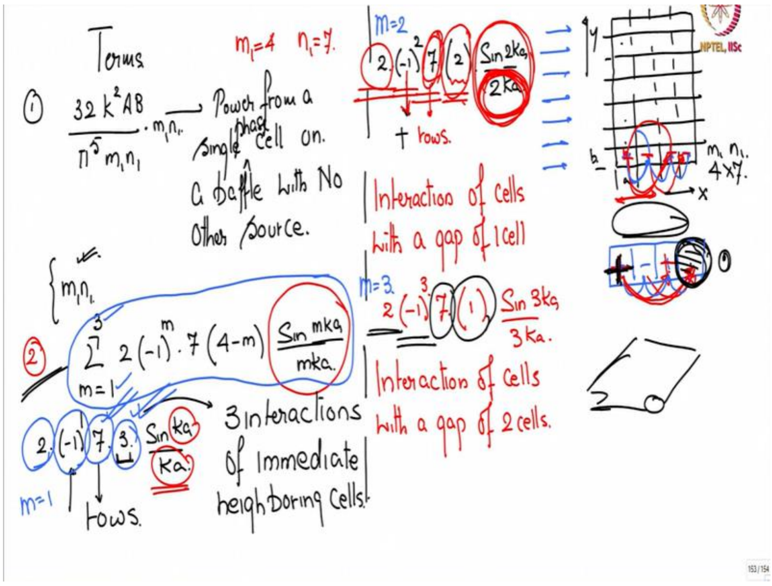
Sound and Structural Vibration
Prof. Venkata Sonti
Department of Mechanical Engineering
Indian Institute of Science – Bengaluru

Module No # 09
Lecture No # 44
Physical of Panel Radiation using Monopole Model Contd.

Good Morning and welcome to this next lecture on sound in structural vibration we were looking at approximating radiating panel using monopoles. And we had formulated the closed form formula for power, which had 4 terms and we were looking at the second term here.

(Refer Slide Time: 00:46)



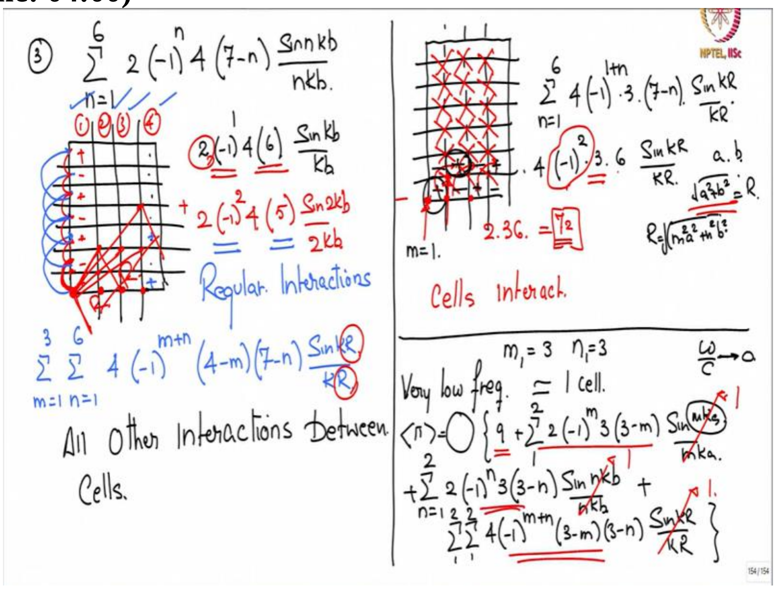
So, second term looks like m goes from 1 to 3 so we had looked at $m = 1$ here and $m = 2$ here and $m = 3$ here. And each of these demonstrate for example if I re-plot redraw m_1 demonstrates interaction of these 2 and these 2 or 3 such interactions and 7 for 7 rows the minus. Because these are alternating in sign and 2 because there are 2 cells involved then for $m = 2$ these are interactions with one gap this interaction and distraction.

There are only 2 such interactions but there are 7 rows and there is no sign change because this is plus and that is plus, so I have a positive and this 2 for 2 cells and lastly $m = 3$ then I have interaction with 2 cells gap, gap of 2 cells. So then again there is a negative because this is plus and this is minus for the $m_1 = 4$ mode. And then there is only one such interaction then there are 7 rows for it and 2 sets so that is the second term. What is an interaction?

One could ask on its either you can think of this cell here applies a pressure generates a pressure which sits on the other cell. So other cells see that pressure in addition to the mean pressure in the medium. And so that cell has to work against the pressure applied by this. So you could think of an interaction of that form or you could think that this panel is a course radiating.

And then this generates a pressure in the receiver point this cell generates a pressure at the receiver point those to add up of you can think of it that has an interaction. But the pressure applied on the cell is what I like so that is what interaction means. There is a pressure seen by 1 cell due to another cell and it has to work against that is at what is interaction now the next third term.

(Refer Slide Time: 04:00)



Term 3 looks like this

$$\sum_{n=1}^{6} 2(-1)^n\, 4\,(7-n)\,\frac{\sin nkb}{nkb}.$$

I will not repeat too much here it is intuitive now. So, if we have the cells $1, 2, 3, 4, 5, 6, 7$. Now let us see the first term n is equal to 1 so 2, $-$ 1 to the power 1 into 4 into 7 $-$ 1 which is 6 and $\sin kb$ over kb.

So, this speaks of interactions of neighbouring cells plus minus So you have neighbouring interactions $1, 2, 3, 4, 5, 6$ that is the 6 number. It happens 4 times 1, 2, 3, 4 columns and the minus because is a sign change across the node line and then 2. Because of 2 cells similarly next term is twice -1 to the power 2, 4 into 5 $\sin 2kb$ by $2kb$ so these are interactions with one cell gap.

So, this is 1 interaction 2 interaction $3, 4$ and 5 there are 5 such interactions 4 such columns and this time there is no sign change this is plus then that is plus and so forth and there are 2 cells and further on. That is the third term now these were regular interaction let me give it that name regular row wise column wise. The last term you have is a summation

$$\sum_{m=1}^{3}\sum_{n=1}^{6} 4\,(-1)^{m+n}(4-m)(7-n)\frac{\sin kR}{kR}.$$

So, what is this now? In simple terms this is all other interactions, all kinds of interactions between cells. For example, we have decided to place the location at corners right this corner and all the left lower corner is the cell location. So this interaction with this cells this interaction with that cell and so forth. There are any repetitions they are accounted for and so this R distance is now so we are looking at this to this that is the R distance.

We are looking at this to this that is the appropriate R. So I will show you one of these and the others should be followed in the same manner. So let us say that $m = 1$ so let us see here. I keep drawing this picture repeatedly $1, 2, 3, 4, 5, 6, 7$. So let us say $m = 1$ so then we get

$$\sum_{n=1}^{6} 4\,(-1)^{1+n}\,3\,\,(7-n)\frac{\sin kR}{kR}.$$

Then suppose n is 1 first term how does it look like I get 4, -1 squared into 3 into 6 and $\sin kR$ by kR.

kR is what square root of $a^2 + b^2$, R that is R because m is 1, n is 1. So R is typically $\sqrt{m^2a^2 + n^2b^2}$ that is R. So now what is this interaction? We have not row wise but this interaction with that. That means this cell interacts with that cell and this cell interacts with this cell this is, this cell interaction, then this cell interact with this cell, and this cell within this interaction.

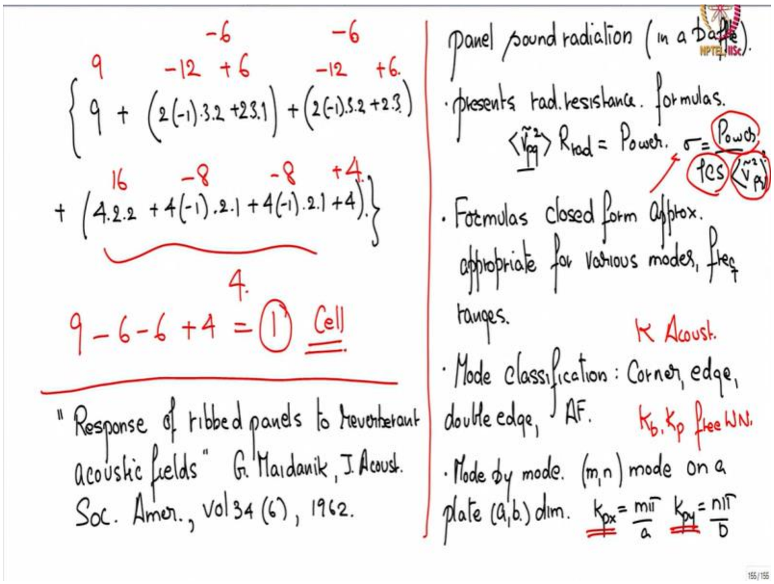
So now how many we have $1, 2, 3$ we have 3 interaction. Now let us look at this we have 3 6 are18, $18, 4, 72$ so if we count 1 to 18 then 19 to 36 and then to 2 cells at 72. And positive why positive because this is so say plus minus, minus plus. So, you are having a plus interaction with a plus interaction and hence plus this sign is plus and the distance is from here to here which is square root of $a^2 + b^2$.

So the fourth term describes all other interactions so this does give insight into how cells interact. Now let us see here I will give you one more insight here suppose I take this case to demonstrate I want a small case to demonstrate m_1 is 3 and n_1 is 3. So, it is a 3, 3 mode and we are at very low frequencies. We said that the entire panel is equal to one cell should we call one cell right. So, if I write the formula let me write the formula for power outside there was a constant representing 1 cell,

$$\langle \pi \rangle = [\ ]\left\{9 + \sum_{m=1}^{2} 2(-1)^m 3(3-m)\,\frac{\sin mka}{mka} + \sum_{n=1}^{2} 2(-1)^n 3(3-n)\,\frac{\sin nkb}{nkb} + \sum_{1}^{2}\sum_{1}^{2} 4\,(-1)^{m+n}(3-m)(3-n)\frac{\sin kR}{kR}\right\}.$$

Now let us say we are at a very low frequency that means k, which is omega by c is very small then I have a sign of a small radian quantity, so it is that argument itself. So, its mka sin of mka is mka then that mka cancels with denominator mka. So, I get a one here similarly I get 1 here and similarly I get one here. So I had to sum these up now I have to sum these up, which I will do on the next page.

(Refer Slide Time: 17:11)



And if you trust me, I will not repeat the formula that is why so that turns out to be here 9 plus the second term gives me 2 into −1 into 3 into 2 + 2 into 3 into 1 in the third term gives me 2 into −1 into 3 into 2 + 2 into 3. The fourth term gives me 4 into 2 into 2 + 4 into −1 into 2 into 1 + 4 into −1 into 2 into 1 + 4 items so these are the 4 terms. So what do I get here?

I get a 9 here, I get a 6 to $12 - 12$ here. I get a $+6$ here, I get a -12 here $+6$ here, I get $16 - 8 - 8 + 4$ is $20 - 16$ I get 4 from here.

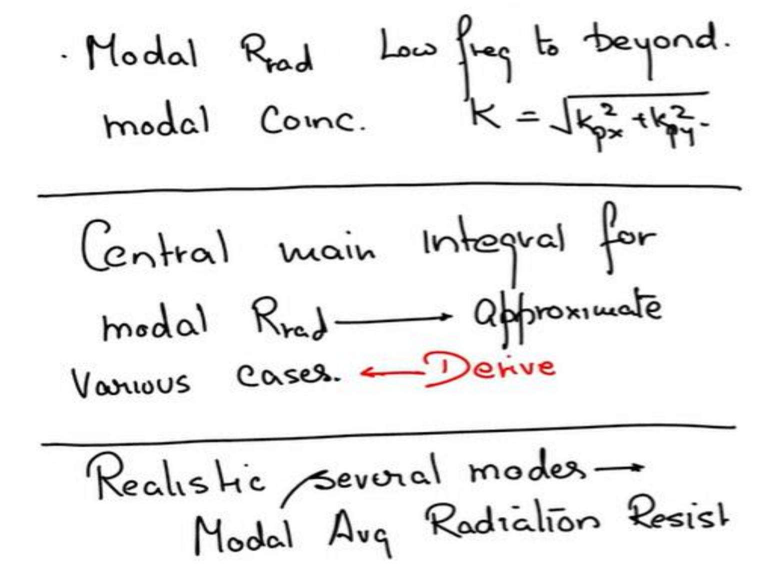
I get a $-6 - 6$ from here so $9 - 6 - 6 + 4$ is 1. So at low frequencies its one cell. So we will see more of this get some further insights. The next topic I would like to start is this paper called I mean the title" Response of ribbed panels to reverberant acoustic fields" by G. Maidanik. It is a journal of acoustical society of America paper volume 34, part 6 year is 1962. It is a very important key paper in panel or plate sound radiation, said in a baffle. This is why Maidanik presents radiation resistance.

So this means square velocity into radiation resistance is sound power that is how radiation resistance is defined. So he presents radiation resistance formulas and formulas obviously closed form approximations, appropriate for various modes and frequency ranges. So now this is also the paper where mode classification was done that is modes differentiated as corner radiators, edge radiators, then double edge radiators then acoustically fast radiators. So here, so this is very similar to radiation efficiency except for some radiation efficiency is power radiated divided by $\rho c s \tilde{V}_{pq}{}^{2}$.

So, power divided by this power divided by this entity is your radiation resistance. So, radiation efficiency into $\rho c s$ is radiation resistance. So he presents radiation resistances, so this is modal, mode by mode modal. So, you have in m, n mode on a plate of dimensions a, b dimensions that means k_{px} mode is $\frac{m\pi}{a}$ the k_{py} mode is $\frac{n\pi}{b}$ the plate lies in the xy plane.

So, these are the modal wave number you should now distinguish modal wave as opposed to the k_p or k_b which is the free plate wave number and k which is the acoustic wave number. You should know these by now.

(Refer Slide Time: 25:37)



So for every modal radiation resistance is presented from low frequency to just beyond the model coincidence. That means k being equal to square root of ${k_{px}}^2 + {k_{py}}^2$just beyond that along with this. So we will see so he first formulates a central main integral for model radiation resistance and from there he starts to approximate various cases and so will derive the central integral because it is a bit convoluted we will derive it from start.

Then what next, let me just mention it is that this is just modal but in a realistic situation several modes contribute the response. So there he computes or presents again a formula for modal averaged radiation resistance. We will see some what of this also later time is up for this lecture I will close here thank you.