Sound and Structural Vibration

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Lecture - 56
Fluid Waves in Rigid-Walled Cylindrical Shells

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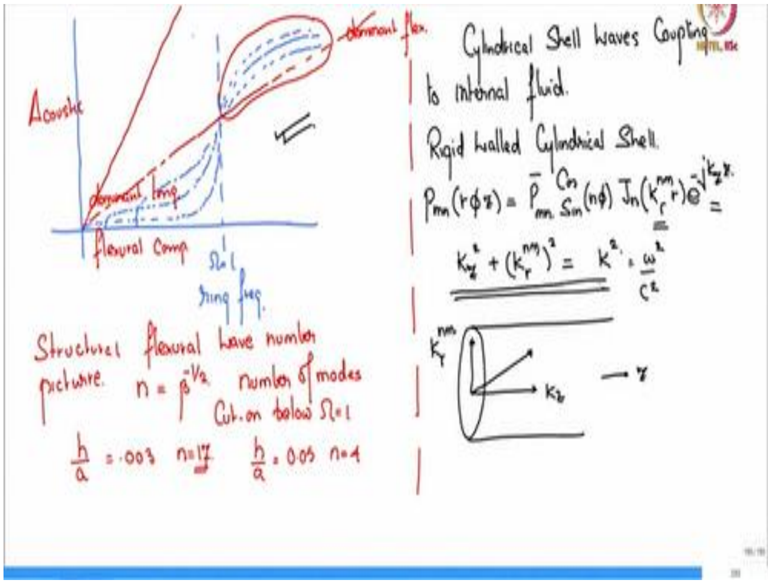

Good morning and welcome to this next lecture on sound and structural vibration. We are looking at this topic of cylindrical shell waves coupling to internal fluid. So, far we have looked at how shell flexural waves behave. So, we have to now bring the fluid aspect in it. So, suppose we have a rigid walled cylindrical shell wall does not move it is a rigid pipe kind of. Then the solution to the cylindrical wave equation has this form let us say

$$P_{mn}(r,\phi,z) = \bar{P}_{mn} \begin{matrix} cos \\ sin \end{matrix} (n\phi)\, J_n(k_r^{nm} r) e^{-jk_z z} \,.$$

So, this is the solution for the mn^{th} mode. Normally it will be a summation over m and n. Now how k is related? The ${k_z}^2 + (k_r^{nm})^2 = k^2$. So, now you are aware of how the wavenumbers add. So, at a given frequency I have a k^2 this is equal to $\frac{\omega^2}{c^2}$. So, at a given frequency in the shell I have a k the shell goes up to infinity I have a k in some direction.

That shall be equal to the k_z in the z direction versus k_r^{nm} in the radial direction. So, that is what this equation is saying. So, now ideally, we are looking at the rigid walled to understand.

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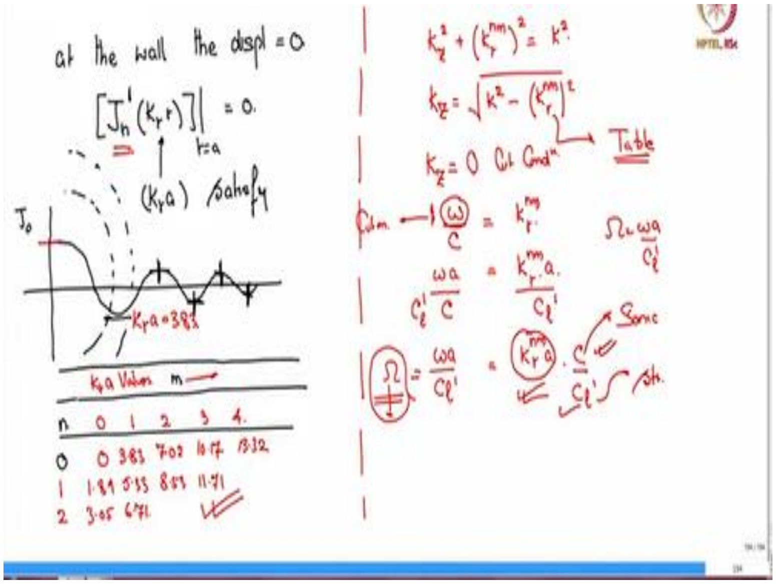

The cut-ons the cylindrical acoustic cut-ons. So, now at the wall the displacement is 0 the normal displacement is 0 which means in the radial direction $r = a$ we will have J_n the Bessel function first kind n^{th} order derivative at $r = a$ shall be 0.

$$[J_n'(k_r r)]|_{r=a} = 0.$$

Now because Bessel function is like a transcendental function there are several $k_r a$ values that will satisfy this condition. Even for a single so let me draw the Bessel function.

So, I will just draw 0 or the J_0, J_0 behaves like this. It is like a decaying sinusoid. So, this is J at 0. So, you can see its derivative goes to 0 here, its derivative goes to 0 here its derivative goes to 0 here this 0 here goes to 0 here goes to 0 here. So, this is kind of the radial direction. You can put this kind of the radial direction. So, this is how the circumference of the shell should go. So, you are trying to find values of k_r such that $k_r a$ lands here, $k_r a$ lands here, $k_r a$ lands here, $k_r a$ lands here.

Why? That is where the J_0 derivative goes to 0. So, for a single n there are several values of m where this happens. So, if I just draw a table now m values are in this here. Then n values 0 and 0, m $0, 1, 2, 3, 4$ etcetera so, m values. For $n = 0$ the first-time derivative goes to 0 is for m

value 0 and the value of the wavenumber is 0. So, that means $k_r a$ is 0 so, here are $k_r a$ values some $k_r a$ values. So, because for J_0 you get a derivative 0 at 0 value.

So, the next value where derivative goes to 0 is 3.83. That is the $k_r a$ value $k_r a = 3.83$. So, if a is known you can find out the actual wavenumber value. Then the next time it goes to 0 at 7.02 these are tabulated. So, these are Bessel functions tabulated. Then for $n = 1$ the first fellow is at 1.84. Second is 5.33. Third is 8.53, 11.71 and equal to 2 it is 3.05, 6.71 etcetera. So, these are the cut on frequency or where J_n' goes to 0.

So, now what we have is we have ${k_z}^2 + (k_r^{nm})^2 = k^2$. Now k_z is of course equal $\sqrt{k^2 - (k_r^{nm})^2}$. This is figured out from the table on the left. So, when k_z goes to 0 that is my cut-on condition which means my k or $\frac{\omega}{c} = \frac{k_r^{nm}}{r}$. So, ω is that cut-on frequency. So, if we put it in non-dimensional form so we have ωa so, I have to do ωa by c this is speed of sound is $k_r^{nm} a$.

But my non-dimensional frequency is what $\frac{\omega a}{c_l'}$. So, I had to put c_l' then I put c_l'. So, what is it?

$$\Omega = \frac{\omega a}{c_l'} = k_r^{nm} a \ \frac{c}{c_l'}.$$

So, c is the speed of sound, sonic speed in that particular medium, c_l' is the speed of longitudinal wave in that medium in the structure. So, that is the cut on non-dimensional frequency now.

So, once I figure out $k_r^{nm} a$ from this particular table on the left the way to figure out the cut on frequency in terms of the standard non-dimensional number that we have been using for frequency is you multiply that by c on top divided by c_l' dash on the bottom. So, now this is how I figure out my cut-on frequency value. I have figured out the cut-on frequencies for the structure. We have figured out after cut-on how these wavenumbers behave.

So, below $\Omega = 1$ they stay low close to 1 they start to rise suddenly. So, now how does this acoustic wavenumber behave? Suppose let me do one thing I need to put some labels. So, this is the Ω axis and let us say this will be $n = 0$, this will be $n = 1$ this will be $n = 2$ and this is $k_z a \beta^{1/2}$.

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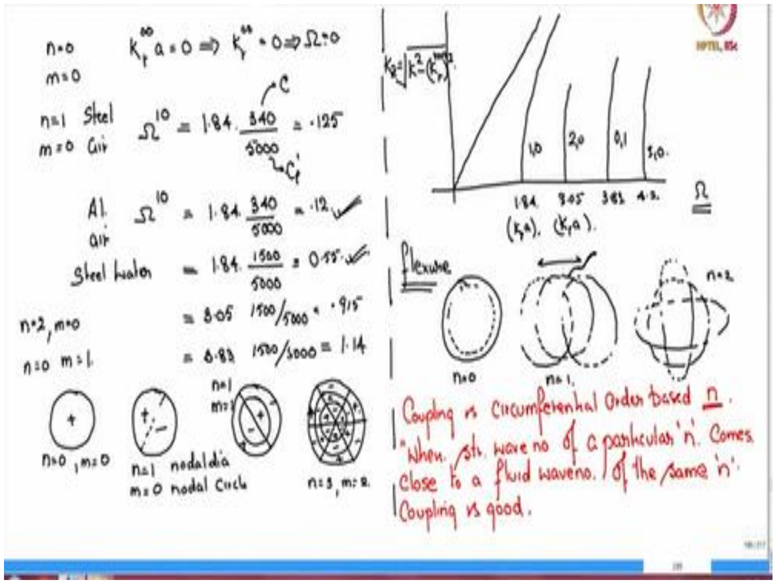

Now based on our table let us look at where the k_z is cut-on. So, for $n = 0$ and $m = 0$ based on the table earlier $k_r^{00}a$ was 0 which means $k_r^{00} = 0$ which implies at the same time that the non-dimensional frequency Ω is 0. Then for $n = 1$ and $m = 0$ let us say for steel and air combination my Ω^{10} will be 1.84 into c which is 340 divided by 5000. This is speed of sound in the air; this is longitudinal wave speed in the medium; this is about 0.125.

Similarly, if we do for aluminium and air the $\Omega^{10} = 1.84$. Similar values 340 by 5000 about 0.12. If we do steel and water then we have 1.84 into 1500 by 5000 which is 0.55. If we do $n = 2$ and $m = 0$ steel water then we have from the table $k_r a$ is 3.05 1500 by 5000 = 0.915. And if we look at $n = 0$ and $m = 1$ from the table $k_r a$ value is 3.83 into 1500 by $5000 = 1.14$. Now circumferentially how do these nm values appear as?

So, if I have this circle $n = 0$ and $m = 0$ looks like this $n = 1, m = 0$ how does it look like? $n = 1$ denotes the nodal diameter then $m = 0$ is nodal circle so, nodal circles. So, we have a nodal 1 nodal diameters plus and minus. How does $n = 1$ and $m = 1$ look like? So, we have 1 diameter 1 circle. So, we will have $+ - + -$. Lastly, we will look at $n = 3$ and $m = 2$. So, $m = 2$ gives me 1, 2, $n = 3$ will be 1, 2 and 3. So, let us say we will do $- + - + - + + - + - + - - + - + - +$.

And we can plot these cut-ons along the frequency axis. So, we get k_z on the y axis and the frequency value here. So, we have $k_z = \sqrt{k^2 - (k_r^{nm})^2}$. So, the k^{00} it cuts on straight from Ω equal to 0. So, it goes like this straight away. So, next we have the $n = 1, \; m = 0$ where the $k_r a$ value is 1.84. So, $k_r a$ value is this is 1, 0 and this is 1.84. This is the $k_r a$ value then we have 2, 0 sequentially 2, 0. So, this is asymptotically go to plane wave.

So, this value was 3.05. The next value is 3.83 the next value is 4.2 so, 3.83 is 0, 1 and 4.2 is 3, 0. There will be a corresponding Ω value from here. So, these are the $k_r a$ values. Now for the structure the flexural behaviour in the circumferential direction looks like this. This is for $n = 0$ is a breathing mode where the shell radially expands and contracts. And for $n = 1$ we will have a cosine theta type variations.

So, the shell profile oscillates back and forth, left to right. So, it moves back and forth left to right. This is the nominal in a circle. It moves oscillates back and force then the $n = 2$ is easier to see. It is the ovaling mode. So, if we take this as a nominal circle and it ovals. So, it in one part it goes like this the other half it goes like this so, it ovals this is $n = 2$ then $n = 1$. And let me say that coupling between fluid and shell is circumferential order based.

Circumferential order base that means n based. So, the n of the structure couples with the n of the fluid. So, that means when the structural wavenumber of a particular n comes close or crosses to a fluid wavenumber in same n same circumferential order then coupling is good.

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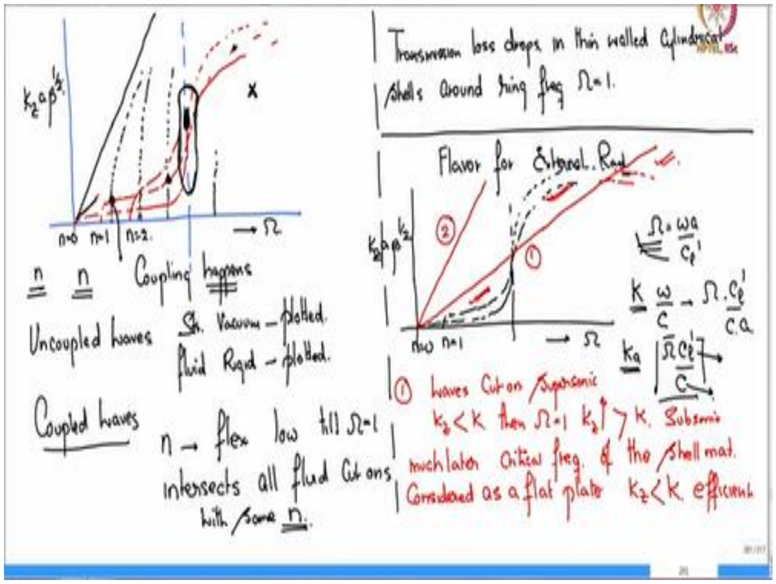

So, if we now plot both the pictures so what we are now going to do is plot both pictures. So, this is my ring frequency. So, let me plot a few structural cut-ons. The first one starts at 0 stays low $n = 0$ and then starts to rise rapidly and then it starts to bend like this. The next one maybe cuts on here and then also stays low and somewhere it starts to rise and behave like this. The third one starts maybe here and does some meandering etcetera.

Now let us say that my plane wave line is going like this let us say plane wave line. The plane wave line goes like this or let me do one thing for clarity let me leave it. Let me say it has a higher angle, it does not matter. Now my acoustic cut on they will rise vertical. The acoustic cut on will rise vertically straight and then they will go. Based on the structure and fluid properties the number will be decided based on the thickness of the shell the shell wavenumbers will be decided etcetera.

So, these fluids cut on rise vertically and as I said the coupling is n based. So, a particular n of a structure let us say can intersect with a certain set of n's maybe somewhere here. So, a particular structural wavenumber carrying a certain n with it will go and intersect all those cut-ons fluid cut-ons which are the same n and that is when coupling occurs strong. Let me say one more thing here that we are plotting uncoupled waves.

So, that means what we have looked at structure as though it is in vacuum and plotted its waves. We looked at fluid and as though the structure is rigid and plotted its waves. This helps us get a picture, this helps us get an idea of what is happening. However actual waves are coupled. So, in actuality when both are present this is not the picture. So, I would like you to understand that. However, this picture of this way of plotting the uncoupled waves together gives us a physical feel for what is happening.

So, we will continue with that. So, what I was saying was that a particular n structural flexural wave cuts on and remains low in its value till Ω = 1. And therefore, along its path it intersects all fluid cut-ons with the same n. And this intersection is a kind of coincidence where the structural wave if it is vibrating will communicate with the fluid or if there is a lot of sound in the fluid it will communicate with the structure. So, these intersections are very important.

Now for certain dimensions of let us say fluid pipes it so happens that a large number of coincidences happened around here. It is possible based on the structural material, the fluid material and the diameters and so forth. It is possible that you have a lot of coincidences. So, it has been observed that transmission loss drops in thin-walled cylindrical shells around ring frequency that is Ω = 1. So, now this is the picture in general.

Now let me just give you a flavour of external radiation. Now you have structural waves that are staying low and then they rise very suddenly. And then they become dominantly flexural and so forth. So, this is what is happening. Now my non-dimensional wavenumber is $\frac{\omega a}{c_l'}$ whereas my acoustic wavenumber k is $\frac{\omega}{c}$. And therefore, $\Omega \frac{c_l'}{ca}$.

Or if I want ka then we will leave it as $\Omega \frac{c_l'}{c}$ on the omega axis. So, this based on the relative values of c_l' the longitudinal speed in the material and the speed of sound in the fluid the acoustic wavenumber line could lie here or it could lie here it could lie here like this. So, in terms of external radiation it is scenario 1. Scenario 1 is that the waves cut on supersonic. Why? Because k_z happens to be below k then around $\Omega = 1$ the k_z rises.

And it becomes greater than k subsonic. And then much later you will reach the critical frequency of the shell material considered as a flat plate. There the flexure again drops there the flexure drops and k_z becomes less than k. So, that the shell is again efficient. Shell starts off as an efficient radiator, loses its efficiency and again becomes efficient that is 1 scenario. Just a minute before I change the page let me put the labels. So, this could be say $n = 0$.

This could be n =1 this is let us say Ω axis this is $k_z a \beta^{1/2}$. Similarly, this is Ω. Let us say this is $n = 0$, $n = 1$, $n = 2$. And this is $k_z a \beta^{1/2}$.

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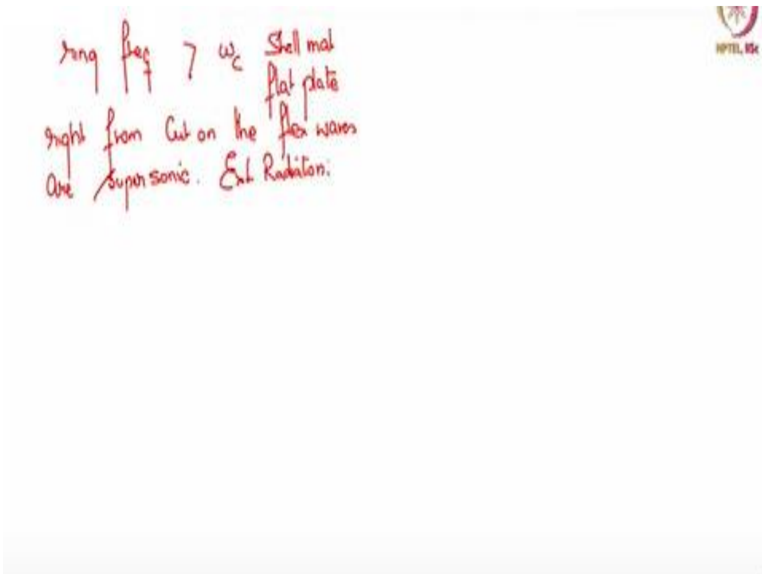

The other scenario is it is possible that the ring frequency is above your coincidence frequency ω_c where you consider the shell material as a flat plate. Then right from cut on the waves or the flexural waves are supersonic. So, this is for external radiation. I will close the lecture here we are running out of time. We will continue next class. Thank you.