Sound and Structural Vibration

Prof. Venkata Sonti

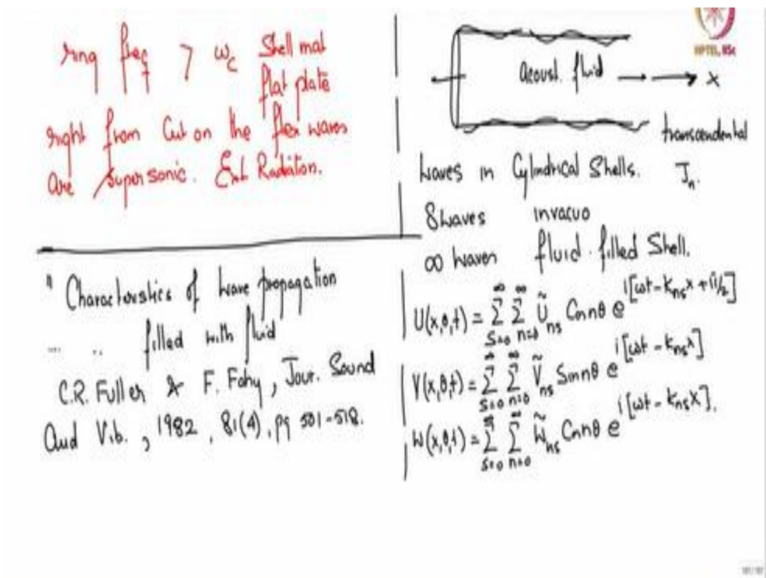
Department of Mechanical Engineering

Indian Institute of Science, Bengaluru

Lecture - 57

Wave Propagation Characteristics in Flexible Cylindrical Shells Carrying Fluid: Fuller's Paper

(Refer Slide Time: 00:27)



Good morning and welcome to this next lecture on sound and structural vibration with regard to coupling of sound and structure in cylindrical geometries. There is this paper it is titled characteristics of wave propagation and so forth filled with fluid by authors C. R Fuller and F Fahy it is a JSV, journal of sound and vibration paper from 1982, volume 81 part 4 pages 501 to 518. Essentially what it discusses is a cylindrical shell infinite carrying an acoustic fluid.

Earlier we discussed waves in cylindrical shells placed in vacuum not in contact with fluids. Now this cylindrical shell vibrates in all its modes and inside it carries an acoustic fluid. So, very similar to the structural acoustic waveguide, the geometry cylindrical here. So, the object is to study the coupling. So, now the fluid carries a transcendental term fluid description carries a transcendental term like a Bessel function and therefore the number of waves becomes suddenly infinite.

Whereas the in-vacuo shell had eight waves for the in-vacuo shell. Now you have an infinity of waves for fluid filled shell and hence the description the displacement axial description is given by so this time I am using x in this direction. So, I have been switching x and z,

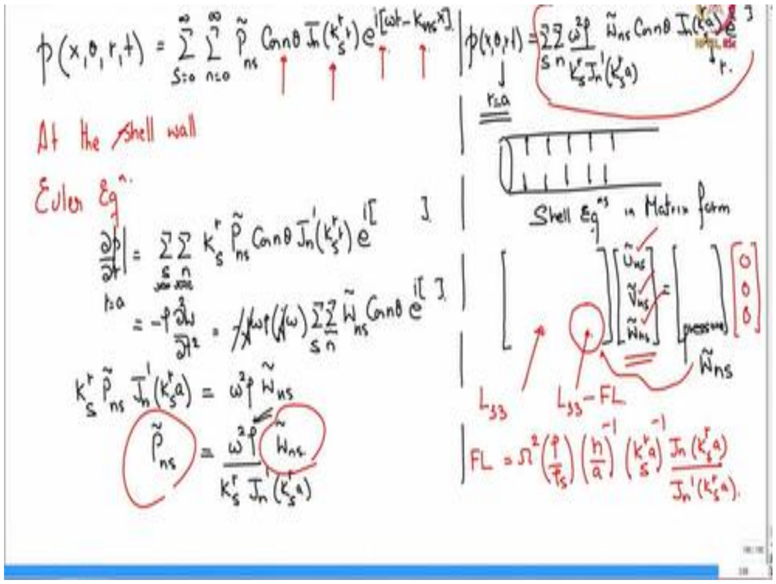
$$U(x,\theta,t) = \sum_{s=0}^{\infty}\sum_{n=0}^{\infty} \tilde{U}_{ns} \cos n\theta \, e^{i\left[\omega t - k_{ns}x + \frac{\pi}{2}\right]},$$

$$V(x,\theta,t) = \sum_{s=0}^{\infty}\sum_{n=0}^{\infty} \tilde{V}_{ns} \sin n\theta \, e^{i[\omega t - k_{ns}x]},$$

$$W(x,\theta,t) = \sum_{s=0}^{\infty}\sum_{n=0}^{\infty} \tilde{W}_{ns} \cos n\theta \, e^{i[\omega t - k_{ns}x]}.$$

This is the displacement of the shell.

(Refer Slide Time: 06:17)



Then we have a descriptor for interior pressure. We have

$$P(x,\theta,r,t) = \sum_{s=0}^{\infty}\sum_{n=0}^{\infty} \tilde{P}_{ns} \cos n\theta \, J_n(k_s^r r) e^{i[\omega t - k_{ns}x]} .$$

So, this describes the theta behaviour, this describes the radial behaviour, this describes the axial behaviour, and this is the time term. Now again at the wall at the shell wall the normal velocity of the shell and fluid must be same which means again our friend Euler equation has to be invoked at the shell boundary.

So, first of all we have

$$\frac{\partial P}{\partial r} = \sum_{s=0}^{\infty}\sum_{n=0}^{\infty} \tilde{P}_{ns} \, k_s^r \cos n\theta \, J_n{}'(k_s^r r) e^{i[\omega t - k_{ns}x]} .$$

J_n is the Bessel function cylindrical first kind of order n.

$$-\rho \frac{\partial^2 W}{\partial t^2} = -j\omega\rho(j\omega) \sum_{s=0}^{\infty}\sum_{n=0}^{\infty} \widetilde{W}_{ns} \cos n\theta \, e^{i[\omega t - k_{ns}x]} .$$

So, if we equate these two and realize that the coupling is first of all n based and then wavenumber based.

So, we will have

$$k_s^r \tilde{P}_{ns} J_n{}'(k_s^r a) = \omega^2 \rho \widetilde{W}_{ns}.$$

$$\tilde{P}_{ns} = \frac{\omega^2 \rho}{k_s^r J_n{}'(k_s^r a)}.$$

And therefore, my pressure now can be given. What is pressure?

$$P(x, \theta, r, t) = \sum_{s=0}^{\infty} \sum_{n=0}^{\infty} \frac{\omega^2 \rho}{k_s^r J_n{}'(k_s^r a)} \cos n\theta \, J_n(k_s^r a) e^{i[\omega t - k_{ns} x]}.$$

Now this is pressure at $r = a$. This is the pressure that the fluid applies on the shell, this is the pressure at $r = a$. Otherwise, we will replace this a with r that is the pressure anywhere inside. Now if you recall the shell equations, we wrote in terms of the matrix shell equations in the matrix form. So, we had a 3 by 3 matrix and the amplitude $\widetilde{U}_{ns}$, $\tilde{V}_{ns}$, $\widetilde{W}_{ns}$. Now here we have a pressure acting on the shell in the normal direction.

So, this will be a pressure coming in there. But this pressure is in terms of $\widetilde{W}_{ns}$ that is what we found this pressure amplitude can be given in terms of $\widetilde{W}_{ns}$. So, it one extra term as a coefficient of $\widetilde{W}_{ns}$ will enter the 3 by 3 element of the matrix. So, the 3 by 3 element originally was L_{33}. Now I get L_{33} minus a fluid loading term that is of this form and that whole set was non-dimensional.

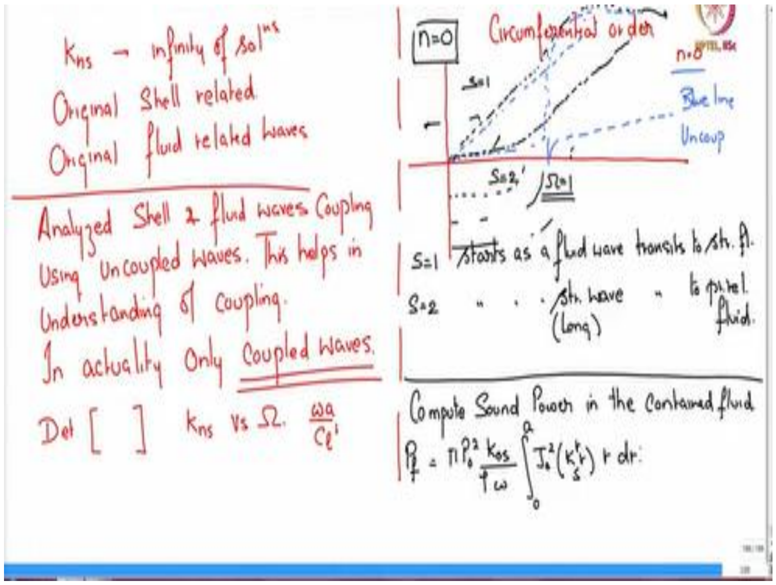
So, I just give you this FL term fluid loading term is

$$\Omega^2 \left(\frac{\rho}{\rho_s}\right) \left(\frac{h}{a}\right)^{-1} (k_s^r a)^{-1} \frac{J_n(k_s^r a)}{J_n{}'(k_s^r a)}.$$

So, this term will be modified. Then again, the right-hand side has zeros because this pressure term in terms of $\widetilde{W}_{ns}$ has come here so this right side will become 0.

And for non-trivial solutions of $\widetilde{U}_{ns}$, $\tilde{V}_{ns}$ and $\widetilde{W}_{ns}$ the determinant again of this matrix must be set to 0.

(Refer Slide Time: 15:14)



But this time you will have an infinity of solutions for k_{ns} infinity of solutions. So, there will be original shell related waves and then original fluid related waves. Now I would like to remind you here that we have analysed shell and fluid waves coupling using uncoupled waves. We have done that on coupled waves in the earlier classes we have just done that. Now we said that this helps in understanding of coupling.

However, in actuality only coupled wave exist, the uncoupled waves do not exist only coupled waves exist. So, when you do an actual calculation, you will do it in terms of coupled wave so that is what we are actually doing now. So, this paper now is talking about the coupled waves. So, now if you set the determinant of that matrix to 0 and you try to figure out the k_{ns} values as a function of this non-dimensional frequency Ω which is $\frac{\omega a}{c_l'}$.

So, you will get several waves and the paper shows that and it is common to describe waves in terms of the circumferential order. So, now the how does it look like. The waves how do I look like for $n = 0$. It is very difficult to describe in general but let us say the blue line is the uncoupled wave. So, this is the uncoupled acoustic plane wave, and we are talking $n = 0$. So, $n = 0$ picture I have given you before.

Then there is this longitudinal dominant wave which comes till here and it very radically transits to a flexure and then comes down and it cuts on again as longitudinal and proceeds in the same direction. So, this is the uncoupled blue line is uncoupled picture whereas the black line I will give you the coupled picture. So, you will have a couple acoustic wave this is called the $s = 1$ branch and then it will become the flexural coupled wave in the shell.

So, based on the terminology from the structural acoustic waveguide is a coupled acoustic plane wave and this is the coupled flexural wave. So, we are retain, the original name but add coupled to it. Now this is the coupled structural wave and right here it will in when you do the coupled calculation it will transit to a pressure release wave just as it did in the structural acoustic waveguide. So, there are two coupled waves mainly at low frequencies.

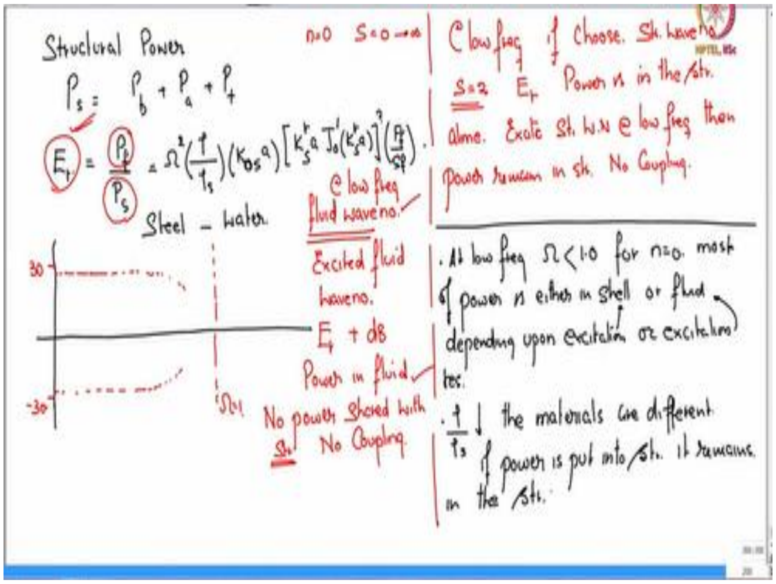
Two propagating that is real valued coupled wave at low frequencies. The nomenclature is $s = 1$ and $s = 2$, $s = 1$ is starts as a fluid wave and transits to a structural wave a flexural wave then $s = 2$ starts as a structural wave dominantly longitudinal. It transits to a pressure release fluid wave. Now similarly for $n = 1, 2$ and so forth and it becomes very complicated very soon. In addition to this there are an infinity of imaginary waves due to the fluid below their cut outs and they start cutting on and then there is you know a lot of detail.

So, this is the ring frequency approximately. So, now what the authors do? So, I am going to keep it just sketchy. So, what the authors do is they compute sound power in the contained fluid, all the details are there and it is a straight forward derivation. So, you get power in the fluid given by something like

$$P_f = \pi {P_0}^2 \frac{k_{0s}}{\rho\omega} \int_0^a {J_0}^2(k_s^r r)\ r\, dr.$$

I am going to show only for $n = 0$ the results.

(Refer Slide Time: 23:35)



Similarly, you can have an expression for $n = 0$ then you have structural power in the waves. It is actually a sum of the power in bending + the power in the longitudinal wave + the power in torsion. These three powers have to be added force into velocity then you can have a ratio power ratio which is power in the fluid by power in the structure and it has this form

$$E_r = \frac{P_f}{P_s} = \Omega^2 \left(\frac{\rho}{\rho_s}\right)(k_{0s}a)[k_s^r a\, J_0{}'(k_s^r a)]^2 \left(\frac{F_f}{S_f}\right).$$

So, now the authors consider steel as the shell material and water as the fluid. So, you can see that there are not too many water cut ons below the ring frequency we have seen that before. So, I am going to show you this E_r picture. So, let us say this is some value 30 or so +30 and this is -30 this is on decibel scale.

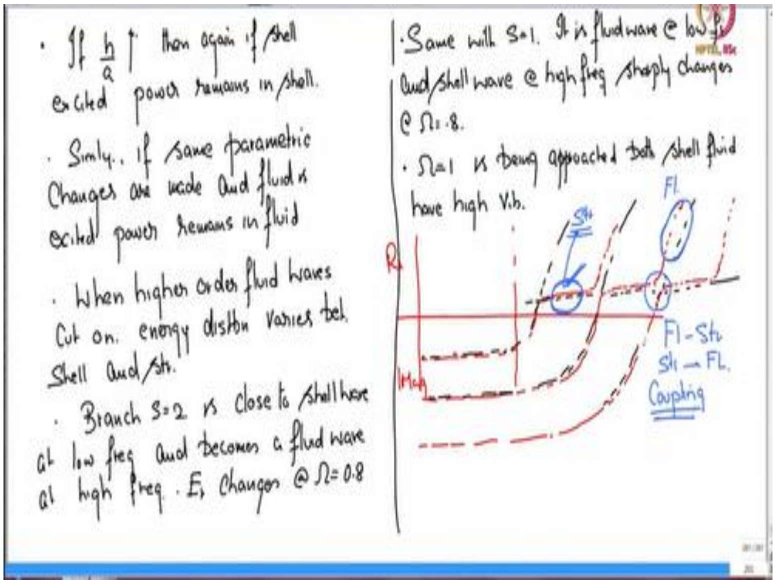
So, if it is positive that means there is more of power in the fluid and negative that means more fluid in the structure. So, now if I choose the fluid wave number there are infinity of wave numbers. So, at $n = 0$ there are a bunch of s wavenumbers that go from 0 to infinity that is why the double sum. So, there is fluid wavenumber I said $s = 1$ is starts off as dominantly fluid. So, if I excite the fluid wave number.

So, if I choose fluid wave number means I have excited the fluid member number. Then what happens? I look at this ratio E_r then I find that it is like this the E_r goes like this so that means E_r has positive decibel values that means what the power lies in the fluid. So, if I manage to excite the fluid wave then the power remains in the fluid that means no power shared with structure. It means no coupling at low frequencies.

So, that is this picture it changes later but and somewhere here is my ring frequency. Similarly, at low frequencies if we choose the structural wave number that is $s = 2$ branch the value related to $s = 2$ and again look at E_r then it behaves like this. It is negative in decibels that mean what power is in the structure alone. So, if I excite structural wavenumber at low frequency then power remains in the structure. So, no coupling that is at low frequency.

Now what happens further? So, let us make some bullet points. So, at low frequencies that is Ω less than 1 and we are just doing $n = 0$ most of the energy or power is either in shell or fluid depending upon excitation, depending upon whether excitation is shell or fluid or excitation is fluid in the fluid respectively. Then if row f or row I have been using just row so rho that means fluid density row by row structure if it is reduced the materials are different or rather their impedances are different. If energy or power is put into structure it remains there in the structure.

(Refer Slide Time: 32:03)



Similarly, if h by a is increased then again if shell is excited power remains in shell, no coupling. Similarly, if same pattern same parameters if same parametric changes are made and fluid is excited, power remains in fluid again no coupling. Now when higher order fluid waves cut on which happens? When higher order fluid waves cut on energy distribution varies now, energy distribution varies between shell and structure. We will see a little more in what manner.

The branch 2 or branch $s = 2$ is close to a shell wave at low frequency and becomes a fluid wave at high frequency. So, the energy ratio changes sharply at $\Omega = 0.8$ as we approach the ring frequency same with $s = 1$. It is a fluid wave at low frequency and shell wave at high frequency and sharply changes again at around $\Omega = 0.8$. And because we are approaching the ring frequency $\Omega = 1$ is being approached.

Both shell and fluid resonate, shell and fluid show have high vibration content. Now when several waves are cutting on what happens? So, we have this picture let us see. So, this is my axis, this is my ring frequency and then there are this fluid wave cuts on. As a fluid wave it sharply changes to a structural wave. The structural wave later starts to follow a fluid wave. Another fluid wave comes it cuts on as a fluid wave then sharply changes to the structural wave.

This structural wave moves a little then it starts to follow the next fluid wave. Another fluid wave is imaginary. This is below imaginary above is real. So, the fluid wave comes as an imaginary value yet to cuts on becomes real comes in and then changes quickly to a structural wave and then the structural wave starts to follow the next fluid wave. So, had there been or if we were plotting just uncoupled waves that mean what this wave actually would cut down and behave like a fluid wave.

This wave would cut on and actually behave like a fluid wave. This wave would cut on and behave like this like a fluid wave and here we have a structural wave that is moving forward. This is the uncoupled picture and drawing the black lines but due to coupling things have changed. So, now if I excite near a structural wave so if I excite this part here pump energy into this wave then the energy will remain in the structure. If I excite the fluid here the energy will remain in the fluid.

Whereas if I excite either of them here where both wavenumbers are close by the red lines are coupled wavenumbers. Then if you excite in the fluid it will share with the structure and if you excite the structure it will share with the fluid so that is where coupling will happen. So, that is the story of shell and fluid coupling. I will close the lecture here. Thank you.