Sound and Structural Vibration

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Lecture - 59

Fluid Loading in a Finite Plate

Good morning and welcome to this next lecture. We were discussing the topic of fluid loading. So, there is a panel in contact with a half space of fluid. The panel is one dimensional and infinite and we have come up with this expression here for the combined impedance.

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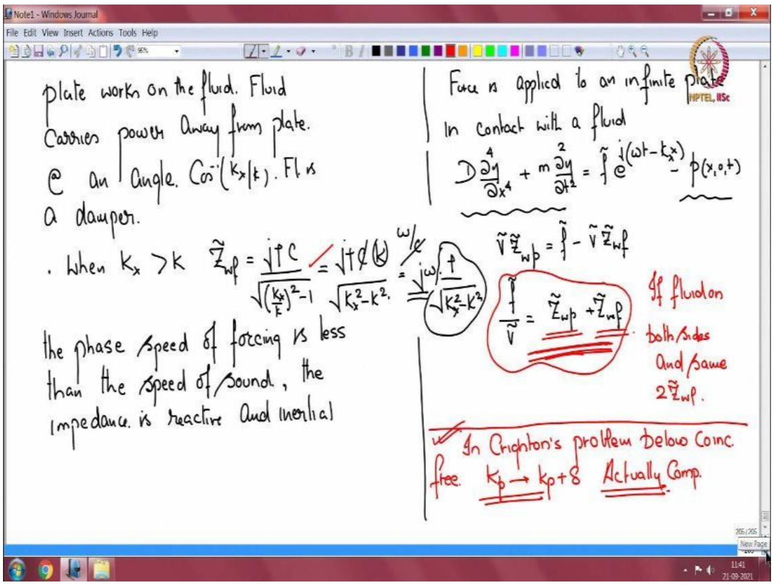

And the idea is to make some heuristic comments. So, the impedance as seen by the force is a combination of plate and the fluid impedance.

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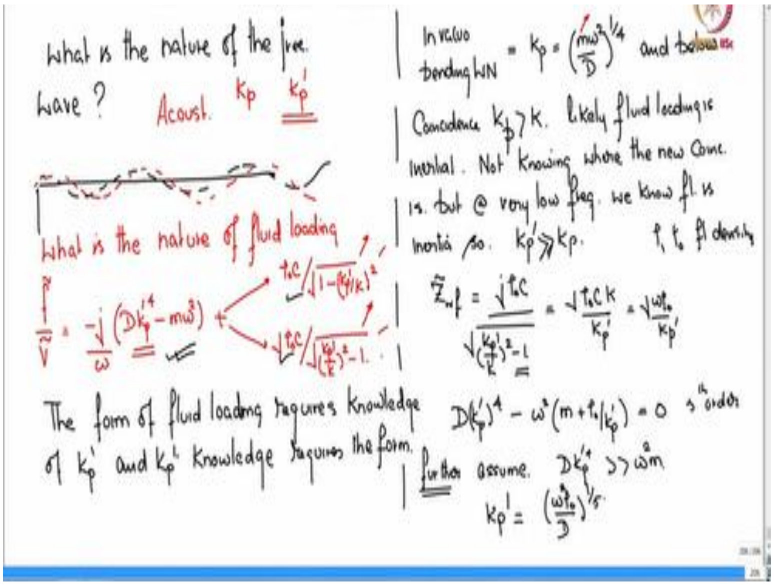

Now the question is what is the nature of the free wave now, what that means is that if I have a remote excitation and there is a in vacuo free wave moving, it is now in vacuum but then I attach or there is an acoustic fluid in place then this free wave will be modified. So, if I say the original in vacuo free wave is k_p. Now, I will call it $k_p{}'$.

So, $k_p{}'$ is the new free wave after the influence of the acoustics, but you see then what is the nature of the fluid loading. Nature of the fluid to the next question of the reason it is relevant now is I have

$$\frac{\tilde{f}}{\tilde{V}} = \frac{-j\left(Dk_p{}'^4 - m\omega^2\right)}{\omega} + ,$$

one form is $\frac{\rho_0 c}{\sqrt{1-\left(\frac{k_p{}'}{k}\right)^2}}$ that is a possibility. The other is $\frac{j\rho_0 c}{\sqrt{\left(\frac{k_p{}'}{k}\right)^2-1}}$.

So, the fluid loading nature depends on $k_p{}'$ and $k_p{}'$ is not yet found, $k_p{}'$ depends on this equation unless we solve this $k_p{}'$ equation which will actually be fifth order we do not know where $k_p{}'$ lies that decides the nature of fluid loading. So, it is circular, the choice of the equation tells me $k_p{}'$ and the value of $k_p{}'$ tells me whether choice is correct or not we are in circular zone.

So, now of course as I said in the Crighton's problem, we computed all of this. So, we are just trying to talk about it under the title of fluid loading. So, let me write what I am trying to say here the form of fluid loading whether this or this requires knowledge of $k_p{}'$ and $k_p{}'$ knowledge requires the form being this or that. So, without so much details if you have to make a simplistic argument, what do we do?

We know that the in vacuo bending wave number is given by $k_p = \left(\frac{\omega^2 m}{D}\right)^{1/4}$ and below coincidence k_p is bigger than k. And so, it is likely that the fluid loading is inertial, inertial meaning the fluid will load with a further inertia. So, not knowing where coincidence will lie right where the new coincidence is or the new let me k_p coincidence is but at very low frequency.

We know fluid behaves as inertia and so we will say that $k_p{}'$ is greater than k_p, that means

$$\tilde{Z}_{wf} = \frac{j\rho_0 c}{\sqrt{\left(\frac{k_p{}'}{k}\right)^2 - 1}} = \frac{j\rho_0 ck}{k_p{}'} = \frac{j\omega\rho_0}{k_p{}'}.$$

Let it be ρ_0 and ρ both are mean fluid density.

Now, if we now go and plug this back here, if I go and plug it back here what I will now get is

$$D\left(k_p{}'\right)^4 - \omega^2\left(m + \frac{\rho_0}{k_p{}'}\right) = 0.$$

So, this is a fifth order equation. So, if we further assume so these are heuristic arguments, further assume that $D\left(k_p{}'\right)^4$ is much greater than $\omega^2 m$ then

$$k_p{}' = \left(\frac{\omega^2\rho_0}{D}\right)^{1/5}.$$

So, these are limit arguments about fluid loading.

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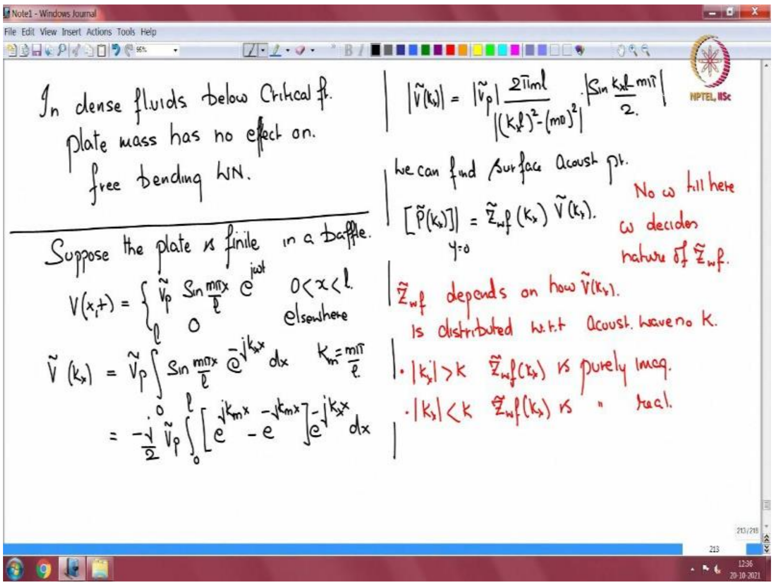

So, we can say now in dense fluids, that is why $k_p{}'$ is so much bigger than k_p in dense fluids below critical frequency or coincidence the plate mass has no effect on the free bending wavenumber is kind of back at the envelope physical arguments. So, that closes the section on infinite plate and acoustic half space. Now, suppose the plate is finite, then again, we have a 1D plate, a finite plate

$$V(x,t) = \begin{cases} \tilde{V}_p \sin\dfrac{m\pi x}{l}\, e^{j\omega t} & 0 < x < l \\ 0 & elsewhere \end{cases}.$$

And this is in a baffle and so we compute the one-dimensional Fourier transform we get

$$\tilde{V}(k_x) = \tilde{V}_p \int_0^l \sin\frac{m\pi x}{l}\, e^{-jk_x x}\, dx\,,$$

$$= \frac{-j}{2}\tilde{V}_p \int_0^l \left[e^{jk_m x} - e^{-jk_m x}\right] e^{-jk_x x}\, dx.$$

Now, if we look at the amplitude

$$\left|\tilde{V}(k_x)\right| = \left|\tilde{V}_p\right| \frac{2\pi m l}{|(k_x l)^2 - (m\pi)^2|} \left|\sin\frac{k_x l - m\pi}{2}\right|.$$

So, we can find the surface acoustic pressure from here. How is that? We know that

$$[\tilde{P}(k_x)]\big|_{y=0} = \tilde{Z}_{wf}\,(k_x)\tilde{V}(k_x).$$

So, far no ω here, no ω involved till here ω now after this stage decides nature of $\tilde{Z}_{wf}$. The $\tilde{Z}_{wf}$ depends on how $\tilde{V}(k_x)$ is distributed with respect to the acoustic wave number. If k_x is greater than k then $\tilde{Z}_{wf}$ is purely imaginary whereas if k_x is less than k then $\tilde{Z}_{wf}$ is real

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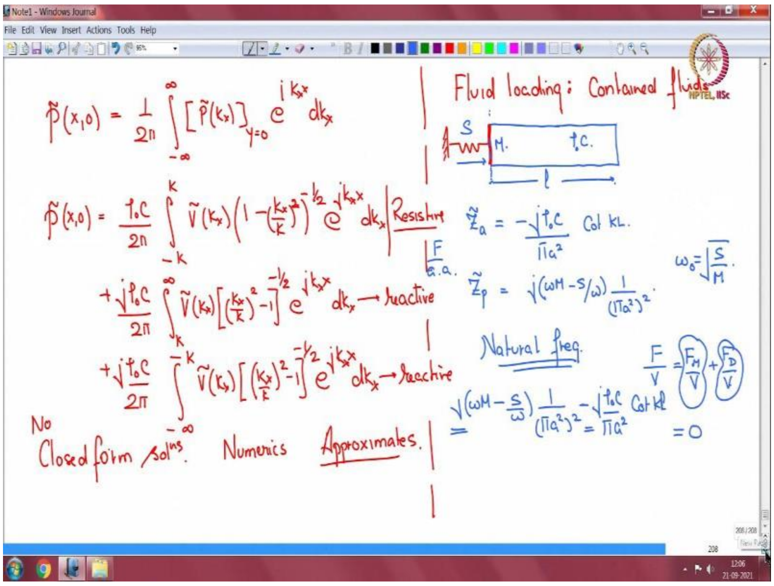

The pressure on the surface is given by

$$\tilde{P}(x,0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\tilde{P}(k_x)]\big|_{y=0} e^{jk_x x}\, dk_x\, .$$

and this infinity now breaks up,

$$\tilde{P}(x,0) = \frac{\rho_0 c}{2\pi} \int_{-k}^{k} \tilde{V}(k_x)\left(1-\left(\frac{k_x}{k}\right)^2\right)^{-\frac{1}{2}} e^{jk_x x}\, dk_x\Bigg|_{resistive}$$

$$+\frac{j\rho_0 c}{2\pi} \int_{k}^{\infty} \tilde{V}(k_x)\left(\left(\frac{k_x}{k}\right)^2 - 1\right)^{-\frac{1}{2}} e^{jk_x x}\, dk_x\Bigg|_{reactive}$$

$$+\frac{j\rho_0 c}{2\pi} \int_{-\infty}^{-k} \tilde{V}(k_x)\left(\left(\frac{k_x}{k}\right)^2 - 1\right)^{-\frac{1}{2}} e^{jk_x x}\, dk_x\Bigg|_{reactive} \, .$$

These mostly do not have closed form solutions. One could possibly try complex variables contour integrations but no close form solutions mostly and you have to do numeric. As I have said this portion I am taking from the book by Frank Fahy and so he approximates these integrals further and makes some comments.

But I think this is good enough as an idea that introduces fluid loading. Now, we have seen for an infinite case an infinite panel in contact with an acoustic half space or a finite panel radiating into an infinite domain. So, the other fluid loading idea the other fluid loading idea comes for contained fluids, fluid is enclosed by the structure. So, what would be a simple system? I have a spring-loaded piston that is kind of the mouth of a duct of length l is exactly your cycle pump.

So, this has spring constant s. This has a mass M and other this is medium with $\rho_0 c$ etcetera. So, if I force this mass over here then there is a specific acoustic impedance of the form $\frac{-j\rho_0 c}{\pi a^2}\cot kl$ then we have this is what force over area squared. This is force over area squared units force divided by area and one more area units.

Then similarly the Z of the piston is $j(\omega M - s/\omega)\frac{1}{(\pi a^2)^2}$, now both are same units. Now, as I said once you talk of contained fluids the question is about natural frequencies, main question where do the natural frequencies lie. So, the original natural frequency of the piston is $\sqrt{\frac{s}{M}}$ and the duct has infinite natural frequencies.

So, if we talk about this combined impedance how does it look like the combined impedance looks like $j(\omega M - s/\omega)\frac{1}{(\pi a^2)^2}$. Now, this is a common velocity junction. This velocity is common which means the total force divided by the velocity is the force taken up by the mass divided by the same velocity as the force taken by the duct divided by the same velocity.

So, the impedances will straight away add, so the other impedance is $\frac{-j\rho_0 c}{\pi a^2}$ $\cot kl$. So, this is a transcendental term $\cot kl$, now fully imaginary this term is imaginary that term is imaginary. So, if we set it to 0, we can find the combined natural frequencies.

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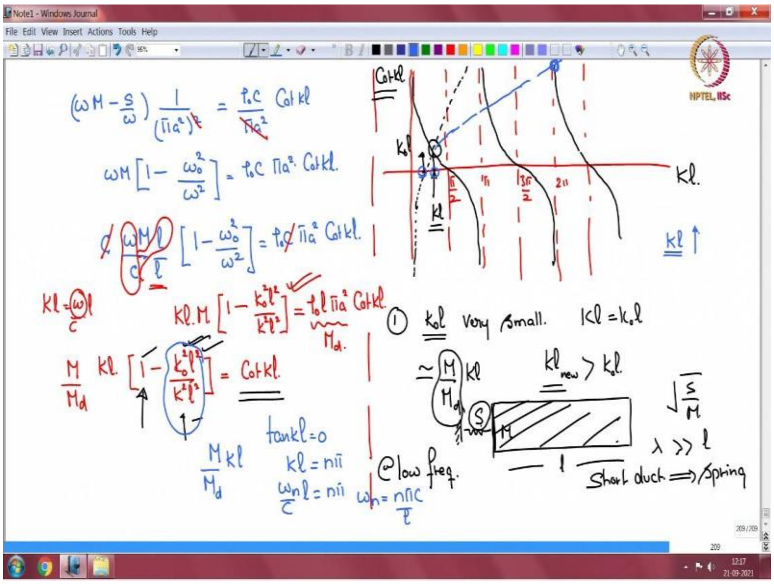

So, let us do that,

$$(\omega M - s/\omega)\frac{1}{(\pi a^2)^2} = \frac{\rho_0 c}{\pi a^2}\cot kl.$$

So, we will make everything $\cot kl$ so first of all let us cancel one area here, so we get

$$\omega M\left[1 - \frac{{\omega_0}^2}{\omega^2}\right] = \rho_0 c\, \pi a^2 \cot kl.$$

Now, here I divide by c and multiply by c, so

$$c\frac{\omega M}{c}\frac{l}{l}\left[1 - \frac{{\omega_0}^2}{\omega^2}\right] = \rho_0 c\, \pi a^2 \cot kl.$$

So, what happens now, ω by c is k into l is kl and this c cancels with this c so I have kl into M this I would like to take it to the other side

$$kl\, M\left[1 - \frac{{k_0}^2 l^2}{k^2 l^2}\right] = \rho_0 l\, \pi a^2 \cot kl.$$

So, this is the mass of air in the duct M_d.

So, I have

$$\frac{M}{M_d}\, kl\left[1-\frac{{k_0}^2 l^2}{k^2 l^2}\right]=\cot kl.$$

Everything is in terms of kl. So, where this goes to 0 or left side equals right side are the new omegas, new resonances why k carries ω, kl is ω by c into l. So, now again this is an opportunity for doing limiting operations and so forth so we will see one or two cases.

So, the answer is basically here there is nothing else to be done but that does not give us much insight. So, we have to do some limiting operations small values of something or large values of something to get a field so that is what we look at one or two cases. So, we will plot $\cot kl$ first if we plot $\cot kl$ what do we get? It starts off at infinity this is $\pi/2$ this is a π this is a $3\pi/2$ the 2π and so forth.

It starts off at infinity goes through 0 goes off to negative infinity starts off at infinity goes through 0, because tan is infinite there and so forth this is the right part. Now, for the left part we will do one approximation, let us say $k_0 l$ the original resonance is very small, then what happens this kl this is the kl axis, I have already plotted $\cot kl$. So, kl will start so we are looking at the left side now.

So, kl starts off at 0 so it $-\infty$ sort starts off here $-\infty$ and as kl starts to increase somewhere kl = $k_0 l$ so this term is 1 that 1 goes off with 1 so you get a 0. So, you get a 0 crossing somewhere 0 crossing. Now as kl goes further and further high, this will be small compared to one because denominator is big then the left side is approximately $\frac{M}{M_d}\, kl$. So, it is a straight line with slope $\frac{M}{M_d}$ and typically piston mass can be heavier than the air.

So, this thing would actually go off at a very sharp angle in a straight line. So, that means what the new so the new value so whereas as the original value may have been here the new value now of resonance is here the new kl value. So, old k value $k_0 l$ was here new value is here where the 2 curves intersect so the new kl is new bigger than $k_0 l$. That means what this piston resonance mounted on the spring, which is M and s originally was $\sqrt{\frac{s}{M}}$ now has gone up.

So, what happened at very low frequencies at low frequencies if you have gone through acoustics a duct whose length is much smaller than acoustic wavelength. If lambda is much greater than the acoustic wavelength then the duct behaves like a spring, a short duct is a stiffness is a spring. At low frequencies a short duct that is closed at the end much smaller than the acoustic wavelength behaves like a spring stiffness.

So, that springiness attaches to the original spring and therefore the resonance goes up, the old resonance is a new resonance or resonance goes up. We can do one more, suppose now we have infinity of resonances. And suppose a resonance kl is very, very high then what happens is that on the right on the left side, this value can be ignored and what we have is $\frac{M}{M_d}\ kl$.

So, it is a straight line at some angle whatever be the angle and the higher and higher values they are intersecting where $\cot kl$ is infinite or $\tan kl$ is 0, so $\tan kl$ is 0 that means kl is the order of some $n\pi$ or $\frac{\omega_n l}{c} = n\pi$ or $\omega_n = \frac{n\pi c}{l}$. So, this is the kind of analysis you do to get a feel for how the system should behave and there are so many cases in between.

So, that is fluid loading infinite systems we have seen a case where a panel interacts with the cavity. So, you there will be fluid loading over there but we did not talk about in that language so this is to give you an idea of what loading fluid loading is in finite systems. Let me close the lectures over here. I am done more or less with this course. I hope you learned something. So, thank you very much.