Sound and Structural Vibration

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Lecture – 47
Radiation Resistance Derivation from Maidanik's Work, Contd

Welcome to this next lecture on sound and structural vibration we ended up last time at this stage of the power integral. So, we are going to move ahead. So, we will do now the b integral.

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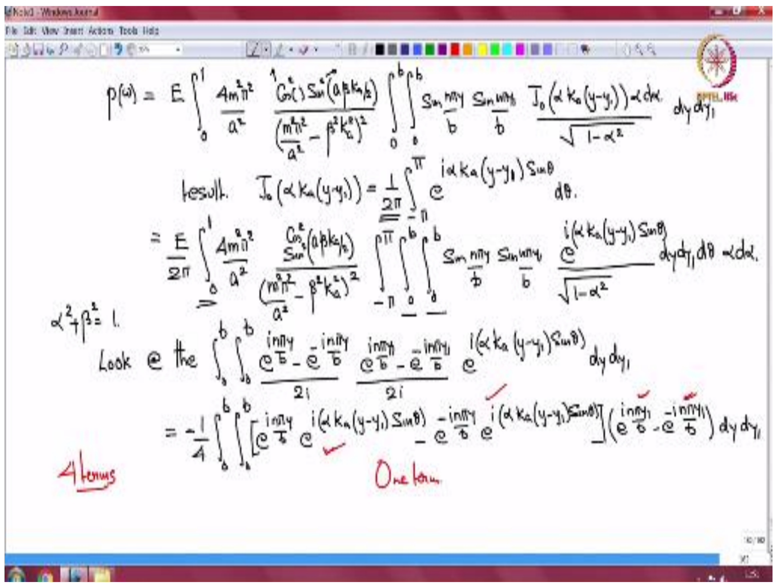

And looking at the b integral we have just the b integral 0 to b integral. So, let us see that so, we have

$$\int_0^b \int_0^b \frac{e^{\frac{in\pi y}{b}} - e^{-\frac{in\pi y}{b}}}{2i} \frac{e^{\frac{in\pi y_1}{b}} - e^{-\frac{in\pi y_1}{b}}}{2i} \; e^{i(\alpha k_a (y-y_1)\sin\theta)} \, dy \, dy_1.$$

So, I get

$$= \frac{-1}{4} \int_0^b \int_0^b \left[e^{\frac{in\pi y}{b}} e^{i(\alpha k_a (y-y_1)\sin\theta)} - e^{-\frac{in\pi y}{b}} e^{i(\alpha k_a (y-y_1)\sin\theta)} \right] \left(e^{\frac{in\pi y_1}{b}} - e^{-\frac{in\pi y_1}{b}} \right) dy \, dy_1.$$

So, this will result in four terms. So, this with this and this gives two terms this with again this and this gives two terms, so you get four terms. So, let us just look at one term. Let us look at one term which is this the first term with the first term here, so there will be many similarities and so I will just go through one term.

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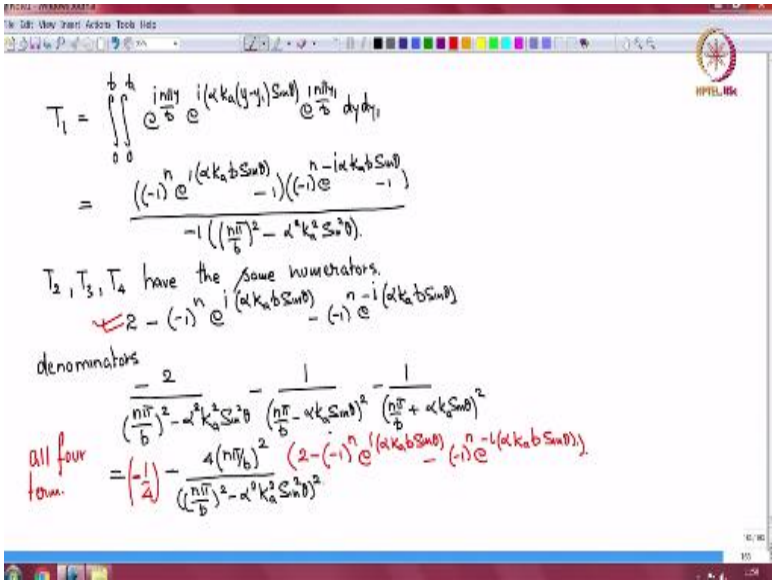

So, the one term is T_1 which is given by

$$\int_0^b \int_0^b e^{\frac{in\pi y}{b}}\, e^{i(\alpha k_a (y-y_1)\sin\theta)}\; e^{\frac{in\pi y_1}{b}}\, dy\, dy_1$$

it is a pretty simple integral y and y_1 they separate out because of the exponents so; they can be done separately. So, I will not show it entirely or you can do it in Maple if you are not confident so, you get

$$= \frac{\left((-1)^n e^{i(\alpha k_a b \sin\theta)} - 1\right)\left((-1)^n e^{-i\alpha k_a b \sin\theta} - 1\right)}{-1\left(\left(\frac{n\pi}{b}\right)^2 - \alpha^2 {k_a}^2 \sin^2\theta\right)}.$$

Then let me just say that T_2, T_3 and T_4 have the same numerators they which simplifies to what each of them have the same numerator

$$2 - (-1)^n e^{i(\alpha k_a b \sin\theta)} - (-1)^n e^{-i(\alpha k_a b \sin\theta)}.$$

And what are the denominators?

The denominators

$$\frac{-2}{\left(\frac{n\pi}{b}\right)^2 - \alpha^2 {k_a}^2 \sin^2\theta} - \frac{1}{\left(\frac{n\pi}{b} - \alpha k_a \sin\theta\right)^2} - \frac{1}{\left(\frac{n\pi}{b} + \alpha k_a \sin\theta\right)^2}.$$

So, if you now combine this; what do you get? You will get

$$= -\frac{4\left(\frac{n\pi}{b}\right)^2}{\left(\left(\frac{n\pi}{b}\right)^2 - \alpha^2 {k_a}^2 \sin^2\theta\right)^2}\left(2 - (-1)^n e^{i(\alpha k_a b \sin\theta)} - (-1)^n e^{-i(\alpha k_a b \sin\theta)}\right).$$

So, this is now all four terms, and you have $\left(-\frac{1}{4}\right)$ in front if you remember minus one forth in front so, this minus will go with this minus this 4 will go with that 4. So, this is what we are left with.

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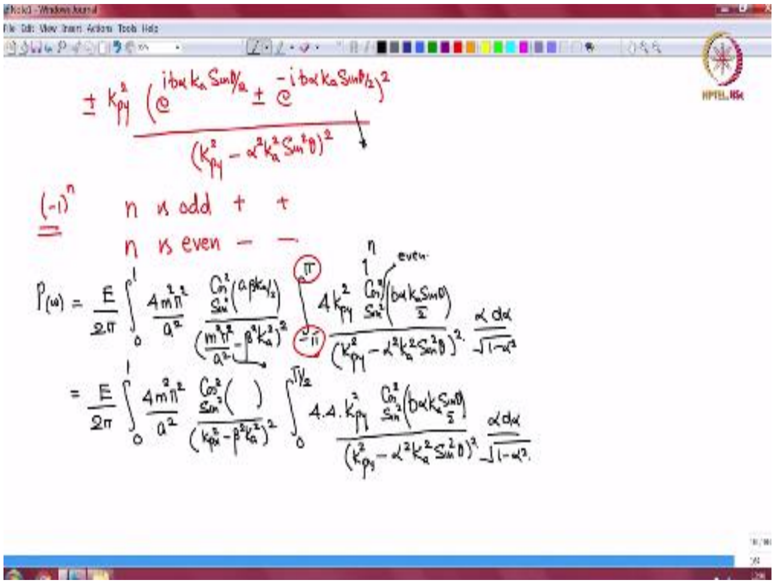

So, let us see what we are left we have $\left(\frac{n\pi}{b}\right)^2$ which I say is my ${k_{py}}^2$,

$$\pm {k_{py}}^2 \frac{\left(e^{ib\alpha k_a \sin\theta/2} \pm e^{-ib\alpha k_a \sin\theta/2}\right)^2}{\left({k_{py}}^2 - \alpha^2 {k_a}^2 \sin^2\theta\right)^2}.$$

Now, what are these plus and minus you remember we have (-1) to the power n everywhere. So, if n is odd you take the plus everywhere, if n is even you take the minus everywhere.

So, that is how this thing goes then now how does this whole thing look like now, it looks like this the power looks like

$$p(\omega) = \frac{E}{2\pi}\int_0^1 \frac{4m^2\pi^2}{a^2}\frac{\frac{\cos^2}{\sin^2}(a\beta k_a/2)}{\left(\frac{m^2\pi^2}{a^2} - \beta^2 {k_a}^2\right)^2}\int_{-\pi}^{\pi}\frac{4{k_{py}}^2 \frac{\cos^2}{\sin^2}(b\alpha k_a \sin\theta/2)}{\left({k_{py}}^2 - \alpha^2 {k_a}^2 \sin^2\theta\right)^2}\frac{\alpha\, d\alpha}{\sqrt{1-\alpha^2}}\, d\theta,$$

Now, finally, this has a certain repeatedness over 0 to π / 2 this integral is going from $-\pi$ to π, so, over full 2π range however it has a certain evenness about 0 to π / 2. So, that it repeats and so, we will write it as 4 times that.

So, when we do that, we get an extra 4 so, we write it as

$$= \frac{E}{2\pi}\int_0^1 \frac{4m^2\pi^2}{a^2}\,\frac{\frac{\cos^2}{\sin^2}(\alpha\beta k_a/2)}{\left({k_{px}}^2-\beta^2{k_a}^2\right)^2}\int_0^{\frac{\pi}{2}} 4.\,4\;\frac{{k_{py}}^2\,\frac{\cos^2}{\sin^2}(b\alpha k_a\sin\theta\,/2)}{\left({k_{py}}^2-\alpha^2{k_a}^2\sin^2\theta\right)^2}\,\frac{\alpha\, d\alpha}{\sqrt{1-\alpha^2}}\;d\theta.$$

So, next if we bring everything all constants we start bringing all constants to the front.

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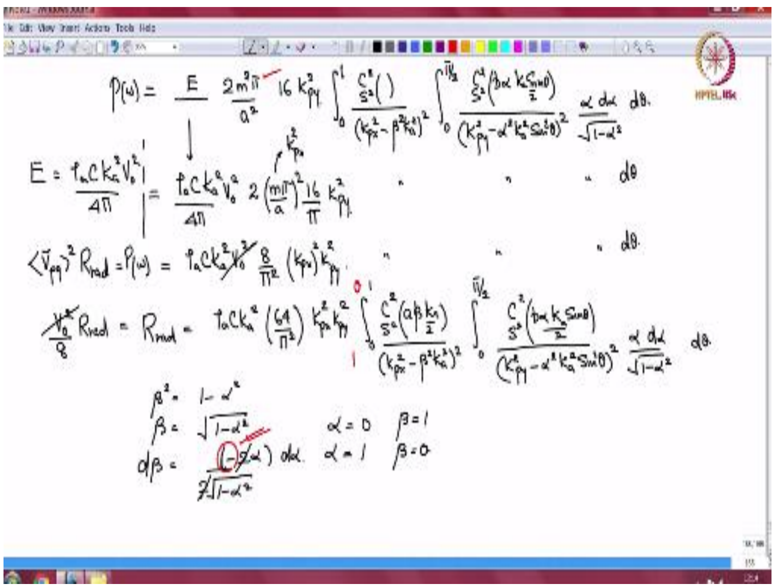

So, my

$$p(\omega) = E\,\frac{2m^2\pi}{a^2}\;16\,{k_{py}}^2\int_0^1 \frac{\frac{\cos^2}{\sin^2}\left(\frac{1}{2}\alpha\beta k_a\right)}{\left({k_{px}}^2-\beta^2{k_a}^2\right)^2}\int_0^{\frac{\pi}{2}} \frac{\frac{\cos^2}{\sin^2}\left(\frac{1}{2}b\alpha k_a\sin\theta\right)}{\left({k_{py}}^2-\alpha^2{k_a}^2\sin^2\theta\right)^2}\,\frac{\alpha\, d\alpha}{\sqrt{1-\alpha^2}}\;d\theta\,.$$

E if you recall E was what?

E was given by $\frac{\rho_a c {k_a}^2 {V_0}^2}{4\,\pi}$ that was my E so, if I bring that in here what do I have

$$p(\omega)$$

$$= \frac{\rho_a c {k_a}^2 {V_0}^2}{4\,\pi}\;2\,\left(\frac{m\pi}{a}\right)^2\frac{16}{\pi}\;{k_{py}}^2\int_0^1 \frac{\frac{\cos^2}{\sin^2}\left(\frac{1}{2}\alpha\beta k_a\right)}{\left({k_{px}}^2-\beta^2{k_a}^2\right)^2}\int_0^{\frac{\pi}{2}} \frac{\frac{\cos^2}{\sin^2}\left(\frac{1}{2}b\alpha k_a\sin\theta\right)}{\left({k_{py}}^2-\alpha^2{k_a}^2\sin^2\theta\right)^2}\,\frac{\alpha\, d\alpha}{\sqrt{1-\alpha^2}}\;d\theta.$$

Now, what did I say earlier I said my $\langle\tilde{V}_{pq}\rangle^2$ is space and time averaged mean square velocity, $\langle\tilde{V}_{pq}\rangle^2 R_{rad} = p(\omega)$. So, for now

$$\langle\tilde{V}_{pq}\rangle^2 R_{rad} = p(\omega)$$

$$= \rho_a c k_a{}^2 V_0{}^2 \frac{8}{\pi^2} k_{px}{}^2 k_{py}{}^2 \int_0^1 \frac{\begin{matrix}\cos^2\\ \sin^2\end{matrix}\left(\frac{1}{2}a\beta k_a\right)}{\left(k_{px}{}^2 - \beta^2 k_a{}^2\right)^2} \int_0^{\frac{\pi}{2}} \frac{\begin{matrix}\cos^2\\ \sin^2\end{matrix}\left(\frac{1}{2}b\alpha k_a \sin\theta\right)}{\left(k_{py}{}^2 - \alpha^2 k_a{}^2 \sin^2\theta\right)^2} \frac{\alpha\, d\alpha}{\sqrt{1-\alpha^2}}\, d\theta.$$

Now, $\langle\tilde{V}_{pq}\rangle^2$ is $\frac{V_0{}^2}{8}$ I have shown you that.

$$\frac{V_0{}^2}{8} R_{rad} = p(\omega)$$

$$= \rho_a c k_a{}^2 V_0{}^2 \frac{8}{\pi^2} k_{px}{}^2 k_{py}{}^2 \int_0^1 \frac{\begin{matrix}\cos^2\\ \sin^2\end{matrix}\left(\frac{1}{2}a\beta k_a\right)}{\left(k_{px}{}^2 - \beta^2 k_a{}^2\right)^2} \int_0^{\frac{\pi}{2}} \frac{\begin{matrix}\cos^2\\ \sin^2\end{matrix}\left(\frac{1}{2}b\alpha k_a \sin\theta\right)}{\left(k_{py}{}^2 - \alpha^2 k_a{}^2 \sin^2\theta\right)^2} \frac{\alpha\, d\alpha}{\sqrt{1-\alpha^2}}\, d\theta.$$

So, if we cancel the $V_0{}^2$ and bring the 8 up there.

Then my R_{rad} becomes equal to

$$R_{rad} = \rho_a c k_a{}^2 \left(\frac{64}{\pi^2}\right) k_{px}{}^2 k_{py}{}^2 \int_0^1 \frac{\begin{matrix}\cos^2\\ \sin^2\end{matrix}\left(\frac{1}{2}a\beta k_a\right)}{\left(k_{px}{}^2 - \beta^2 k_a{}^2\right)^2} \int_0^{\frac{\pi}{2}} \frac{\begin{matrix}\cos^2\\ \sin^2\end{matrix}\left(\frac{1}{2}b\alpha k_a \sin\theta\right)}{\left(k_{py}{}^2 - \alpha^2 k_a{}^2 \sin^2\theta\right)^2} \frac{\alpha\, d\alpha}{\sqrt{1-\alpha^2}}\, d\theta.$$

Now $\beta^2 = 1 - \alpha^2$ or $\beta = \sqrt{1-\alpha^2}$ or $d\beta = \frac{(-2\alpha)}{2\sqrt{1-\alpha^2}}\, d\alpha$. So, now, when α is 0, β is 1, when α is 1, β is 0. So, the sign change on β, on the limit change on β, β now goes from 1 to 0. So, if I use the sign change and 1 to 0, I will still get here.

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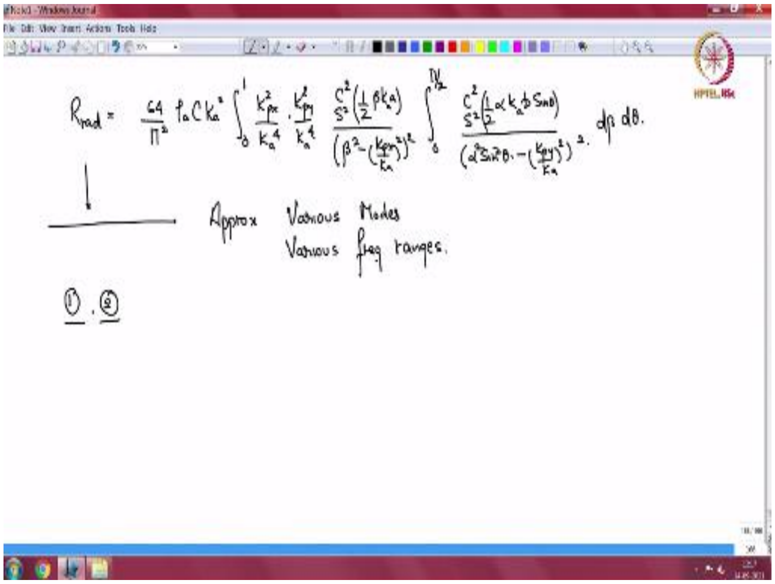

So, let me write

$$R_{rad} = \frac{64}{\pi^2}\rho_a c k_a{}^2 \int_0^1 \frac{k_{px}{}^2}{k_a{}^4}\frac{k_{py}{}^2}{k_a{}^4}\frac{\frac{\cos^2}{\sin^2}\left(\frac{1}{2}a\beta k_a\right)}{\left(\beta^2 - \left(\frac{k_{px}}{k_a}\right)^2\right)^2}\int_0^{\frac{\pi}{2}} \frac{\frac{\cos^2}{\sin^2}\left(\frac{1}{2}b\alpha k_a \sin\theta\right)}{\left(\alpha^2 \sin^2\theta - \left(\frac{k_{py}}{k_a}\right)^2\right)^2} d\beta\, d\theta.$$

So, this is a form that the paper by Maidanik's has so this is the basic or the central radiation resistance integral.

So, from here it starts to make approximations for various modes and various frequency ranges and largely they are these approximations are intuitive. So, at least in the paper, only a few details are there I am closing the lecture here. Thanks.