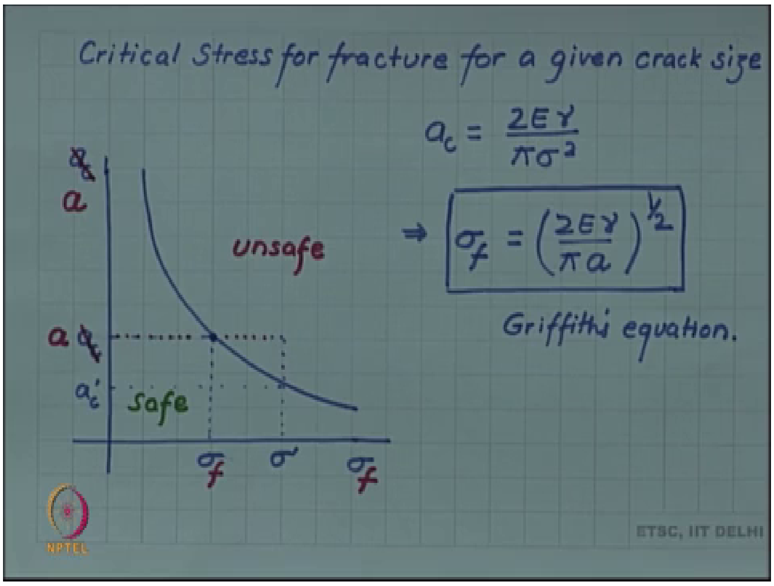
So, this region is unsafe let me mark it with red and this will be true for everywhere above this black line, so this is an unsafe region. So, in the a c sigma domain this line a c versus sigma line divides the a c sigma space into 2 domains; the unsafe domain and the safe domain.

We can also write this as unstable crack because this crack will propagate, whereas, this we can write as stable crack; this crack will not propagate. Thus here the component will break where here although the crack may be present, but the component may not break and if it is a component like a pressure vessel then it will have leak. So, it will leak, but it will not break and here above the curve it will break.

So, this is an important information coming from Griffith's analysis of the crack problem. And you can see that the stress here in from this expression you can see a stress was the driving force for crack propagation. So larger is the stress smaller is the critical size.

So, if you have larger driving force even smaller cracks can propagate but the surface energy was in a sense a barrier or obstacle to crack propagation. So, larger is the surface energy higher is the critical crack size. So, if surface energy is larger only a small cracks will not be able to propagate only correspondingly larger cracks will be able to propagate.

(Refer Slide Time: 25:27)



We can look at this curve in a slightly different light. Instead of looking at it instead of looking at it as critical crack size as a function of given a stress which is what we have drawn here, we can change our point of view and try to look at stress as a function of crack size, critical stress as a function of crack size. And this interpretation is possible because again if you look we said that for a given stress, for a given stress this was the critical crack size, this was the critical crack size.

Now, suppose I change my point of view and I simply call not this as a critical crack size, I do not look at it as critical crack size and I say that I have a component in which this is the crack size and I want to know and I am gradually apply higher and higher stress at what stress it will fracture. So, what is the critical stress for fracture for a component with this crack size?

Now we have already seen that at this stress this was the critical crack size. Now if I start with this stress if I start for a given crack size and I start with 0 stress and gradually increase the stress you can see that I will remain in the safe region we had seen that this was a safe region below the curve. So, I remain in the safe region till I hit the curve again beyond that if I go beyond that if I go I will enter the unsafe region or the crack propagation region.

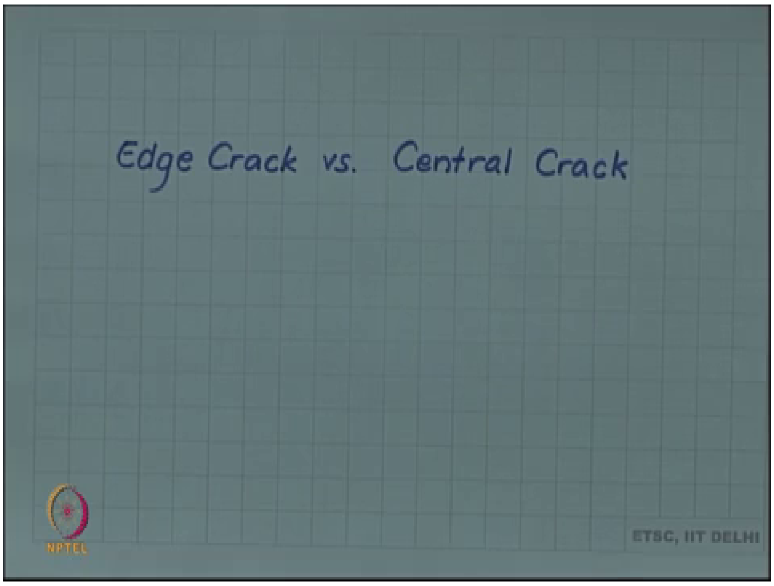
Because for those stresses you can look at it another way that if I have an applied stress this then the critical crack size was much smaller, a c was much smaller and then this a is larger than a c for this applied stress so you will be in the unsafe region. So, the stress dividing the safe and unsafe region is again given by the same curve which we have plotted as a c versus sigma.

So, now, for this given a, this becomes sigma critical which we label as sigma f. So, now, I change the interpretation of this curve instead of calling this an ac versus sigma I call this as a versus sigma f. So, for a given crack size I can find what is the critical crack stress and it is the same equation same curve.

So, I can I can write an expression for sigma f simply by algebraic manipulation of this equation, all I have to do is now solve this for sigma. So, I get 2E gamma by pi a square root. And I write now since I am my interpretation has change. So, sigma has now become the critical stress sigma f and a c has become the given crack length a. So, this expression also is known as Griffith's equation. So, both are Griffith's equation either in terms of the critical crack size or in terms of the fracture stress.

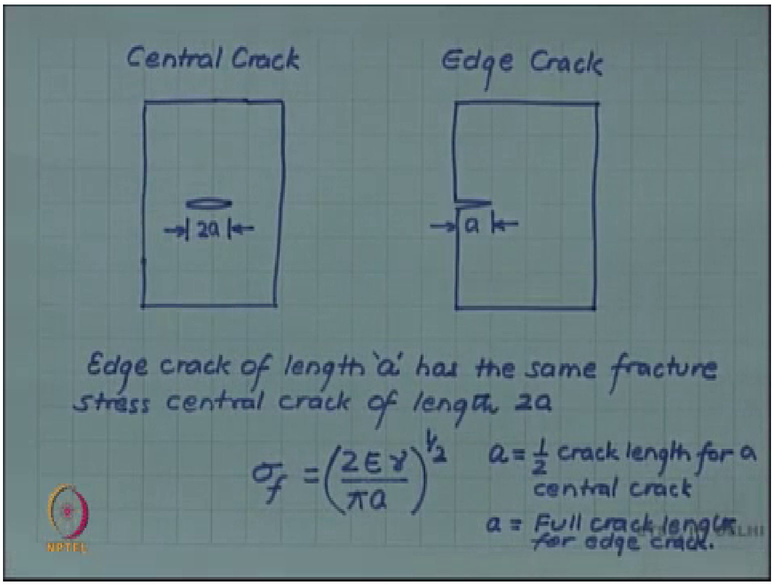
So, to summarize it was Griffith central contribution to realize the importance of surface energy as an important factor in the analysis of crack. So, the mechanical energy this is the driving force for the crack reduction in mechanical energy whereas, increase in surface energy is obstacle to the crack propagation and it is the balance of the two which determines a critical crack size or a critical fracture stress.

(Refer Slide Time: 30:30)



Let us consider one particular situation edge crack versus central crack.

(Refer Slide Time: 30:38)



In the Griffith plate which we analyzed the crack of length 2a, was in the centre of the plate but the paper experiment which I did was with edge cracks. So, how the 2 situations compare? It turns out that actually edge cracks are more dangerous because edge cracks of length a has the same fracture stress at central crack of length 2a.

So, a smaller crack on the surface is as effective in bringing down the fracture stress at a longer crack in the center of the plate. So, we can use the Griffith's formula sigma f is equal to 2 E gamma by pi a square rooted both for either for the central crack or for the edge crack, only the interpretation for a will change and a will be interpreted as half crack length if it is a central crack, but will be full crack length if it is the edge crack.