

**Sound and Structural Vibration**  
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**Lecture – 49**  
**Model Average Radiation Efficiency Contd**

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The image shows a handwritten derivation in a software window. The derivation starts with an expression for radiation efficiency, involving terms like  $\frac{A_p \rho_a c k_a}{h k_p^2} \left\{ 1 + \frac{k^2}{(k_p C_n \theta)^2} \right\}$ . It then proceeds through several steps of algebraic manipulation and integration, leading to a final result for the common radiation efficiency  $R_{rad}^c = \frac{8 \rho_a c k_a^2}{\pi k_p^2 k^2}$ .

So, good morning and welcome to this next lecture on sound structural vibration. So, we have been looking at modal average radiation efficiency and we found the average value for the X edge radiators are we are in the process of finding the value for the X edge radiators. So, we continue with this. So, the result is here

$$\frac{A_p \rho_a c k_a}{b} \left\{ \frac{k_p}{\mu^3} \sin \theta_1 + \frac{1}{k_p \mu} \ln[\sec \theta_1 + \tan \theta_1] \right\}.$$

Now, we use the relations we found earlier and substitute here.

So, that becomes equal to

$$\frac{A_p \rho_a c k_a}{b} \left\{ \frac{k_p}{\mu^3} \frac{k}{k_p} + \frac{1}{k_p \mu} \ln \left[ \frac{k_p}{\mu} + \frac{k}{\mu} \right] \right\}.$$

So, if we do the cancellations the final answer is

$$\frac{A_p \rho_a c k_a}{b} \left\{ \frac{k}{\mu^3} + \frac{1}{k_p \mu} \ln \left[ \frac{k_p + k}{\mu} \right] \right\}$$

this is part 1. Then the  $b$  portion of this the  $b$  portion so, there you have corner modes corner radiators and if you look at Maidanik's approximate formula the  $R_{rad}$  for the corner modes is something like  $\frac{8\rho_a c k_a^2}{\pi k_{px}^2 k_{py}^2}$ .

So, this ends up as what

$$\int_{\theta_1}^{\theta_2} \frac{8\rho_a c k_a^2}{\pi (k_p^2 \sin^2 \theta k_p^2 \cos^2 \theta)}.$$

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Handwritten mathematical derivations on a whiteboard. The left side shows the evaluation of an integral from  $\theta_1$  to  $\theta_2$ , resulting in a logarithmic expression involving  $k$ ,  $\mu$ , and  $k_p$ . It also defines the  $y$ -edge radiator  $R_{rad}$ . The right side shows a similar derivation for the  $x$ -edge radiator, leading to a final expression for  $R_{rad}^{avg}$ .

So, if we now proceed, we will get

$$= \frac{8\rho_a c k_a^2}{\pi (k_p)^4} \left[ \frac{1}{\sin \theta \cos \theta} - \frac{2 \cos \theta}{\sin \theta} \right]_{\theta_1}^{\theta_2}.$$

Now, we can put the limits take the difference and we have already found expressions for

$$\sin \theta_1 \cos \theta_1 = \sin \theta_2 \cos \theta_2 = \frac{k\mu}{k_p^2},$$

so, the result will end up looking like

$$= \frac{16\rho_a c k_a^2}{\pi k_p^4} \left\{ \frac{\mu}{k} - \frac{k}{\mu} \right\}.$$

And of course, lastly the Y strip radiators the c portion a b c the c portion of the strip we have Y edge radiators, and they will mirror the X edge radiator except that the  $k_{py}$  is replaced by  $k_{px}$ . So, we have

$$R_{rad}^y = \frac{A_p \rho_a c k_a^2 \{1 + (k_p^2 - k_a^2)/k_{px}^2\}}{k_a a \{(k_p^2 - k_a^2)/k_{px}^2\}^{3/2}}$$

it will look exactly the same except now the integral limits are different.

So, this looks like

$$\frac{A_p \rho_a c k_a}{a} \left\{ \frac{k_p}{\mu^3} \sin \theta + \frac{1}{k_p \mu \sin \theta} \right\}.$$

So, if we now put the integration the limits are as follows

$$\frac{A_p \rho_a c k_a}{a} \left\{ \frac{k_p}{\mu^3} \int_{\theta_2}^{\pi/2} \sin \theta d\theta + \frac{1}{k_p \mu} \int_{\theta_2}^{\pi/2} \frac{1}{\sin \theta} d\theta \right\}.$$

So, this becomes equal to

$$\frac{A_p \rho_a c k_a}{a} \left\{ \frac{k_p}{\mu^3} \cos \theta_2 - \frac{1}{k_p \mu} \ln[\operatorname{cosec} \theta_2 - \cot \theta_2] \right\}.$$

If you again use the relations, we found earlier we have

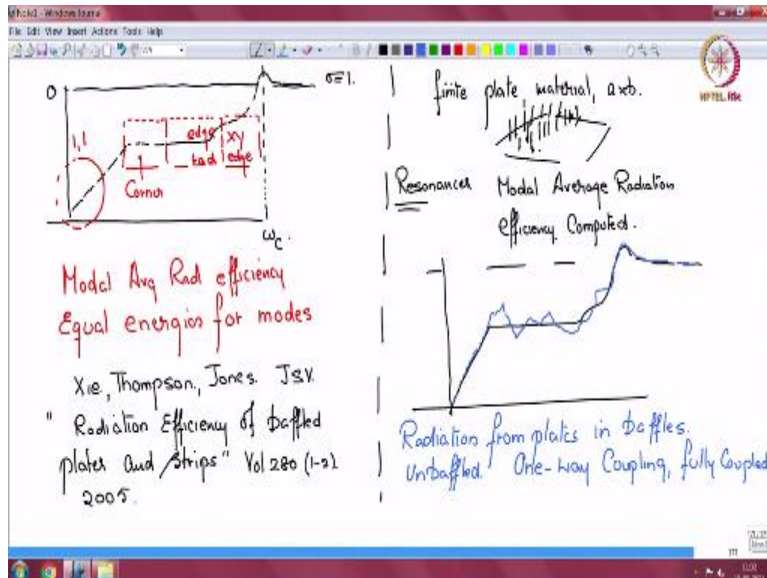
$$\frac{A_p \rho_a c k_a}{a} \left\{ \frac{k_p}{\mu^3} \frac{k}{k_p} - \frac{1}{k_p \mu} \ln \left[ \frac{k_p}{\mu} - \frac{k}{\mu} \right] \right\}.$$

So now if we add all the three so I did not put numbers so let this 2 this is 3. So, we then multiply by  $2/\pi$  we add 1 + 2 + 3 and further you will get a place where you will have terms like  $2(a + b)$  so  $2a + 2b$  is the panel perimeter I am going to make it sketchy from now on here. So, then the average has a form given by

$$R_{rad}^{avg} = A_p \rho_a c \left\{ \left[ \frac{p_r \lambda_p}{A_p} \right] g_2 + \frac{\lambda \lambda_p}{A_p} g_1 \right\}$$

and this is below coincidence below the general panel coincidence frequency so, you can do it close to coincidence you can do it above coincidence. I will not go into that it is already a little too much for this course.

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Except now, finally you get a picture that looks somewhat like this. At low frequency you have a straight line then a flat line and then you have an increment, and you have a peak that settles to the value 1 asymptotically that value is of course this is a decibel scale I am plotting so you get 0 here. So, this is  $\sigma = 1$  and this peak happens to be the coincidence frequency, now this will be largely dominated by the 1,1 mode of the panel.

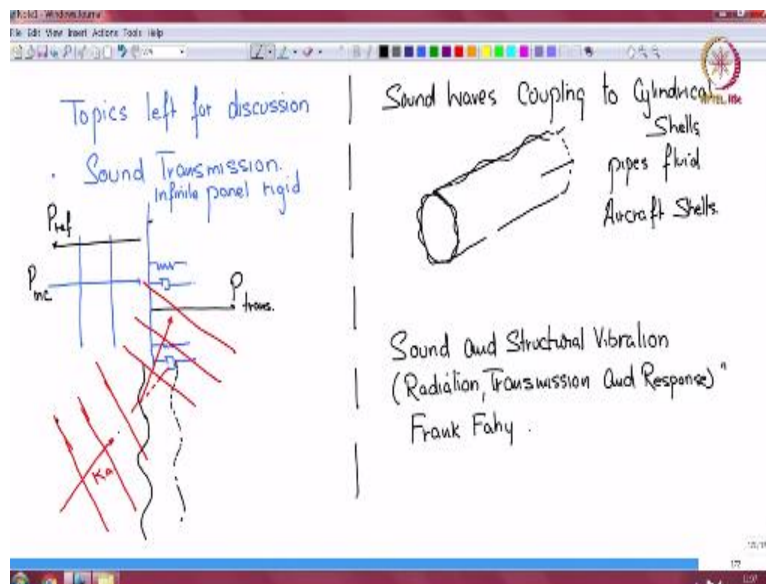
As you come here, you will have corner modes at dominate. Corner modes dominate as you come more and more over here. You will have edge modes radiators and as you come further over here you will have double edged radiators or XY edge radiators beyond these modes are acoustically fast so, this is how the average model average radiation efficiency looks like of course now this is a averaged picture assuming equal energies for or equal vibrational energies for all modes participating that is not true.

Some will be at resonance some will be at end resonance etcetera. So, the last piece of the whole story is this that a paper by Xie Thompson and Jones it is a JSV paper journal of sound and vibration paper it is titled radiation efficiency of baffled plates and strips volume 280 parts 1 and 2 and year is 2005 so, this particular paper now addresses in actuality what happens so, you have a finite plate of some material etcetera some dimensions excited you know excited in an average manner an averaged over all the points on the plate excited every point is excited.

And then so, the resonances are accounted for the panel resonances are accounted for and a modal average radiation efficiency computed numerically computed for the panel and it will turn out that if the curve by Maidanik's for an idealized situation shows this trend then the curve by Xie Thompson will lie somewhere around it and account for the panel resonances and follow this, so that is I think adequate for now as far as radiation from plates in baffles is concerned in placed in baffles.

There is a lot more you can have plates not placed in baffles unbaffled and moreover this is we have done one way coupling or what we call uncoupled so then you could study fully coupled like a plate of steel placed in water so the water pressures are applied back to the panel so you can study fully coupled. So, those are for future, now so a major this was a major portion of the course.

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So, there are now only a few topics that are left for discussion, one of those is sound transmission so, if we have say one case if we have a panel let us say an infinite panel that is rigid but that is placed periodically on springs and there is a plane sound wave incident on it. So, the question is how much of the sound will be reflected and how much of the sound will be transmitted. This is  $p$  incident, this is  $p$  reflected, this is  $p$  transmitted, so we will look at that case is of course I say idealize because this is rigid.

And it is placed on mounts the other is an infinite panel that can now flex an infinite panel that can flex. And then on top of that you have a plane wave incident at any angle you have a sound wave incident which is a plane wave incident at a particular angle. So, now as it is how will it get transmitted into the other medium? We will take different media find a general expression, so there is a plane wave in the medium beyond the plate. So, we will find a general expression for what we call transmission loss for this case.

Now, there can be more studies when it comes to transmissions you could have a double panel which is actually very useful in reducing sound from one room to the other and so forth. But those are advanced studies. This is to give a basic idea of sound transmission. Then this will be followed by a study of sound waves and they are coupling to cylindrical shells. It is a very important topic. So, you have a cylindrical shell which is flexible, let us say it is an infinite shell goes off to infinity both directions, but it can flex the thin flexible shell.

So, you could have a pipe carrying a fluid, you can have pipes that carry fluid, water pipes that carry water or you could have systems like aircraft shells with fluid inside it acoustic air inside. And so, we will look at sound and structure interaction in this cylindrical shell geometry. So, this is one very important topic, I will follow textbook sound and structural vibration, radiation transmission and response. It is written by Frank Fahy. So, we will start looking at these two topics in the next class I will close it for here. Thank you.