Sound and Structural Vibration
Prof. Venkata Sonti,
Department of Mechanical Engineering
Indian Institute of Science - Bengaluru

Lecture – 51
Frequency Dependence of Sound Transmission

Good morning and welcome to this next lecture on sound and structural vibration. We were looking at a normal incidence of a plane wave on a partition and transmission from there.

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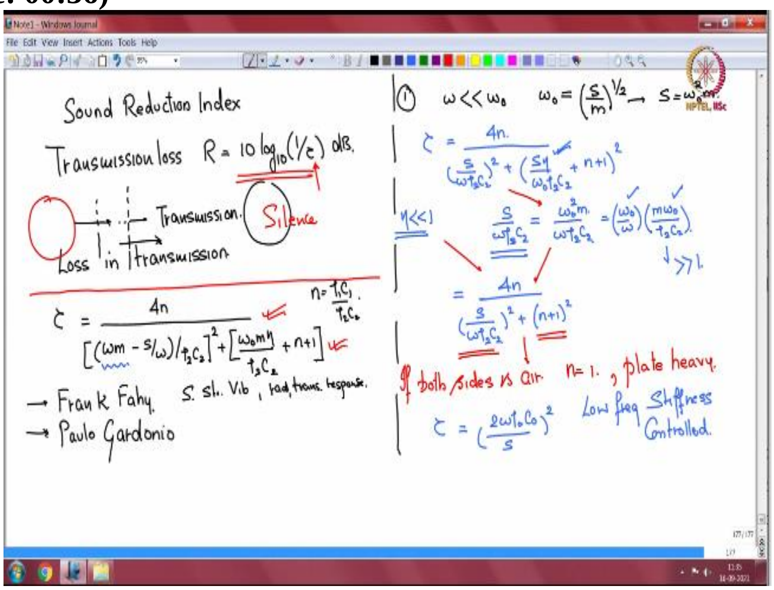

So, we reached this point where we define the transmission loss. So, let me just put that expression that comes again and again. So, the transmission coefficient is

$$\frac{4n}{\left[\left(\omega m-\frac{s}{\omega}\right)/\rho_2c_2\right]^2+\left[\frac{\omega_0 m\eta}{\rho_2c_2}+n+1\right]^2}.$$

So, n was the ratio of fluid impedances. I should probably mention that I am taking this from the book Frank Fahy, almost entire in its entirety I am taking it from their sound and structural vibration radiation transmission response is in the book.

As I said there is a second author now, I think Paulo Gardonio. In the later versions I am taking this portion directly from that. So, this is my transmission coefficient. So, now we will see the answer is there. We have found the answer there is nothing more to be done you use your values

ω, m and s values and plot it and look at it, we are done. But we will try to get more insights through limiting processes.

So, first limiting processes, what is this value when ω is much less than ω_0? ω_0 is that natural frequency of the panel on its mounts $\left(\frac{s}{m}\right)^{1/2}$ that is the natural frequency. So, if we are way below natural frequency what happens? This also amounts to s being equal to $\omega_0{}^2$. So, if omega is way below ω_0, we are at low frequencies then this term will be weak.

So, as a result we get τ given by $\frac{4n}{\left(\frac{s}{\omega\rho_2 c_2}\right)^2+\left(\frac{s\eta}{\omega_0\rho_2 c_2}+n+1\right)^2}$. Now typically, η is a very small entity, for materials it is very small loss factor. So, also, $\frac{s}{\omega\rho_2 c_2} = \frac{\omega_0{}^2 m}{\omega\rho_2 c_2} = \left(\frac{\omega_0}{\omega}\right)\left(\frac{m\omega_0}{\rho_2 c_2}\right)$. Now, $\frac{\omega_0}{\rho_2 c_2}$ itself is actually quite bigger than 1, typically for normal occurrences of air and other materials, on top of that we are below resonance.

So, this term dominates and this term dominates, so, this is a dominating term. So, and the loss factor we are considering to be small enough. So, this should look like $\frac{4n}{\left(\frac{s}{\omega\rho_2 c_2}\right)^2+(n+1)^2}$, if further. So, step by step we are doing it. So, we put some assumptions on this we reached here this assumption, and this assumption gives us this and further we make one more, if both sides is air, then $n = 1$.

And say the plate is heavy, then what will happen is this term will dominate over that because n is 1 here. So, then the τ will acquire this value of $\left(\frac{2\omega\rho_0 c_0}{s}\right)^2$. So, this low frequency is stiffness control. On the other hand, again looking at the; same formula.

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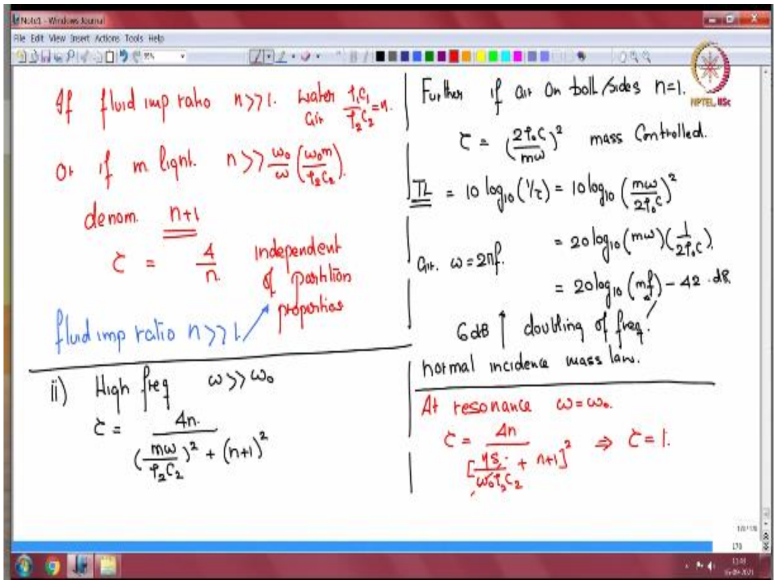

If fluid impedance ratio is large that means n is large, that means the incident side is what are let us say the receiving side is air let us say and this $\rho_1 c_1 \ / \ \rho_2 c_2$ which is n, so n is a large quantity or if the material of the panel is light or n is such that it is much greater than $\frac{\omega_0}{\omega}\left(\frac{m\omega_0}{\rho_2 c_2}\right)$, then basically in the denominator the n term dominates, you have $n + 1$. So, n dominates so, my transmission coefficient is given by $\frac{4}{n}$.

So, this is now independent of partition properties. So, where do we get if fluid impedance is heavy or not heavy large, fluid impedance ratio and n is large then the transmitted coefficient is independent of partition properties. So, that is the low frequency story. So, we have done the low frequency story. What about the high frequency story? All about the high frequency which means ω is greater than ω_0.

You about the panel mounting resonance then τ is given by $\frac{4n}{\left(\frac{m\omega}{\rho_2 c_2}\right)^2 + (n+1)^2}$ and further if we have air on both sides that is $n = 1$ and the mass of the panel is quite heavy compared to air then my τ starts to look like $\left(\frac{2\rho_0 c}{m\omega}\right)^2$, this is mass controlled region. Now let me just bring in the idea of transmission loss or sound reduction index is given by $10 \log_{10}(1/\tau)$. So, then that is $10 \ \log_{10}\left(\frac{m\omega}{2\rho_0 c}\right)^2$ or equal to $20 \log_{10} m\omega \left(\frac{1}{2\rho_0 c}\right)$.

So, we can do many things we can do for air medium, and we write ω in terms of hertz if we do that, we get $20\log_{10}(mf) - 42$ in terms of dB. So, now what this says is that if my frequency doubles I get a 6 dB increments for doubling of frequency in transmission loss. So, this is called the normal incidence mass law. Just so that you know this term comes up again and again mass law. Now so, obviously is the mass control region beyond resonance high frequencies.

Now what about at resonance which is $\omega = \omega_0$. At $\omega = \omega_0$ the 2 impedance terms $m\omega$ and the stiffness term are equal, and they disappear. So, we are left with certain terms which are $\frac{4n}{\left[\frac{\eta s}{\omega_0 \rho_2 c_2} + n + 1\right]^2}$ and if the fluid is the same and loss factor is very small what do we get we get loss vector is small we drop it fluid is same and is 1. So, I get 1, $\tau = 1$.

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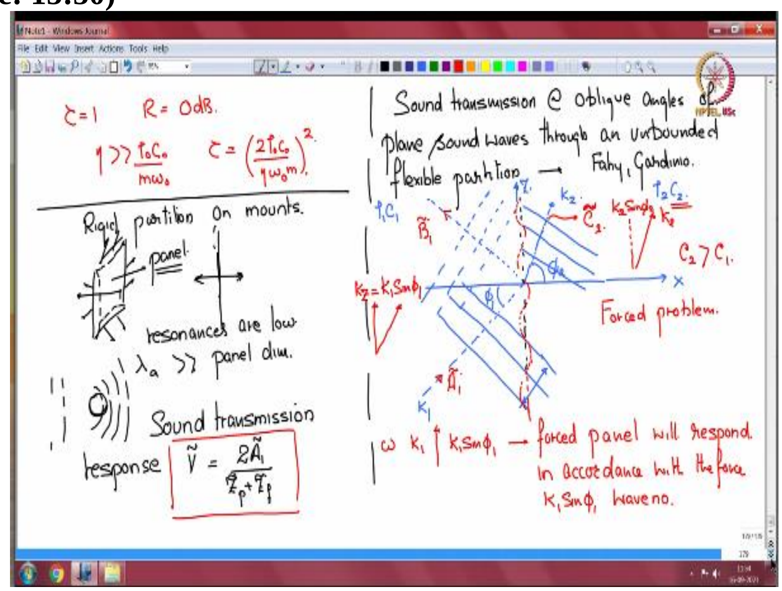

On the other hand, you could have which means $\tau = 1$ which means $R = 0$ dB, no transmission loss, good transmission that makes on the other hand you could have the loss factor term dominate and then the transmission coefficient is $\left(\frac{2\rho_0 c}{\eta\omega_0 m}\right)^2$. So, this is the story let me say this is the story of a rigid partition on mounts and we are trying to model the very simplest model of a plane wave transmission.

So, if you want to look at the usefulness of this model it would actually be that you have kind of some window. And now this partition vibrates in unison, back and forth. There are no waves on it. So, it is rigidly oscillating back and forth. So, that would kind of seem to be the 1,1 mode of some reasonable size panel. But the thing is that the resonances of these panels let us say windows that you use to study transmission across rooms, their resonances are very low where the wavelength of sound is much bigger than the panel dimensions.

So, that means you have something like transmission through a hole. So, it is very much dominated by the diffraction effects. So, you have diffraction effects. So, here also you would have diffraction effects dominate diffraction of the aperture effect dominate or be equal in addition to the panel dynamics. So, we have just looked at panel dynamics. So, in some sense this is not very useful except that in sound transmission the response can be seen as twice the blocked or the blocked pressure twice the incident value over the panel impedance plus the fluid impedance.

$$\tilde{V} = \frac{2\tilde{A}_1}{\tilde{Z}_p + \tilde{Z}_f}.$$

This is one idea that we should take away from the derivation. We will use this so, having obtained this idea, let us now look at a more realistic case of a sound transmission at oblique angles of plane sound waves through an unbounded flexible partition. Again, I am taking this almost verbatim kind of from that book by Fahy and Gardonio. So, now what are we talking about we have let us say to begin with a partition that can carry a flexural wave and let us say that this happens to be my x axis and that is my z axis.

Now at some angle at some angle you have a plane wave hitting the panel, if plane wave hits the panel is supposed to be 90 degrees. So, this angle is ϕ_1 and this side is $\rho_1 c_1$ to begin with that side is $\rho_2 c_2$ to begin with and so, by Snell's law you will have a reflection happening that is another plane wave, the wave fronts are normal to the wave vector. So, this is one wavelength actually. Now, because the medium is different there is going to be a refraction the wave will bend.

So, let us say it bends like this and so, now you have wave fronts which are going normal that wave vector. So, this is K_1 this is K_2 and this angle is ϕ_2. Now we have a flexural wave on this panel. So, let us say that is the flexural wave on this panel. Now this is a forced problem and so, at ω if K_1 happens to be the wavenumber in this direction, then wavenumber in that direction is $K_1 \sin \phi_1$. And so, the forced panel will kind of follow or obey or respond in accordance with force and therefore, it will be forced to have $K_1 \sin \phi_1$ as the wavenumber.

So, let us see we have K_1 over here. So, that is going to be K_z which is $K_1 \sin\phi_1$ similarly will have K_2 and this will be $K_2 \sin\phi_2$ and we will for obvious reasons actually c_2 will be greater than c_1. So, now, let us say this is my incident amplitude, this is my reflected amplitude, and this is my transmitted amplitude. So, again tilde will be amplitude and no tilde is speed of sound.

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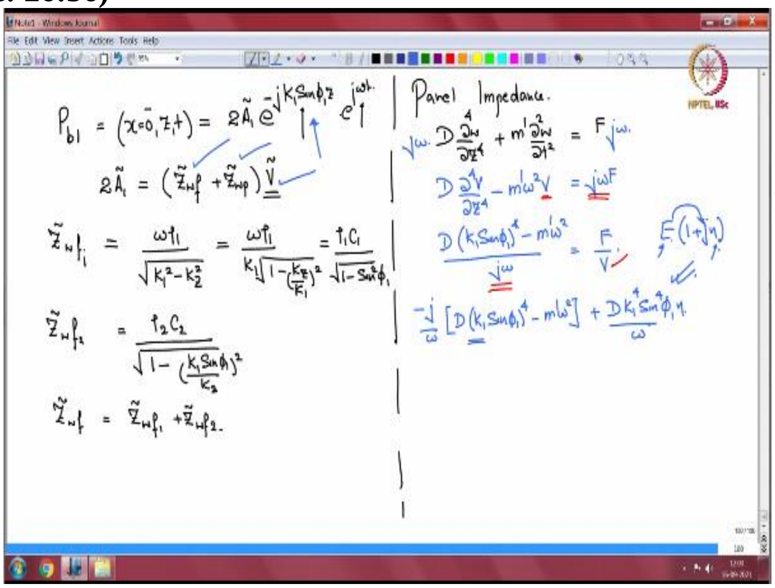

So, now let us see we will straight away go for the blocked pressure P blocked. What is that equal to

$$p_{b1}(x = 0^-, z_1, t) = 2\tilde{A}_1 e^{-jK_1 \sin\phi_1\, z} e^{j\omega t},$$

this is my blocked pressure. So, what is my velocity now, we can straight away write the velocity. So, that is given by

$$2\tilde{A}_1 = (\tilde{Z}_{wf} + \tilde{Z}_{wf})\tilde{V}.$$

The phasor part and the temporal part are common to both sides the velocity of the panel has the same z wavenumber, z temporal behaviour and therefore, it cancels so, this is actually the velocity amplitude. So, what is the fluid impedance? If you remember the fluid impedance was of this form earlier with the plus minus sign we had done if you remember, the wave impedance of a panel was carrying a wavenumber K_z as given by

$$\tilde{Z}_{wf1} = \frac{\omega\rho_1}{\sqrt{{K_1}^2 - {K_z}^2}}.$$

Now K_1 and K_z that are already arranged in a manner that K_1 is bigger than K_z, so this is fluid impedance of the ρ_1 side first side. So, this can be written as $\frac{\omega\rho_1}{K_1\sqrt{1-\left(\frac{K_z}{K_1}\right)^2}}$ which is $\frac{\rho_1 c_1}{\sqrt{1-\sin^2\phi_1}}$.

Similarly, the other side the other impedance is

$$\tilde{Z}_{wf2} = \frac{\rho_2 c_2}{\sqrt{1-\left(\frac{K_1 \sin\phi_1}{K_2}\right)^2}}.$$

K_2 is the wavenumber in the second medium.

And therefore, the total fluid impedance is $\tilde{Z}_{wf1} + \tilde{Z}_{wf2}$. Now, what about the panel impedance? The panel impedance is a one-dimensional panel again. So, if we write the equation of motion, it is

$$D\frac{\partial^4 w}{\partial z^4} + m'\frac{\partial^2 w}{\partial t^2} = F.$$

Now, if we make it into a velocity, I multiplied by $j\omega$ then this becomes equal to

$$D\frac{\partial^4 V}{\partial z^4} - m'\omega^2 V = j\omega F.$$

Then the wave number in the z direction which is $-jK_1\sin\phi_1\ z$. So, it will fall out 4 times. So, that will result in

$$\frac{D(K_1\sin\phi_1)^4 - m'\omega^2}{j\omega} = \frac{F}{V}.$$

Last, I have done some juggling here. So, I have taken this velocity put it in the denominator here taking this $j\omega$ put it in the denominator here and I have the rest here.

So, if I now take the j upstairs, what do I have? I have $\frac{-j}{\omega}[D(K_1\sin\phi_1)^4 - m'\omega^2]$ and we add a small damping term $\frac{DK_1{}^4\sin^4\phi_1\,\eta}{\omega}$ it is of the same form as your stiffness term you can take it as if it has come from the complex Young's modulus,

$$\frac{-j}{\omega}[D(K_1\sin\phi_1)^4 - m'\omega^2] + \frac{DK_1{}^4\sin^4\phi_1\,\eta}{\omega}.$$

And it has it is perpendicular to the stiffness term, so it does not have the j. So, it is a damping term it has a real part, we do this now we have Young's modulus, and we do $1 + j\eta$. So, the stiffness part has an η loss factor.

So, we have put it in that fashion over here. So, now this is my panel impedance. So, I had a fluid impedance, I have panel impedance now the whole thing is there. So, I know now my velocity, I can completely define my velocity. So, now what do we get? We time is actually up. So, I will close the lecture here. We will continue from the next class. Thank you.