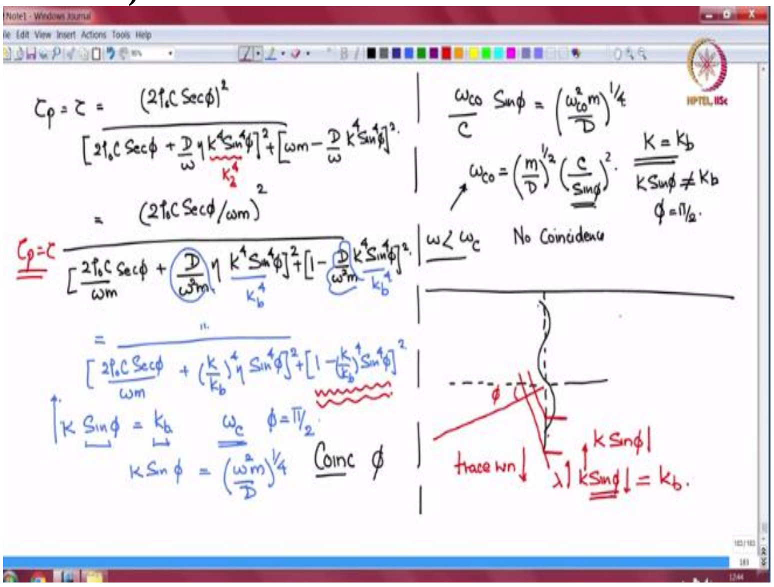
Sound and Structural Vibration
Prof. Venkata Sonti
Department of Mechanical Engineering
Indian Institute of Science – Bengaluru

Lecture - 53
Transmission Loss in Different Situations

(Refer Slide Time: 00:36)

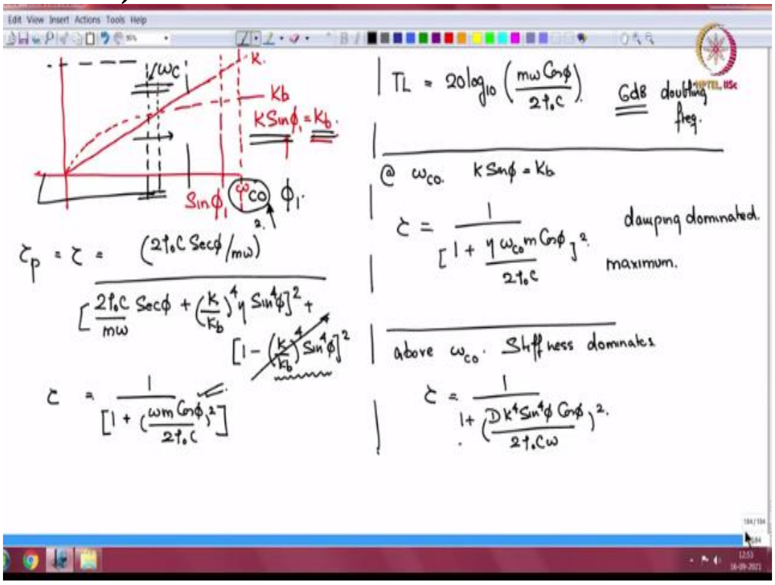


Good morning and welcome to this next lecture on sound and structural vibration. We are looking at transmission through flexible panel and we have reached the stage where you can see this is my power or intensity transmission coefficient because media on both sides are same. Just want to reiterate one thing here that let us say the free wavenumber on the panel or free wavelength also is like this, we can only show the wavelength, and this is let us say the incidence axis.

Now if and this is ϕ, so ϕ is becoming smaller, what will happen is that the trace wavelength will become bigger, so that the trace wave number will become smaller. So, $K \sin\phi$ will become smaller as ϕ reduces, I am just giving the picture here. So, this is a trace wavelength which is becoming bigger whereas wavelength becomes bigger the corresponding trace wavenumber becomes smaller.

So, now that has to match your free wavenumber and therefore, the frequency angle is fixed the frequency has to go up so that you can actually match the free wavenumber, we are talking so much because there are these terms where you have 1 minus the ratio forced to free wave numbers.

(Refer Slide Time: 03:08)



So, in another way if we want to see it again if we have a free acoustic wavenumber line here and this is my K_b panel free wave number line over here then let us say I have a certain angle at which I am incident, then let us say this has to be the frequency for that angle certain ϕ_1 such that I have a certain $\sin\phi_1$. So, I have to be here at this frequency for matching $K\ \sin\phi_1 = K_b$.

If ϕ_1 is reduced then $\sin\phi_1$ is reduced then I have to be at a higher frequency to match the free wavenumber, this is my free wavenumber line and this is my acoustic wave line. So, now let us look at this picture which we have or this expression that we have

$$\tau_p = \tau = \frac{(2\rho_0 c\ \sec\phi\ /\omega m)^2}{\left[\frac{2\rho_0 c\ \sec\phi}{\omega m} + \left(\frac{K}{K_b}\right)^4 \eta \sin^4\phi\right]^2 + \left[1 - \left(\frac{K}{K_b}\right)^4 \sin^4\phi\right]^2},$$

so by the same accord, if I am at such a frequency that I am just barely at coincidence.

Then ϕ_1 has to be 90 degrees there is no other choice and so, if I am falling below this basic coincidence frequency then I have no coincidence possible. So, for a certain ϕ_1 I could be at a certain frequency higher than the basic coincidence frequency, this is my basic coincidence frequency here then I could be at some particular coincidence frequency that is related to ϕ_1 such that $K\ \sin\phi_1 = K_b$.

So, when we mean low frequency we will talk of low frequency so, we could mean that we are wave below the basic coincidence, or we could say that we are wave below ω_{co} the currently possible coincidence.

So, now we are going to talk of low frequency which means below the possible coincidence at a certain ϕ angle. So, now what happens then that means this term is small and if we say that the loss factor term is also small then my τ is given by $\frac{1}{\left[1+\left(\frac{\omega m\cos\phi}{2\,\rho_0 c}\right)^2\right]}$ and based on materials placed in gases are materials metals placed in air and so forth this term can be larger than one.

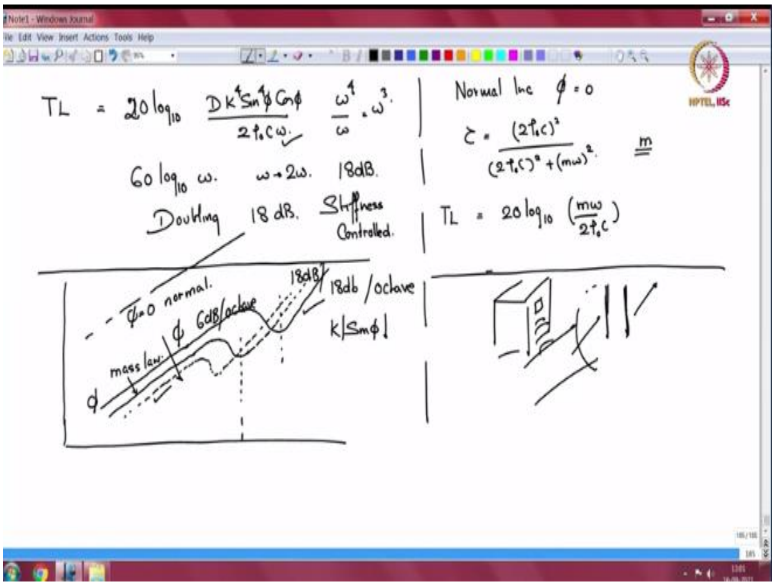
In which case the τ depends only on that term so, that my transmission loss or sound reduction index is $20\log_{10}\left(\frac{m\omega\ \cos\phi}{2\,\rho_0 c}\right)$ you can see this is again the 6dB increment per octave or doubling of frequency this is the mass law. Now as I said low frequency can be taken in two ways for a given ϕ if you are about the basic coincidence then there is a good enough range where we call low frequency because at a certain ϕ the coincidence frequency is higher.

Now, at coincidence the low frequency we are done at coincidence at ω_{co} what happens? Now my $K\ \sin\phi = K_b$ this term is actually exactly 0. Now this term τ after cancelling all the terms can be written as $\frac{1}{\left[1+\frac{\eta\omega_{co}m\cos\phi}{2\,\rho_0 c}\right]^2}$ therefore, this region is now damping dominated at coincidence and this is the minimum value this happens to be them or rather this happens to be the maximum value this is the maximum value.

Why the denominator becomes a minimum? So, there is a maximum value so maximum transmission value that means transmission loss goes to a minimum. Finally above coincidence the stiffness term dominates, so that what do we get now? We get τ given by $\frac{1}{1+\left(\frac{D\,K^4\sin^4\phi\ \cos\phi}{2\,\rho_0 c\,\omega}\right)^2}$.

So, this stiffness term can be bigger than one.

(Refer Slide Time: 12:31)



And therefore, the transmission loss or sound reduction index becomes equal to

$$20\log_{10}\left(\frac{D\ K^4 \sin^4\phi\ \cos\phi}{2\ \rho_0 c\ \omega}\right),$$

and then in a fourth we have ω^4 and below we have an ω here so this is ω^3. So, finally in terms of frequency we get $20\log_{10}\omega$ so that means what when frequency doubles when ω becomes 2ω we get log 2 times 60 which is 18 decibels.

So, in this range for doubling of frequency we get 18dB increment in transmission loss this is the stiffness control region so what is the picture now? So, the picture now is this here so we have transmission loss so at a certain angle ϕ we have a mass law and let me pull this a little longer then you have a dip this is coincidence and you are at 18dB per octave let me do this one once you have, so this is a certain angle I get a dip and then I get 18dB per octave this is the stiffness control region this is a smaller angle.

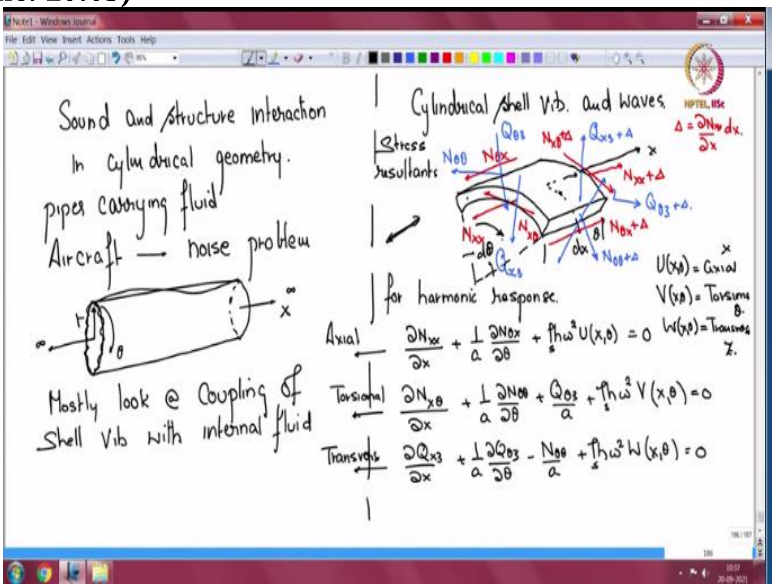
Now what happens? At a higher angle I have because angle is small coincidence occurs at highest frequency $\sin\phi$ pulls it down, so frequency has to be higher as my angle increases, I have this line I have this line then I have a dip here and then the dip starts to go parallel to again 18dB per octave at the stiffness control. So, this is the coincidence at a higher frequency and it continues.

So, now if you take a further higher angle then I will have a dip here and then the dip will continue at 18dB. Smaller angle higher coincidence higher angle lower coincidence. So, this is incrementing ϕ this is 18dB per octave this is 6dB per octave. And finally, if you actually do normal incidence then ϕ_0 and τ is going to be $\frac{(2\,\rho_0 c)^2}{(2\,\rho_0 c)^2+(m\omega)^2}$ and so, if which typically m is a bigger entity mass dominated.

Then we will have transmission loss given by $20\log_{10}\left(\frac{m\omega}{2\,\rho_0 c}\right)$ which will be some here which is parallel to the 6dB curve but keeps going parallel to the 6dB curve but it keeps going this is $\phi = 0$ normal so there are many more studies that can be done in transmission actually when you have a building and there is a window and there is traffic going by and that is sound reaching your window, so there is going to be a transmission of sound into your room.

Then you have a double walled indoors, two layers of windows that are used. So, now that could be a study on how to layer could have two layers? So, two flexible partitions and the sound waves incident. So, what is the sound wave transmitted? So, there are such studies possible and also averaging across angles what is a angularly average transmission? But those are at this stage beyond this course so I will not get into that. So, I think to give a basic idea of sound transmission this is good enough section.

(Refer Slide Time: 20:05)



The next important topic is sound and structure interaction in cylindrical geometries so cylindrical objects like pipes carrying fluid or quite common and of course we have aircraft jar approximately cylindrical and there is a noise problem associated with aircraft, so we will look at how cylindrical shell vibrations coupled with the fluid inside or the fluid outside and we look at it for this course infinite geometries.

So, this is a cylindrical shell which is flexible it can vibrate carry waves and it extends to infinity in this direction most likely that will be my x direction this will be θ direction and then that is radial. Now one more thing we will mostly we talk about coupling of shell vibrations with internal fluid not so much with exterior is a very elaborate topic more or less you can spend a lifetime in it.

And so, it is just if based of what it is? This could become actually a separate topic entirely for a course. So, we will just skim through some details so, firstly we will just look at cylindrical shell vibrations and waves in an infinite cylindrical shell. So, if I draw a shell element it looks like this it is a place my coordinate here, this is my x axis and let us say this is my θ direction now I will just put my force or stress resultants.

So, if that is the x direction I will have this will be N_{xx} this will be N_{xx} plus an increment will not write the increment all the time increment will be of the form $\frac{\partial N_{xx}}{\partial x}\ dx$ of this form there is no space that is why. Then we will have $N_{\theta x}$ in the x direction and $N_{\theta x} +\ \Delta$ then we have let us see $N_{\theta\theta}$ and this will be $N_{\theta\theta}$ plus increment then we will have vertical shears resultant Q_{x3}, $Q_{x3} + \Delta$ then $Q_{\theta 3}$ and $Q_{\theta 3} +\ \Delta$.

Then we have let me $N_{x\theta}$ and $N_{x\theta} + \Delta$ these are the stresses resultant in terms of this stress resultant we can write the shell equations for a harmonic response which you look like

$$\frac{\partial N_{xx}}{\partial x} + \frac{1}{a}\frac{\partial N_{\theta x}}{\partial \theta} + \rho_s h\omega^2\ U(x,\theta) = 0.$$

Then

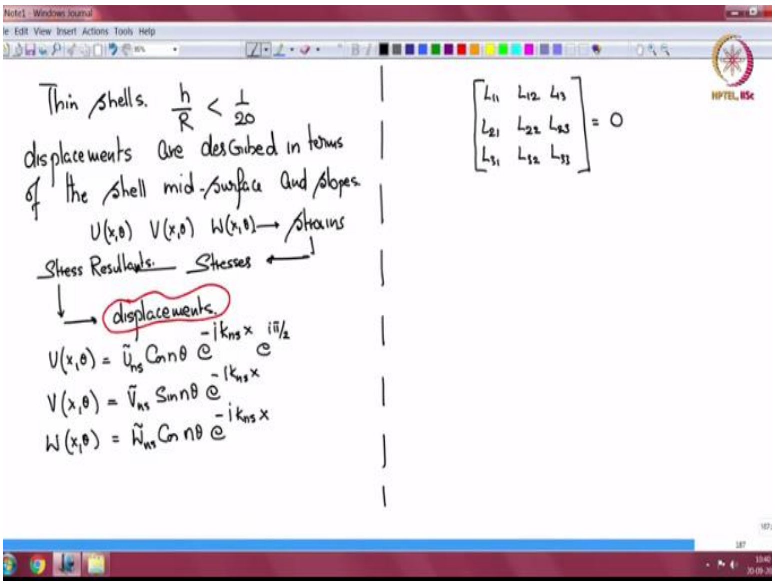
$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{a}\frac{\partial N_{\theta\theta}}{\partial \theta} + \frac{Q_{\theta 3}}{a} + \rho_s h\omega^2\ V(x,\theta) = 0,$$

and

$$\frac{\partial Q_{x3}}{\partial x} + \frac{1}{a}\frac{\partial Q_{\theta 3}}{\partial \theta} - \frac{N_{\theta\theta}}{a} + \rho_s h\omega^2\ W(x,\theta) = 0\ .$$

So, $U(x,\theta)$ is axial displacement of a point in the x direction axial displacement then $V(x,\theta)$ is torsional this way V displacement is this way then W displacement is transverse there is in the x direction to the θ direction we can make this in z direction.

(Refer Slide Time: 31:02)



And we are looking at thin shells theory were the thickness of the shell to the radius of curvature is in some thumb rule less than 1 / 20 and so that the displacements are described in terms of the shell mid-surface and slopes so, there are these approximations So, these approximations of displacements are described $U(x,\theta)$, $V(x,\theta)$, $W(x,\theta)$ and then from there you compute your strains linear and then you compute your stresses and then from the stresses you come to stress resultants.

It means we can put the stress resultants in terms of mid plane displacements then what happens is? You have the entire 3 equations. So, I did not say it I suppose, so this describes the axial motion of the shell element, this describes the θ direction or torsional force balance in the torsional direction, this is force balance in the transverse direction. So, this is dx this distance is dx and this angle is $d\theta$.

So, if we put back the displacements into these 3 equations put the stress resultant in terms of displacements and how do we do that?

$$U(x,\theta) = \widetilde{U}_{ns}\ \cos n\theta\ e^{-ik_{ns}x}e^{i\pi/2}\ ,$$

$$V(x,\theta) = \tilde{V}_{ns}\ \sin n\theta\ e^{-ik_{ns}x},$$

$$W(x,\theta) = \widetilde{W}_{ns}\ \cos n\theta\ e^{-ik_{ns}x}\ ,$$

these are the mid plane displacements.

After this if this is substituted back into the equations that are in terms of displacement derived from stress resultants then what we get is a matrix? We get a matrix which has

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} = 0.$$

We are running out of time so I will close the lecture here we continue next class.