Sound and Structural Vibration

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Module No # 08
Lecture No # 41
Simultaneously Radiation from Several Modes

Good morning and welcome to this next lecture on sound and structural vibration.

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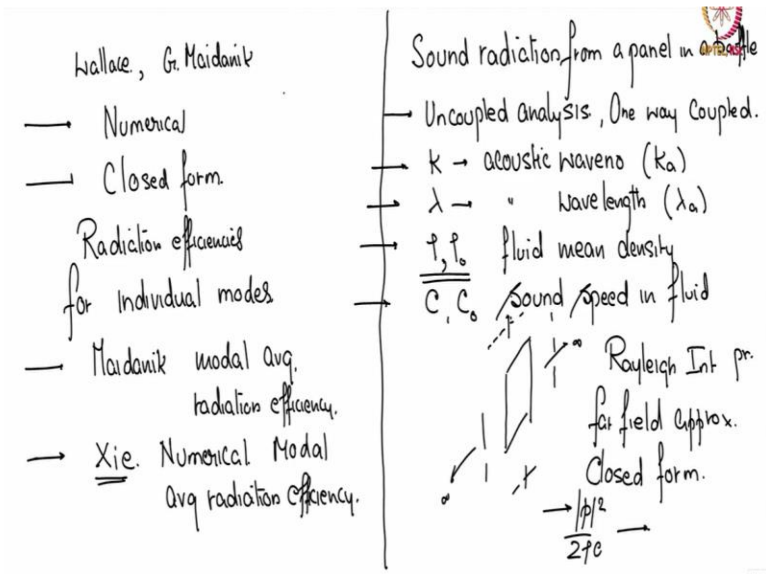

So, we have been looking at a case of sound radiation from a panel set in a baffle. A few things I would like to mention it is never too late this is what we call uncoupled analysis. We have seen coupled analysis in case of the classical problem the waveguide and the panel back by a cavity. But this is uncoupled analysis or one way couple analysis. The other thing I want to mention I use k for acoustic wave number I could at time use k_a also.

Similarly, I use lambda for the acoustic wavelength and similarly I could use with a subscript a. Then for medium density which is mostly air I use ρ or ρ_0 so the mean fluid density either and similarly so the fluid, mean density. And similarly, I use c or sometimes c_0 sound speed or sonic speed in the fluid. Now what have we seen so far; we have seen a case of a rectangular panel finite placed in a rigid wall of baffle.

Baffle extends to infinite in all directions, and we formulated the Rayleigh integral for pressure. Then from there we made a far field approximation in a closed form for closed form integration.

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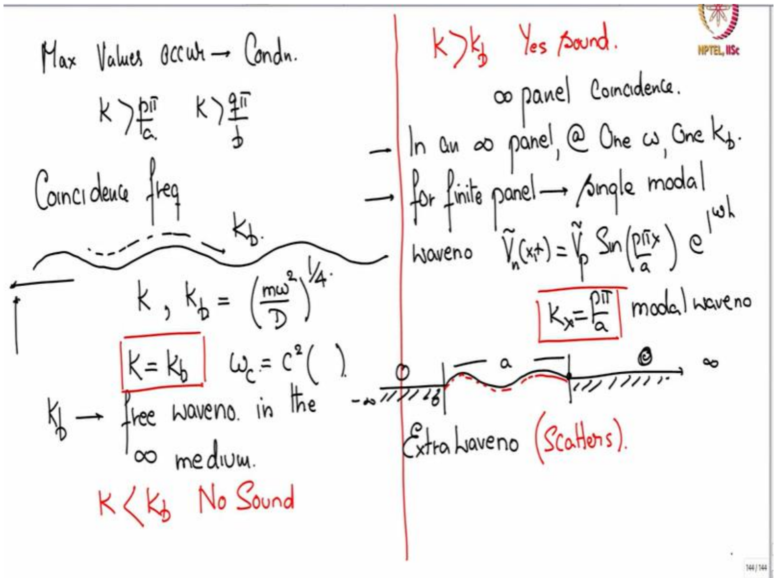

Then we went to, far flied intensity and we looked at maximum values where maximum values occur. And condition for maximum values which is k greater than $\frac{p\pi}{a}$ and k greater than $\frac{q\pi}{b}$. So here I would like to clarify one thought when we spoke of coincidence frequency right in the beginning in the infinite 1D plate case then we had the fluid wave number. And the plate wave number k_p or k_b and $k_b = \left(\frac{m\,\omega^2}{D}\right)^{1/4}$.

So, this and the coincidence is $k = k_b$ and we also found the coincidence frequency, so it was equal to c square something. Now this is the k_b is the free wave number in the infinite medium that means somewhere remotely far a way there is a localized excitation given and what now propagates in the medium is k_b. And for this infinite medium the coincidence condition is this. So, k less than k_b it is absolutely no sound and k greater than k_b suddenly yes sound the far field so it is a zero one situation, so this is we call it the infinite panel coincidence.

And there is only one wave number and there is an infinite panel in an infinite panel at one frequency at one omega there is only one free plate wave number k_b. These changes for finite panel the single modal wave number so what did we do for a 1D case? We said $\tilde{V}_n(x,t) = \tilde{V}_p \sin\left(\frac{p\pi x}{a}\right)\, e^{j\omega t}$. So, $\frac{p\pi}{a}$ is the model wave number and now because this placed in a baffle.

So, I know the velocity from 0 to a and I know the velocity from a to ∞ which is 0 and I know $-\infty$ to 0 velocity again 0. So, along this plane I know the velocity and so Fourier transform can be taken. Because I know the velocity Fourier transform can be taken and that generates extra wave numbers. So, this finite model wavenumber because it sits only in a finite region it scatters and generates extra wavenumbers. Whereas in an infinite panel there is no such scattering and only one wavenumber remains.

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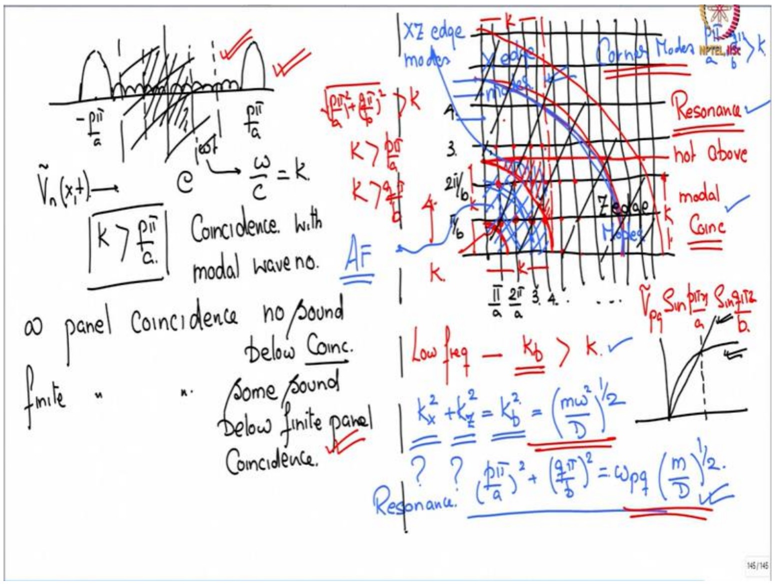

So, the coincidence now we speak of, so we found the Fourier spectrum for this $\tilde{V}_n$ excluding the time term we found the Fourier transform of this and it had a peak at $\frac{p\pi}{a}$ and again at $-\frac{p\pi}{a}$ and then this scattered wave numbers. So here we said when $\tilde{V}_n$ happens at a certain omega that omega translates to omega divided by c a certain k and then we draw the k line and integrate between them and that generates sound.

And as, k grows higher and higher omega is taken and we get more and more sound radiation from these scattered waves. And finally, when k crosses $\frac{p\pi}{a}$ so this is another coincidence with the model wavenumber this is not the coincidence in the infinite panel sense. So, there are 2 coincidences one has one has to understand. So, in the infinite panel case infinite panel coincidence no sound below coincidence.

In the finite panel some sound below finite panel coincidence. So, for finite panels we are looking this coincidence so please note that. Now let us see what is this full picture once again? So, we have a full regular 2D plate and will make it oblong this way. So, this the wave number space in 2 dimensions so this is $\frac{\pi}{a}$, this is $\frac{2\pi}{a}$, $\frac{3\pi}{a}$, $\frac{4\pi}{a}$ and so forth. And this is $\frac{\pi}{b}$, this is $\frac{2\pi}{b}$, $\frac{3\pi}{b}$, $\frac{4\pi}{b}$, and so forth.

And these crossings are modes of the plate and so as the plate is vibrated at low frequencies, we start at low frequencies then we have our acoustic quarter circle this is k acoustic wave number circle. Now at low frequencies the infinite panel wave number k_b is greater than the acoustic wave number so this is the let us say blue line, this is the infinite parallel wavenumber. Now we also know the $k_x{}^2 + k_z{}^2 = k_b{}^2$.

In an infinite panel remotely if there is an excitation at a single frequency you will get k_b and that relates to $k_x{}^2$ and $k_z{}^2$ which are in some way unknown some boundary condition will decide what it is? But and this is equal to $\left(\frac{m\,\omega^2}{D}\right)^{1/2}$. But we also know that $\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2 = \omega_{pq}\sqrt{\frac{m}{D}}$.

So now let us see on this picture what happens? At low frequencies k_b happens to be higher than k and in general k_b is obeys this law, However, when this k_b line passes through modal points then it obeys this law which is modes are in resonance this is the equation for resonance of a mode let us check.

So, when this k_b line passes these modal points then those modes in resonance. But the k quarter circle is below and so you will have initially modes in resonance but not above coincidence their modal coincidence is mode related coincidence. But what is happening just as the 1D case here because it is finite panel and you have $\tilde{V}_{pq} \sin\left(\frac{p\pi x}{a}\right) \sin\left(\frac{q\pi z}{b}\right)$ occurring on a finite dimension. So you will have scattered wave number like this picture.

So, some of those scattered numbers will be radiating sound and if its resonance of course there will be some amplification and you get more, sound. So, in this range its range its resonant radiation dominated but modes are below coincidence So now let us say the case the k circle moves up a little here and I draw the bound of k, so this is k. So now what happens is these modes they are $\frac{q\pi}{b}$ values which is this direction values are below k.

So, in this range these are Z edge modes similarly here the $\frac{p\pi}{a}$ values are below this k value so these are X edge modes. And these here are corner modes here $\frac{p\pi}{a}$ and $\frac{q\pi}{b}$ are greater than k so they are corner modes. So, in the beginning k quarter circle is small all the modes are corner they begin in the corner. And slowly based on the dimensions a and b they either end up in X edge modes or Z edge modes.

Then there is a region here where $\sqrt{\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2}$ greater than k but k is greater than $\frac{p\pi}{a}$ and k is greater than $\frac{q\pi}{b}$. So, this region is double edge modes XZ edge modes and finally modes enter the acoustic circle these modes are between acoustic circle these became acoustically fast modes. So now as frequency is raised at some instance what will you have is that the k quarter circle and the blue k_b line they coincide.

So, when they coincident those node points where this blue line crosses are resonant and now they are just below their model coincidence. So, then you have the resonant and coincidence. So, you get from those modes the most efficient radiation and the further increase you will have the k quarter circle dominating. So, we have to draw this picture somewhere in the beginning where acoustic wave number is like this, and k_b line is like this repeatedly seen it so somewhere this is now infinite panel coincidence.

So, beyond the infinite panel coincidence wave number or frequency you will have the acoustic wave number above the infinite panel wave number. When that happens just at that instance you those modes that this goes through or passes nearby the blue line or resonant and as well as just above coincidence. And beyond that what happens is all these modes below the red line become coincident I mean it become they move about their respective coincidences and some of them become resonant also.

So, you have some modes in the resonant and above coincidences radiation so that how the panel radiation picture looks like.

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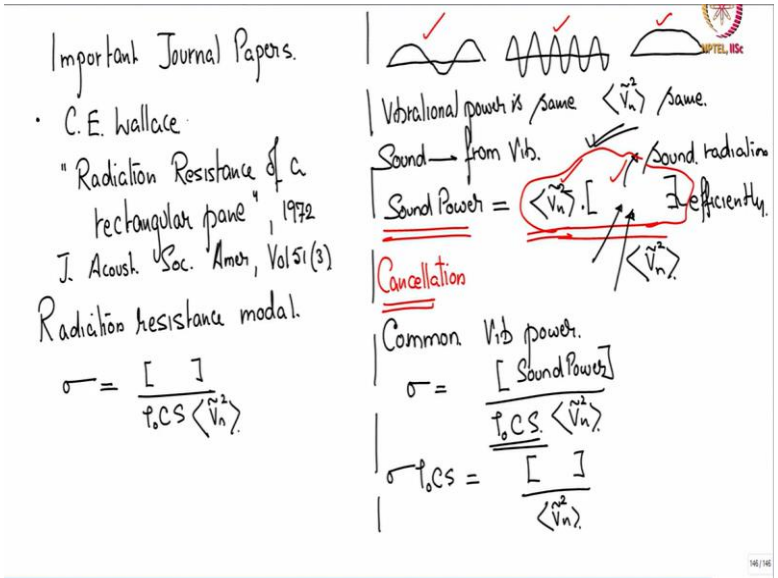

Now there are few important journal papers that are contributed to this understanding one of the key ones is by on C.E Wallace and the paper title is radiation resistance of a rectangular panel published in 1972. The journal of acoustic society of America volume is 51 part 3. So now in here what he does is he discusses radiation resistance mode by mode modal radiation resistance. What is that?

We have radiation efficiency given by the actual power radiated by the panel divided by $\rho_0 cs$ and mean square space time average velocity so let me say a word about this once again. So let say there are few panels with this vibration pattern there is another with. Let us say this vibration pattern and there is a third panel with this vibration pattern just looking at one 1D suppose.

Then let us say that their vibration power is same that means the mean square velocity is same mean square velocity is same. Now sound is related now sound is first of all generated from vibration and therefore if we want sound power, we talk of sound power hypothetically it will be this vibrational power into some other terms which talks about sound radiation. Mathematical expression that presents sound radiation that means these carries how efficiency effect how efficiently the radiator radiates.

So, it will be different for different radiator so it will be different now for this case even though the term is common. And what is the efficiency effect it is cancellation? Right now, how are we saying 1 radiator is more effective, and one is less effective is because of cancellation occurring on that. So, if sound power is known that means this term is known as composite not as this individual terms. We know the sound radiation so dividing let us there should be a square here.

So, dividing first of all by this takes out the common vibrational power effect that is removed all that reminds is the radiation factor how efficiently the source is radiating. So, if we now have the radiation efficiency it is obvious that it will be defined as this sound power which is the entire term divided mainly by this $\langle \tilde{V}_n{}^2 \rangle$ the vibration power. So, what should effectively remain is the other factor which discusses radiations of course there are these terms placed in there for making this a power form a circular piston.

So now radiation resistance is $\sigma \rho_0 c s$ that is equal to the sound power divided by mean square velocity. So that is what Wallace discusses in this paper modal mode by mode. So now how do the curves look like?

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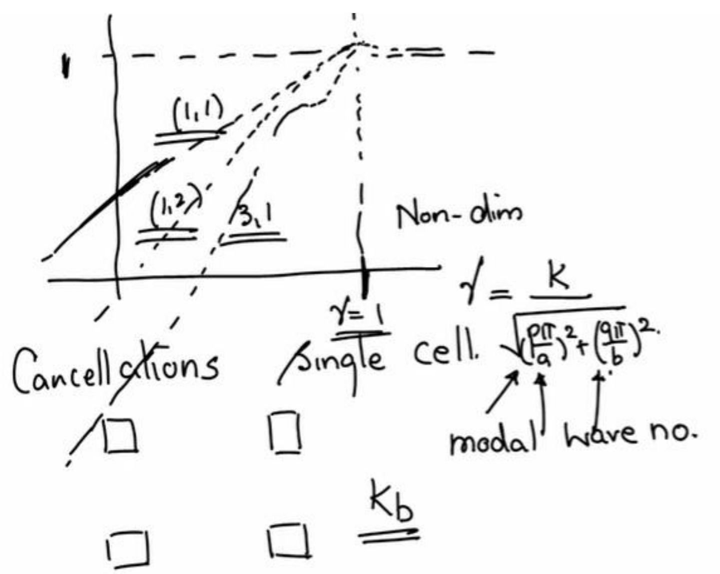

The curves look like these are radiation resistance curve they start of, and they reach with a value one somewhere and at times they cross over also and then asymptotically back down to one. So let us say this is some mode 1, 1 mode and 1, 2 mode and 1, 3 mode or 3, 1 mode and so forth. Now at very low frequencies based on the cancellations every mode this is can be 1, 2 lets this could be 3, 1.

Every mode starts of as we saw with a single cell at low frequencies and single cell dimension could be different based on the aspect ratio and mode number. So, you can see 1, 1 mode is the biggest quarter at the 4 corners. So, its starts of higher and cancellations with it are less so it has the highest radiation efficiency or radiation resistance. Whereas all other modes with higher number of half circle patterns at low frequencies the area of a single panel is much smaller and therefore the radiator less sound and so they start off from a lower value.

And what is this non dimensional frequency is given the term γ and it is the acoustic wave number divided by the $\sqrt{\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2}$ which is the modal wave number. So non dimensionally this is γ equal to 1 but dimensionally it is different because each mode has it is own p and q. So, this is not coincidence with k_b the infinite panel wave number it is coincidence with the modal wave number.

So, each mode as it approaches it is modal coincidence becomes efficient. So, this is Wallace's work and time has run has run out so let me stop here we will continue next class thank you.