Sound and Structural Vibration
Prof. Venkata Sonti
Department of Mechanical Engineering
Indian Institute of Science – Bengaluru

Lecture - 52
Sound Transmission through a Flexible Partition

So, good morning and welcome to this next lecture on sound and structural vibration, we are looking at oblique plane wave transmission through an unbounded flexible panel. So, we have reached up to this point where the blocked pressure gives me the response.

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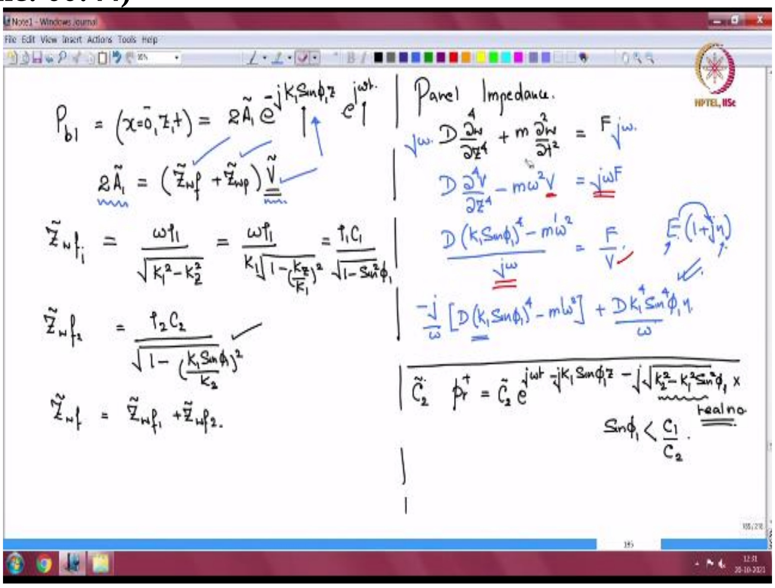

Now, let us see on the transmitted side. So, we are interested in $\tilde{C}_2$ should not forget that we are interested in $\tilde{C}_2$ so, the pressure on the transmitted side can be written as

$$p_r^+ = \tilde{C}_2 e^{j\omega t - jK_1 \sin\phi_1 z - j\sqrt{K_2{}^2 - K_1{}^2 \sin^2\phi_1}\, x}.$$

So, for far field transmission this should be a real number or far field transmission so that is one condition. So, what does that give me let us say? That says if you compute $\sin\phi_1$ has to be less than $\frac{c_1}{c_2}$. I would like to make a correction here I am using m everywhere so this dash should not be there.

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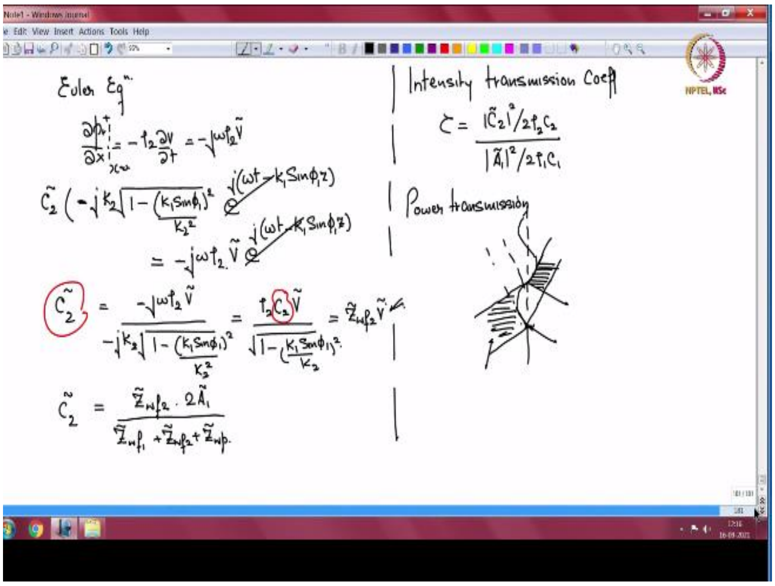

Next, if I again use my Euler equation

$$\left.\frac{\partial p_r^+}{\partial x}\right|_{x=0} = -\rho_2 \frac{\partial V}{\partial t} = -j\omega\rho_2\tilde{V},$$

then that means

$$\tilde{C}_2 - jK_2\sqrt{1-\left(\frac{K_1 \sin\phi_1}{K_2}\right)^2}\, e^{j(\omega t - K_1 \sin\phi_1\, z)} = -j\omega\rho_2\tilde{V}e^{j(\omega t - K_1 \sin\phi_1\, z)}.$$

Now, this phasor part goes away.

So that my

$$\tilde{C}_2 = \frac{-j\omega\rho_2\tilde{V}}{-jK_2\sqrt{1-\left(\frac{K_1 \sin\phi_1}{K_2}\right)^2}} = \frac{\rho_2 c_2\tilde{V}}{\sqrt{1-\left(\frac{K_1 \sin\phi_1}{K_2}\right)^2}} = \tilde{Z}_{wf2}\tilde{V}.$$

Now so we have found $\tilde{V}$ from blocked pressure. So, $\tilde{C}_2$ which is our interest again let me remind you that this is the transmitted pressure amplitude this is the velocity of sound in the right-side medium.

So,

$$\tilde{C}_2 = \frac{\tilde{Z}_{wf2}\, 2\tilde{A}_1}{\tilde{Z}_{wf1} + \tilde{Z}_{wf2} + \tilde{Z}_{wp}}$$

Now, we want the intensity transmission coefficient and then the power transmission coefficient. The intensity transmission coefficient τ is given by $\frac{|\tilde{C}_2|^2/2\rho_2 c_2}{|\tilde{A}_1|^2/2\rho_1 c_1}$ because the media are different there is refraction.

Now, once we bring in, we want to get to the power transmission that means some area term has to come in now that area term is going to be what there is a slight difference in the beam width. So, if I say this is my panel let us say and this is my wave vector. So, these are the wave fronts. So, this is the beam width here but if the wave vector is in this direction changes here than the normal is like this. So, now this beam width is different.

So, the beam width that is working on the half-cell of a panel that beam works on half-cell of this panel so, the widths are different. So that width has to be brought into account for the power transmission, so far as intensity transmission but once you talk of power you have to bring that beam width in.

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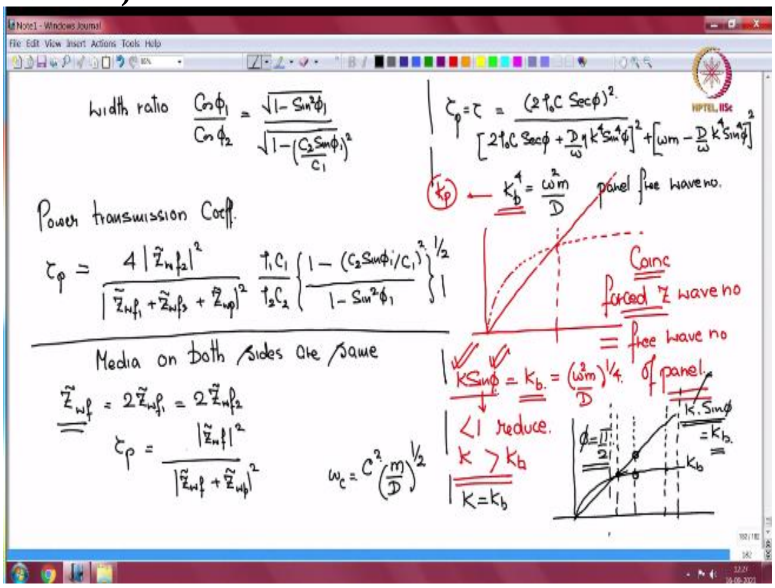

So, if you bring that beam width into the picture the width ratio happens to be

$$\frac{\cos\phi_1}{\cos\phi_2} = \frac{\sqrt{1-\sin^2\phi_1}}{\sqrt{1-\left(\frac{c_2\sin\phi_1}{c_1}\right)^2}}.$$

So, now we write the power transmission coefficient as this given by let us call it

$$\tau_p = \frac{4\left|\tilde{Z}_{wf2}\right|^2}{\left|\tilde{Z}_{wf1}+\tilde{Z}_{wf2}+\tilde{Z}_{wp}\right|^2}\frac{\rho_1 c_1}{\rho_2 c_2}\left\{\frac{1-(c_2\sin\phi_1/c_1)^2}{1-\sin^2\phi_1}\right\}^{1/2}.$$

So, this is the true picture when you have two different media. So, we will not continue with this thought for from here onwards. So, first thought is media are same the media on both sides are same that simplifies certain things immensely. So that the $\tilde{Z}_{wf}$ is twice $2\tilde{Z}_{wf1}$ or $2\tilde{Z}_{wf2}$, so we will use this notation. So, what is now the power transmission coefficient

$$\tau_p = \frac{\left|\tilde{Z}_{wf}\right|^2}{\left|\tilde{Z}_{wf} + \tilde{Z}_{wp}\right|^2}.$$

If you put in all the terms now, what the impedances stand for if we put the expression now comes out to be

$$\tau_p = \tau = \frac{(2\rho_0 c\ \sec\phi)^2}{\left[2\rho_0 c\ \sec\phi + \frac{D\eta}{\omega}\ K^4 \sin^4\phi\right]^2 + \left[\omega m - \frac{D}{\omega}\ K^4 \sin^4\phi\right]^2}.$$

Now please remember this that ${K_b}^4 = \frac{\omega^2 m}{D}$ the panel free wave number so let us looks at this picture once.

So, I have my acoustic wavenumber going this wave straight up and this is the panel free wave number as I said I use K_p and K_b interchangeably we will stick with K_b for now. So, the panel free wave number goes like this. Now the coincidence what is coincidence now? It is the forced z wavenumber being equal to the free wavenumber of the panel that is coincidence it is like resonance what I am forcing is equal to what is naturally liked by the system.

So, forced z wave number is $K\ \sin\phi$, so

$$K\ \sin\phi = K_b = \left(\frac{\omega^2 m}{D}\right)^{1/4}.$$

Now $\sin\phi$ is a reducing factor $\sin\phi$is less than 1 always so it will reduce so that means what K has to be greater than K_b. So now, there are two terms that you can control the frequency and the angle these are the two terms you can control so that the product is equal to K_b.

So, if I am at a high frequency suppose I am at a very high frequency let me redraw in this way this is my wavenumber acoustic. So, let us say this is my panel wavenumber. So, this goes here so I am at a very high frequency that means K is much bigger than I am here much bigger than K_b and K. So, I have a certain angle of ϕ which will give me equality then I have smaller and smaller angles of ϕ which will also give me equality with K_b.

If I can go to a higher frequency, then phi can be even smaller so that $K \ \sin\phi = K_b$ because K is high and $\sin\phi$ brings it down. So, smaller $\sin\phi$ so that $K \ \sin\phi = K_b$ that said higher and higher frequencies. But if I now come down in frequency then my K is not so much bigger than K_b. So, my $\sin\phi$ cannot be that small and in the limit when $K = K_b$ then ϕ has to be equal to $\pi/2$ at which you can have come to coincidence.

Because $\sin\pi/2$ is 1 then $K = K_b$ but if you now go below at means $K = K_b$below this which is basic coincidence frequency. If you go below the basic coincidence frequency which you should know it is equal to $c^2\left(\frac{m}{D}\right)^{1/2}$ this is a basic panel fluid coincidence frequency if we go below that, then there is no ϕ angle for which you will have coincidence so that should be remembered. So, now what did we do here let us see.

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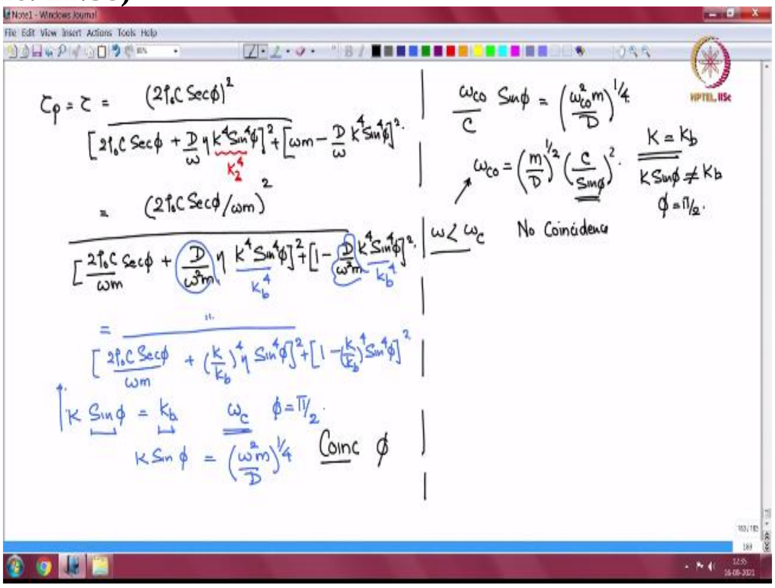

So, if we now use these let me rewrite actually, we have τ_p or τ intensity and power are same because angles have turned up to be same. So,

$$\tau_p = \tau = \frac{(2\rho_0 c \ \sec\phi)^2}{\left[2\rho_0 c \ \sec\phi + \frac{D\eta}{\omega} K^4 \sin^4\phi\right]^2 + \left[\omega m - \frac{D}{\omega} K^4 \sin^4\phi\right]^2}$$

So, if we divide by ωm what do we get?

$$= \frac{(2\rho_0 c \ \sec\phi/\omega m)^2}{\left[\frac{2\rho_0 c \ \sec\phi}{\omega m} + \frac{D\eta}{\omega^2 m} K^4 \sin^4\phi\right]^2 + \left[1 - \frac{D}{\omega^2 m} K^4 \sin^4\phi\right]^2},$$

remember $\frac{D}{\omega^2 m}$ is $K_b{}^4$ so, we have the ratio of the forced wavenumber and free wavenumber here. So, I think I have written like that again.

So, I have

$$= \frac{(2\rho_0 c \ \sec\phi \,/\omega m)^2}{\left[\frac{2\rho_0 c \ \sec\phi}{\omega m} + \left(\frac{K}{K_b}\right)^4 \eta \sin^4\phi\right]^2 + \left[1 - \left(\frac{K}{K_b}\right)^4 \sin^4\phi\right]^2}.$$

So, based on the frequency because again we are saying that you have to equal $K \ \sin\phi$ with K_b based on how high you are in frequency you have a range of $\sin\phi$ to match K_b but once $K = K_b$ that means your current frequency is the coincidence frequency only $\phi = \frac{\pi}{2}$ can match.

So, if we look at this equation $K \ \sin\phi \ = \left(\frac{\omega^2 m}{D}\right)^{1/4}$ then and we are talking of coincidence with angle ϕ. So, we have

$$\frac{\omega_{co}}{c} \sin\phi = \left(\frac{{\omega_{co}}^2 m}{D}\right)^{1/4}$$

or the coincidence frequency is now a function of

$$\omega_{co} = \left(\frac{m}{D}\right)^{1/2} \left(\frac{c}{\sin\phi}\right)^2,$$

as $\sin\phi$ goes down the coincidence frequency has to go up for to give a match with the bending wave number.

Whereas if ϕ is 90 degrees, then you are back to your original basic coincidence, which also means that if you are at such a frequency that K and K_b are already equal then no other $\sin\phi$ will give you a match except $\phi = \pi/2$. So, $\phi = \pi/2$ is the lowest frequency, if you are below the basic coincidence ω_c if you are below this then you cannot get no coincidence. So, now we will look at various regimes based on this formula over here meanwhile, the time is actually up. So, I will close the lecture here and continue in the next class.