Sound and Structural Vibration

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Module No # 02
Lecture No # 07
Uncoupled Solution Continued

Good morning welcome to this next lecture on sound and structural vibration. Last time we stopped at this juncture where we found the uncoupled plate vibration.

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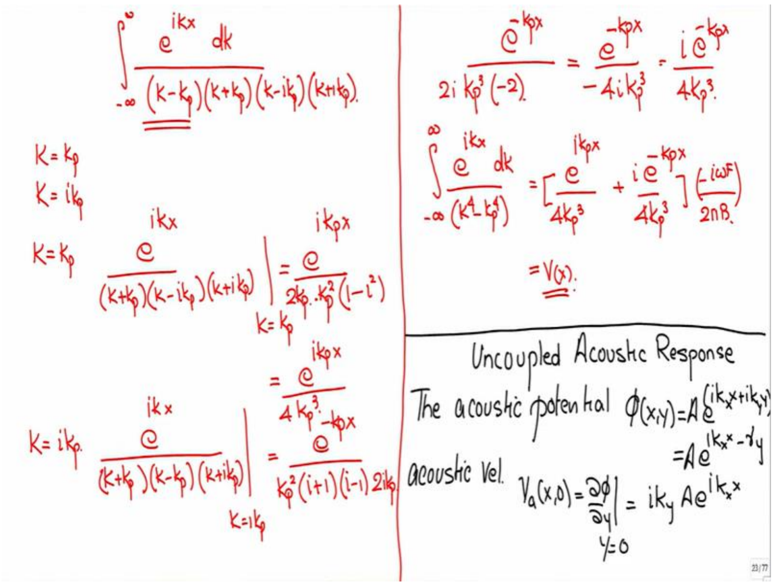

So now we are going to find the uncoupled acoustic response, the acoustic potential was given

$$\phi(x,y) = A\, e^{(ik_x x + ik_y y)} = A\, e^{ik_x x - \gamma y}.$$

So, γ and k_y are related in that manner, the acoustic particle velocity at the plate surface which is

$$v_a(x,0) = \left.\frac{\partial \phi}{\partial y}\right|_{y=0} = ik_y\, A\, e^{ik_x x}.$$

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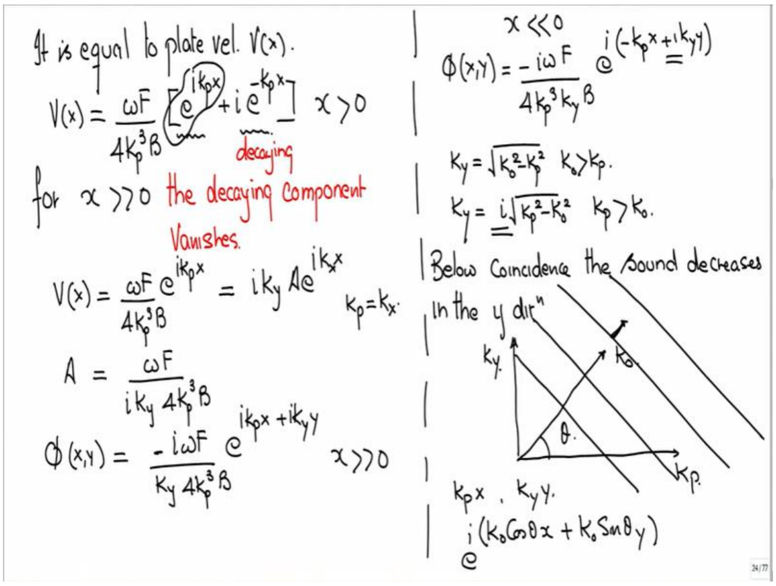

Now this is equal to the plate velocity $v(x)$ and we just found the plate velocity what was that? $v(x)$ was given by

$$v(x) \;=\; \frac{\omega F}{4{k_p}^3 B}\left[e^{ik_p x} + i\, e^{-k_p x}\right]\; for\;\; x > 0.$$

Now there are 2 terms here and we have to find the sound field related to both. But for the moment for x much greater than 0 there is a decaying component.

This component is decaying so we ignore that component the decaying or evanescent decaying component vanishes. So, for the time being we will ignore it so my $v(x)$ is finally

$$v(x) \;=\; \frac{\omega F}{4{k_p}^3 B}\; e^{ik_p x}.$$

And that is at the surface there is a plate velocity as a surface obviously and this is equal to $ik_y\, A\, e^{ik_x x}$,the acoustic particle velocity at $y = 0$.

And of course, we know that k_p must be equal to k_x right the plate wave number must be the x wave number of the acoustic field. And therefore, A turns out to be

$$A \;=\; \frac{\omega F}{4{k_p}^3 B\; ik_y}.$$

which means what my $\phi(x, y)$ is given by

$$\phi(x, y) \;=\; \frac{-i\omega F}{k_y 4{k_p}^3 B}\; e^{ik_p x \,+\, ik_y y}\;\; x \gg 0.$$

If we want for x much less than 0, we have

$$\phi(x, y) \;=\; \frac{-i\omega F}{4{k_p}^3 k_y B}\; e^{-ik_p x \,+\, ik_y y}\;\; x \ll 0.$$

Here we have k_y given by $\sqrt{{k_0}^2 - {k_p}^2}$ when $k_0 > k_p$ and $k_y = i\sqrt{{k_p}^2 - {k_0}^2}$ when $k_p > k_0$ below coincidence. So below coincidence the sound field decreases in the y direction. This i and this i will make it minus and $k_y y$ will decrease in the y direction. Now what do we have? We have essentially a k_x wave number which is k_p and a k_y wave number and both combine to give me the acoustic wave number k_0 and this angle is θ.

So, what we can write is we have $k_p x$ as the phase part of x we have $k_y y$ for the phase part of y so we have $k_0 \cos\theta x + k_0 \sin\theta y$ as the expression for the propagator. So, the sound field in the case of propagation moves as a plane wave at an angle θ to the plate. The k_0 is normal to the wave front and makes an angle θ with the plate this is the part related to the propagating portion of the plate response. And that is the dominant response at larger and larger values of x.

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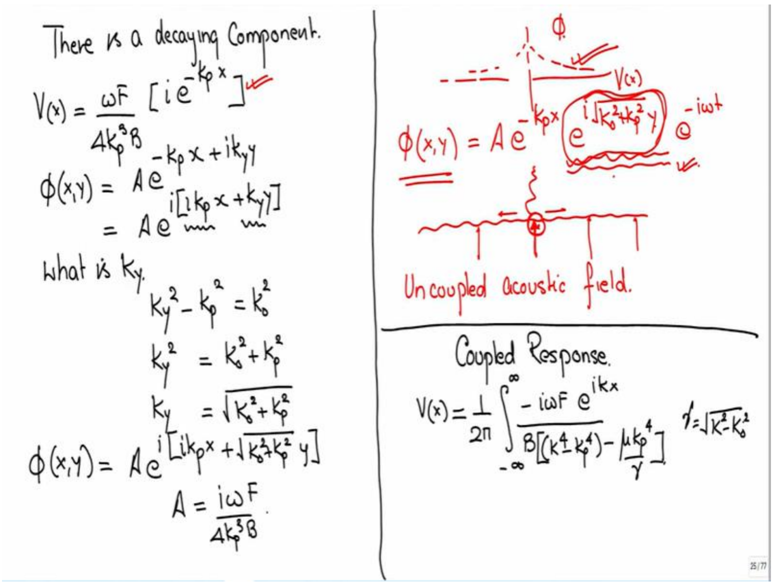

But there is a decaying component there is a decaying component. So, what is that? That is

$$v(x) = \frac{\omega F}{4{k_p}^3 B}\left[ie^{-k_p x}\right].$$

Now phi was given as what?

$$\phi(x,y) = A\, e^{-k_p x + ik_y y} = A\, e^{i[ik_p x + k_y y]}.$$

Now what is k_y? What I have basic wave equation again will give me this square plus this square should be equal to my acoustic wave number.

So, I will get

$$k_y{}^2 - k_p{}^2 = k_0{}^2,$$

$$k_y{}^2 = k_0{}^2 + k_p{}^2,$$

$$k_y = \sqrt{k_0{}^2 + k_p{}^2}.$$

Which means my $\phi(x, y)$ is

$$\phi(x, y) = A\, e^{i\left[ik_p x + \sqrt{k_0{}^2 + k_p{}^2}\, y\right]}.$$

A of course, is given by

$$A = \frac{i\omega F}{4k_p{}^3 B}.$$

Now what has happened is this I have a decaying field from here on the plate from the forcing location I have a decaying field.

This is the velocity field corresponding to that I found the acoustic field and what does; it says the acoustic field has an amplitude and a decaying component just as the plate. But it has a propagating component in the y.

$$\phi(x, y) = A\, e^{-k_p x}\, e^{i\sqrt{k_0{}^2 + k_p{}^2}\, y}\, e^{-i\omega t}.$$

So, the decaying velocity field has produced a propagating field in the y direction in some sense it is not significant because we said we are looking at far field.

However, if you look at a wave coming in on a plate and a discontinuity placed over here like a rib. So, the common sense says that presence of a rib you know improves impedance creates discontinuity reduces vibration reduces the sound it is common sense idea. It is well worth looking into of course but now watch, here the same thing a kind of a discontinuity or a rib placed here produces a propagating component of a sound field.

This decays away in the x direction but there is a propagating component due to a discontinuity that is what this tells you. So, it is now always true that if you rib a vibration plate you will get reduction you have to keep this fact in mind at a discontinuity there can be a y direction propagating field. So that is the sound field due to the decaying part, so the sum of the fields is both.

So that closes the sound field which is uncoupled acoustic field now we have found that my coupled response which is the main interest for us in this problem. The coupled response both in velocity and potential are given by

$$v(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{-i\omega F\, e^{ikx}}{B\left[\left(k^4 - {k_p}^4\right) - \frac{\mu {k_p}^4}{\gamma}\right]}.$$

γ has the form gamma specifically has the form $\sqrt{k^2 - {k_0}^2}$.

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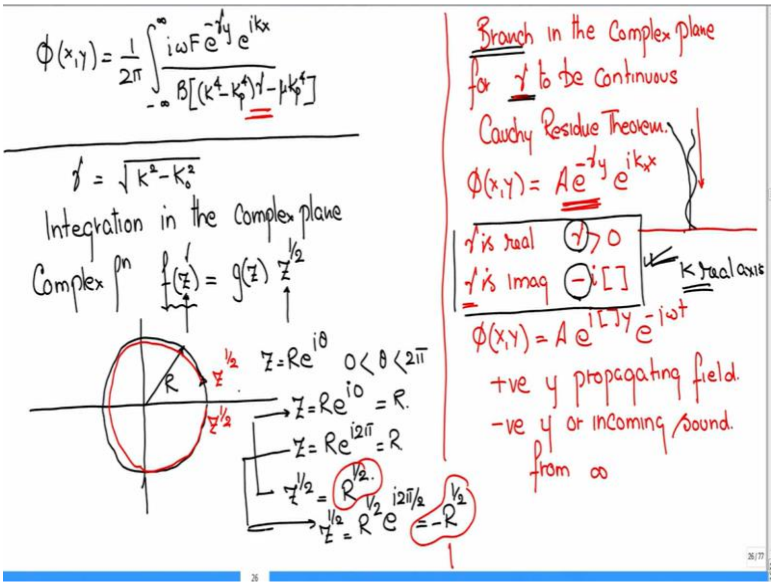

Similarly, the

$$\phi(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{i\omega F\, e^{-\gamma y}\, e^{ikx}}{B\left[\left(k^4 - {k_p}^4\right)\gamma - \mu {k_p}^4\right]}.$$

Now before we get into integrating these we have, look at the γ issue γ is a square root function.

$$\gamma = \sqrt{k^2 - {k_0}^2}.$$

And we are going to resort to integration in the complex domain in the complex plane.

Now to give you a taste of this if I have a complex function let us say $f(z)$ and it has nice continuous analytic functions let us say some $g(z)$ but it has a square root of $z^{\frac{1}{2}}$ also.

$$f(z) = g(z)\, z^{\frac{1}{2}}.$$

So now let us look at it this function $f(z)$ in the complex plane $g(z)$ is well behaved. So, it does not create any problem but look at $z^{\frac{1}{2}}$. So, at this point suppose I am going to integrate on a circular arc full circular I am going to integrate.

So, my z this what parameterized as

$$z = Re^{i\theta} \quad 0 < \theta < 2\pi.$$

Now just above 0 my z the independent variable is R so let us say this is Re^{i0} which is R and z at $Re^{i2\pi}$ = R. So independent variable is continuous now look at the function which is $z^{\frac{1}{2}}$ at z with $\theta\ = 0$ we get $z^{\frac{1}{2}}$ as $R^{\frac{1}{2}}$.

But at z with $\theta\ = 2\pi$ that means you have come full circle

$$z^{\frac{1}{2}} = R^{\frac{1}{2}} e^{i\frac{2\pi}{2}} = -R^{\frac{1}{2}}.$$

So as my function goes around the circle and comes back the functional value $z^{\frac{1}{2}}$ here to $z^{\frac{1}{2}}$ here has changed sign it is discontinuous. And that is the result of square root so now γ has a square root in it both in velocity and acoustic potential.

So, we have to first find a branch in the complex plane explain for γ to be continuous because we will finally use the Cauchy residue theorem and it requires that the function be continuous in analytic in most of the region except at isolated singularities. So, we have to find a branch where γ is going to continuous. In addition, my ϕ had let us say

$$\phi(x, y)\ =\ A\ e^{-\gamma y}\ e^{ik_x x}.$$

And of course, time if you want to put time however the y function the y propagator was like this. So now a γ is real we want the sound field to decay in the y direction. So, then γ must be positive whereas the γ is imaginary then γ must be negative imaginary. What does that do negative imaginary makes it

$$\phi(x, y)\ =\ A\ e^{i[\ ]y} e^{-i\omega t},$$

which is a positive y propagating field.

If γ is imaginary and we take the positive square root that means positive i something it means, there is an incoming negative coming or incoming sound field or incoming wave from infinity. That means there is a plate vibrating but there is a sound field coming towards it from infinity which is not possible. Therefore, the negative sign has to be chosen when it is imaginary.

And for a similar reason the positive value must be chosen when it is real so that the field is decaying if we chose the negative when it is real it will imply that this is blowing up in the y direction. So, at infinity it actually blows up so both are a must this 2 are physical condition that has to be imposed on γ. So, one is we have to choose a branch such that γ is continuous that is one so there are several ways to choose a branch that γ is continuous.

In addition, such a branch has to be chosen where this is also valid when k is moving on the real axis these 2 conditions should be valid. So, we have we will look at the square root function in the complex plane to get an understanding. So let me stop here for today for this lecture we will continue from there in the next class thank you.