Sound and Structural Vibration
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Lecture 45
Response of Ribbed Panels to Reverberant Acoustic Fields

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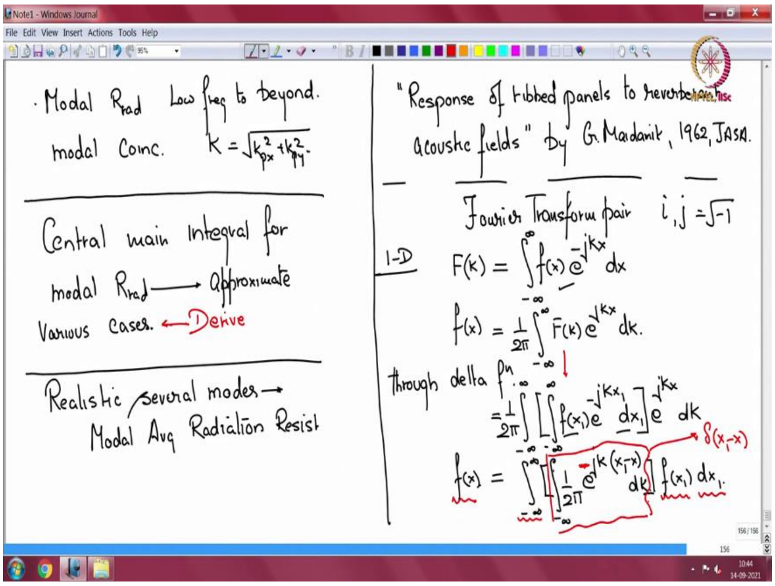

Welcome to this next lecture on sound and structural vibration last class we ended up saying we will study this paper, or we will derive some paper results from this paper Response of Ribbed Panels to Reverberant Acoustic Fields by Maidanik its a 1962, JASA paper. So, let us see we need a Fourier transform pair definition. Fourier transform pair. So, initially let us see 1D, for the 1D case the forward transform.

I am doing the space version. So, it is called the wavenumber transform also but name does not matter from space to wavenumber. So, this is the spatial function I take a transform and I will use i and j freely interchangeably as the square root of minus 1

$$F(k) = \int_{-\infty}^{\infty} f(x)\, e^{-jkx}\, dx\,,$$

that is the forward transform. Then the inverse is

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(k)\, e^{jkx}\, dk\,.$$

So, we will just set up our convention through the delta function. So, let us see. So, this now if I try to get back using circular arguments. So, I have

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(x_1)\, e^{-jkx_1}\, dx_1\right] e^{jkx}\, dk\,.$$

So, if I take $-\infty$ to ∞ here then

$$f(x) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} \frac{1}{2\pi}\, e^{-jk(x_1-x)}\, dk\right] f(x_1)dx_1.$$

So, note this here that I have $f(x_1)dx_1$ here I have $f(x)$ here and an integral over infinite domain. So, this has to be the delta function ok. So, this has to be the delta function this part Dirac's delta function.

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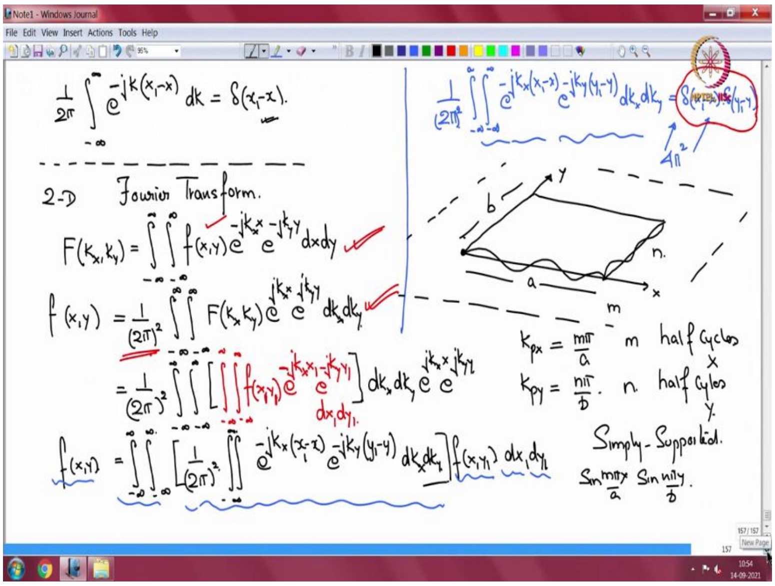

Which means

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-jk(x_1-x)}\, dk = \delta(x_1 - x)\,.$$

So, this is the this is how you define delta then the Fourier transform pair must be defined as it was that is the idea. If we use this definition for delta function then the way we define forward inverse transform conform to this definition.

Let us look at 2D now 2D Fourier transform we have the forward

$$F(k_x, k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-jk_x x} e^{-jk_y y}\, dx\, dy\,.$$

And the inverse let us say we define as

$$f(x,y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(k_x, k_y)\, e^{jk_x x} e^{jk_y y}\, dk_x\, dk_y\,.$$

Now again we will try to get back to $f(x,y)$. So, I have

$$f(x,y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1,y_1)e^{-jk_x x_1}e^{-jk_y y_1}\,dx_1\,dy_1\right] e^{jk_x x}e^{jk_y y}\,dk_x\,dk_y\,.$$

So, now we will do whatever we did last time

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-jk_x(x_1-x)}e^{-jk_y(y_1-y)}\,dk_x\,dk_y\right] f(x_1,y_1)\,dx_1\,dy_1\,.$$

So, again what happens here we have $f(x,y)$ here $f(x_1,y_1)$ here and an integral over an infinite domain.

So, what is inside here must be a delta function double delta function ok. So, what is inside here?

$$\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-jk_x(x_1-x)}e^{-jk_y(y_1-y)}\,dk_x\,dk_y = \delta(x_1-x)\delta(y_1-y)\,.$$

So, therefore this part just this part here is $4\pi^2$ taken up into this delta function ok.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-jk_x(x_1-x)}e^{-jk_y(y_1-y)}\,dk_x\,dk_y = 4\pi^2\delta(x_1-x)\delta(y_1-y).$$

So, that part the reason I am saying is that if you use this definition somewhere if this definition is used somewhere.

Then the corresponding forward and inverse Fourier transform pair must be this. That is why I am stressing this. Now let us see here. So, we are going to derive as I said the main central integral of Maidanik. So, the geometry is this I have a plate placed in the xy plane. Let us say my origin is here and this dimension is a and this dimension is b it is placed in a baffle.

We know so, that communication with backside is prevented then there are individual modes individual modes on the panel ok. So, this direction is mode number is m the y direction the mode number is n. So, the k_{px} modal wavenumber in the x direction for the plate or panel is $\frac{m\pi}{a}$. Similarly, the modal wavenumber in the y direction is $\frac{n\pi}{b}$ ok, m is the number of half cycles of the mode in the x direction and n is the number of half cycles on the plate in the y direction.

And the plate for now is simply supported. So, that the mode shapes are $\sin\frac{m\pi x}{a}\ \sin\frac{n\pi y}{b}$.

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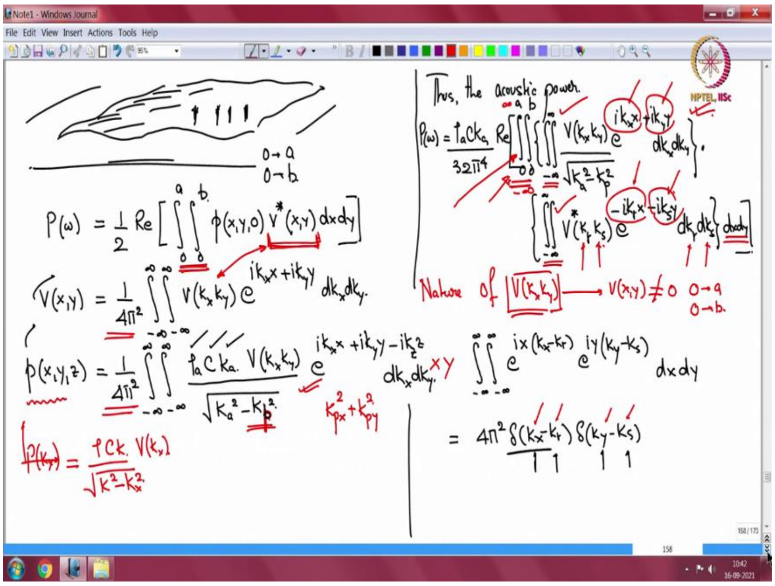

So, now somewhere in the paper we have a power definition sound radiated from the panel. So, if you consider this to be the panel ok. So, the sound radiated is pressure seen by the panel multiplied by the velocity of the panel ok and the panel vibrates has nonzero displacement only on the plate not on the baffles. So, baffle is rigid. So, outside of a and b outside of zero to a and zero to b there is no displacement ok. So, a pressure times velocity over the panel will give you the power ok.

So, we will use this definition the power is given by

$$p(\omega) = \frac{1}{2}\ Re\left[\int_0^a\int_0^b p(x,y,0)v^*(x,y)\ dx\ dy\right].$$

Now the V velocity of the panel

$$V(x,y) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V\left(k_x,k_y\right) e^{ik_xx+ik_yy}\ dk_x\ dk_y.$$

This is the inverse double Fourier transform inverse and then the pressure

$$p(x,y,z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\rho_a ck_a\, V(k_x,k_y)}{\sqrt{{k_a}^2-{k_p}^2}}\ e^{ik_xx+ik_yy-ik_zz}\ dk_x\ dk_y\ ,$$

ρ_a is the density of the acoustic medium, c is the speed of sound in the medium and k_a is the wavenumber in the acoustic medium. If you remember that in 1D panel also we had a similar expression pressure x and y direction was given by $\frac{\rho c k}{\sqrt{k^2 - {k_x}^2}} V(k_x)$.

We had this expression pressure in k_x at k_x wavenumber if you recall in the 1D case. So, this is just an extension to 2D ok. Next hence the acoustic power I will need space here. Let us see thus the acoustic power is given by if I put this these two back in there in the equation above.

So, that

$$p(\omega) = \frac{\rho_a c k_a}{32\,\pi^4} Re\left[\int_0^a \int_0^b \left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{V(k_x, k_y)}{\sqrt{{k_a}^2 - {k_p}^2}}\, e^{ik_x x + ik_y y}\, dk_x\, dk_y\right\}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V^*(k_r, k_s)\, e^{-ik_r x - ik_s y}\, dk_r\, dk_s\right\} dx\, dy\right].$$

So, one thing I will say here that the nature of $V(k_x, k_y)$ is such that it has been derived from a $v(x, y)$ which is not equal to zero only on the panel outside it is 0 such is the nature of $V(k_x, k_y)$. And therefore, this spatial integral over 0 to a and 0 to b this spatial integral can be replaced with an infinite integral because this $v(x, y)$ is zero outside anyway this and this are tied in such a manner that $V(k_x, k_y)$ arrives from $v(x, y)$ which is zero outside the panel.

So, this integral here can be replaced by infinite plus minus infinity which is what I will do ok. So, if we do that ok. So, if we do that suppose I have replaced them with minus infinity to infinity. Now let us see where x and y exist in the integrals. So, I have x here I have y here I have x here I have y here and this is the $dx\, dy$ integral that is the only place I have xy and $dx\, dy$ and this integral I have replaced by infinite limits.

We now look at just the xy integral. So, that is

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{ix(k_x - k_r)} e^{iy(k_y - k_s)}\, dx\, dy = 4\pi^2 \delta(k_x - k_r)\delta(k_y - k_s).$$

So, the delta function which will force k_x and k_r and k_y to be equal to k_s, now if this result is brought in into this integral above then if k_x is force to k_r and k_y is force to k_s then the remaining double integrals on k_r k_s and k_x k_y will collapse to one integral. So, we will see that now.

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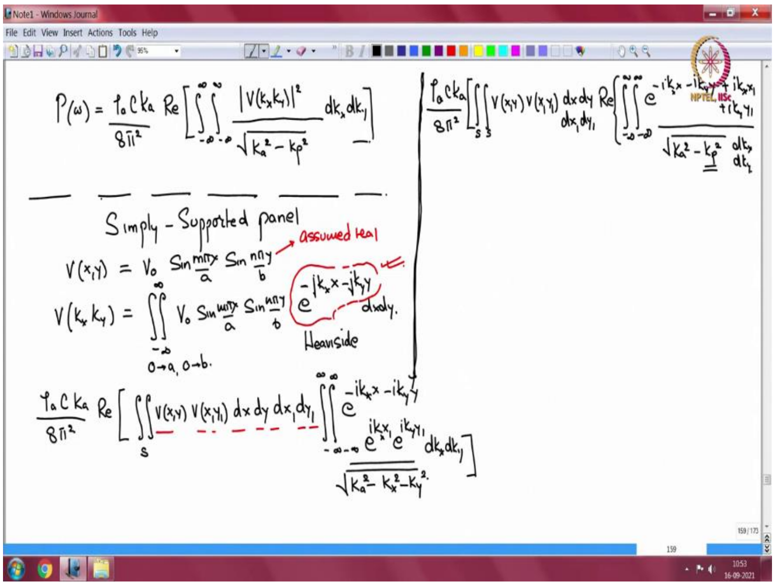

So, we get

$$p(\omega) = \frac{\rho_a c k_a}{8\,\pi^2}\, Re\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\left|V(k_x, k_y)\right|^2}{\sqrt{{k_a}^2 - {k_p}^2}}\; dk_x\, dk_y\right].$$

Now here this is one stage we have arrived at now the mode shape assumed is on a simply supported panel and therefore my

$$v(x, y) = V_0\; \sin\frac{m\pi x}{a}\; \sin\frac{n\pi y}{b}\,.$$

And this is assumed as real this is assumed as real then the transform is given by

$$V\left(k_x, k_y\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V_0\; \sin\frac{m\pi x}{a}\; \sin\frac{n\pi y}{b}\; e^{-jk_x x - jk_y y}\, dx\, dy\,.$$

So, what I have done is I have shortened the presentation a little bit. So, there should be a heavy side function actually which says that my panel vibrates only between 0 to a and 0 to b then this infinite integral and the heavy side will give us what we want ok.

Now the complex part is only here this is the complex part ok. So, this becomes

$$\frac{\rho_a c k_a}{8\,\pi^2}\; Re\left[\int_0^a\int_0^b V(x, y)V(x_1, y_1)\; dx\; dy\; dx_1\; dy_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-ik_x x - ik_y y}\, \frac{e^{ik_x x_1 + ik_y y_1}}{\sqrt{{k_a}^2 - {k_x}^2 - {k_y}^2}}\; dk_x\; dk_y\right].$$

Now this whole part is real this whole section this whole portion these are all real ok. So, the real here can be shifted inwards real here can be shifted inwards. So, we get

$$\frac{\rho_a c k_a}{8\,\pi^2}\left[\int_0^a\int_0^b V(x,y)V(x_1,y_1)\,dx\,dy\,dx_1\,dy_1\;\; Re\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-ik_x x-ik_y y+ik_x x_1+ik_y y_1}}{\sqrt{{k_a}^2-{k_p}^2}}\,dk_x\,dk_y\right\}\right].$$

So, we will stop this lecture here thank you.