Sound and Structural Vibration

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Lecture - 38

Derivations in the Frequency Domain:1-D

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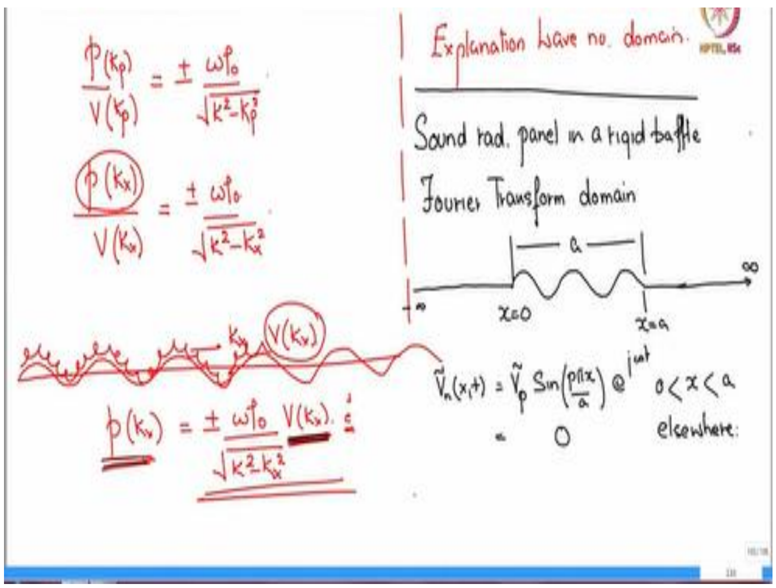

Good morning and welcome to this next lecture on sound and structural vibration. We have been looking at sound radiation from a panel set in a rigid baffle and we have come to understand the cancellation effect. Now we will try to understand the same phenomenon in the frequency domain that means the Fourier transform domain and we will do this for a one-dimensional case. That means there is an infinite baffle that comes from $-\infty$ up till $x = 0$ and there is a 1D panel.

That is, vibrating goes off to ∞ and it extends till $x = a$. So, now let us say that the velocity is

$$\tilde{V}_n(x,t) = \tilde{V}_p \sin\left(\frac{p\pi x}{a}\right) e^{j\omega t} \qquad 0 < x < a$$

$$= 0 \qquad\qquad elsewhere.$$

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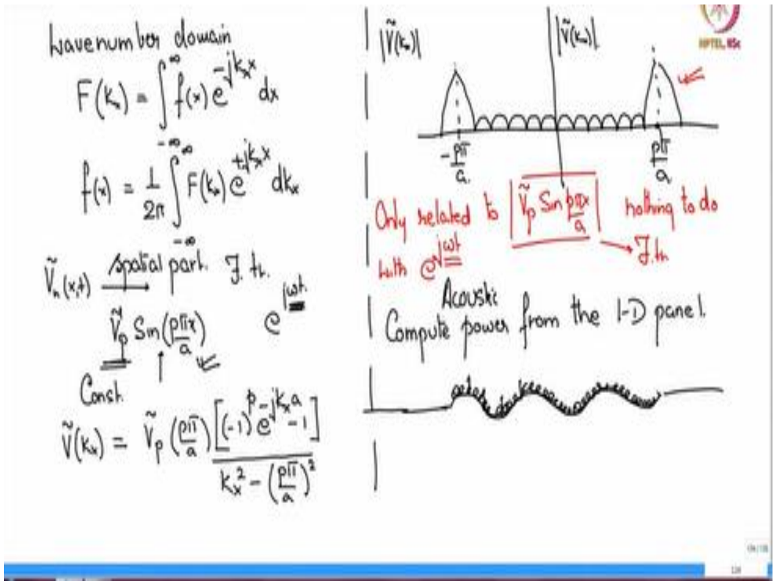

Now we look at this in the Fourier domain or we call it the wave number domain and the definition we choose is the forward transform

$$F(k_x) = \int_{-\infty}^{\infty} f(x) e^{-jk_x x}\, dx.$$

And the inverse

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k_x)\, e^{+jk_x x}\, dk_x.$$

So, now if we just take the spatial part of $\tilde{V}_n$, we just take the spatial part and apply the wave number transform or Fourier transform.

That means the Fourier transform is applied to $\tilde{V}_p \sin\left(\frac{p\pi x}{a}\right)$, $e^{j\omega t}$ is left out. By the way as before $\tilde{V}_p$ is the amplitude which is held constant it does not change with frequency. It is held constant, and this mode is being driven at this frequency any particular frequency. So, now if we apply the Fourier transform to this, we will get

$$\tilde{V}(k_x) = \tilde{V}_p \left(\frac{p\pi}{a}\right) \frac{\left[(-1)^p e^{-jk_x a} - 1\right]}{{k_x}^2 - \left(\frac{p\pi}{a}\right)^2}.$$

So, if we plot the magnitude of this, we get $+\frac{p\pi}{a}$, $-\frac{p\pi}{a}$ we get a peak here then these bumps we get a peak here. We get these bumps this is how this looks like magnitude $\tilde{V}(k_x)$. Now let me tell you that this is only on purely related to $\tilde{V}_p \sin\left(\frac{p\pi x}{a}\right)$. It has nothing to do with ω, it has nothing to do with ω. The moment you have this shape in the spatial domain and you put it through a Fourier transform.

This is what you will get. Now we would like to compute an expression for power from the 1D panel. And so, we will compute that using acoustic power we mean acoustic power we will compute that using the normal velocity and the pressure on the panel. We computed using these two expressions.

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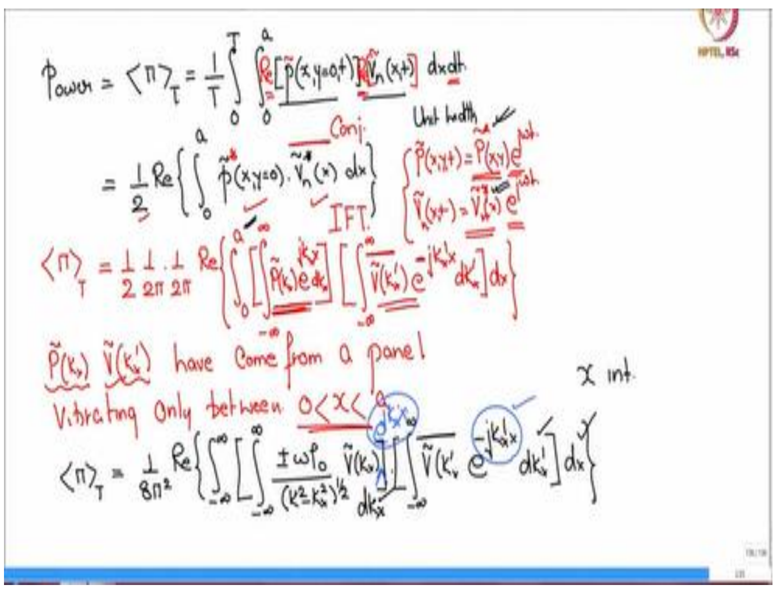

So, now let us see here how does it look like. The power space averaged power which I will I mean time averaged power is given by

$$\langle \pi \rangle_T = \frac{1}{T}\int_0^T \int_0^a Re[\tilde{p}(x, y = 0, t)]\, Re[\tilde{V}_n(x, t)]\, dx\, dt,$$

and let us assume a unit width perpendicular to the paper. So, this is kind of power per unit width. Now we know that the time can be removed,

$$= \frac{1}{2} Re\left\{\int_0^a \tilde{p}^*(x, y = 0)\overline{\tilde{V}_n^*(x)}\, dx\right\}.$$

So, here we took the pressure and velocity to be real so just to clarify we took the real pressure, let me do this. We will make this real of $\tilde{p}$ and will make this real of $\tilde{V}_n$ that is better. So, real part of $\tilde{p}$ real part of $\tilde{V}_n$. So, that I can have $\tilde{p}(x, y, t) = \tilde{p}^*(x, y)e^{j\omega t}$and I can have $\tilde{V}_n(x, t) = \tilde{V}_n^*(x)e^{j\omega t}$, this $e^{j\omega t}$ is the thing that always messes up.

So, we start with this we take the real parts, and we multiply, and we integrate over time that is now equivalent to one here, this is equivalent to this. So, let us say $\tilde{p}^*,\ \tilde{V}_n^*$ now I will make conjugate as a bar over here this is conjugate. So, here the time is removed using the $p\ v$ conjugate relation. Now this time averaged power is equal to the one half which is already there and for this and this I am going to bring in inverse Fourier transforms.

So, which means I get

$$\langle\pi\rangle_T = \frac{1}{2}\,\frac{1}{2\pi}\,\frac{1}{2\pi}\ Re\left\{\int_0^a \left[\int_{-\infty}^{\infty} \tilde{p}(k_x)e^{jk_x x}\, dk_x\right]\left[\int_{-\infty}^{\infty} \overline{\tilde{V}(k_x')}\, e^{-jk_x' x} dk_x'\right]\, dx\right\}.$$

This minus again because it is conjugated and again dx at the end. Now the

$\tilde{p}(k_x)$ and $\tilde{V}(k_x')$ have come from a panel vibrating only between this. The form of $\tilde{V}(k_x')$ and the corresponding $\tilde{p}(k_x)$ comes up only because the panel vibrates between 0 and a. So, these inverse Fourier transforms will lead to those pressures which correspond to a panel vibrating between 0 and a and no vibration of sight.

Therefore, I can safely change this limit to $-\infty$ to $+\infty$. So, how do what do I get? I get power per unit width of the panel,

$$\langle \pi \rangle_T = \frac{1}{8\pi^2}\, Re \left\{ \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \frac{\pm \omega \rho_0}{\left(k^2 - {k_x}^2\right)^{1/2}} \tilde{V}(k_x) e^{jk_x x}\, dk_x \right] \left[\int_{-\infty}^{\infty} \overline{\tilde{V}(k_x{'})}\, e^{-jk_x{'} x} dk_x{'} \right] dx \right\}.$$

There are there is a dk_x and a $dk_x{'}$ integral and a dx integral. So, we have one do one of these first and we will do the x integral occur. Why? Because x occurs here

So, the x occurs only here and only here nowhere else. So, we will do the x integral first. So, let us see.

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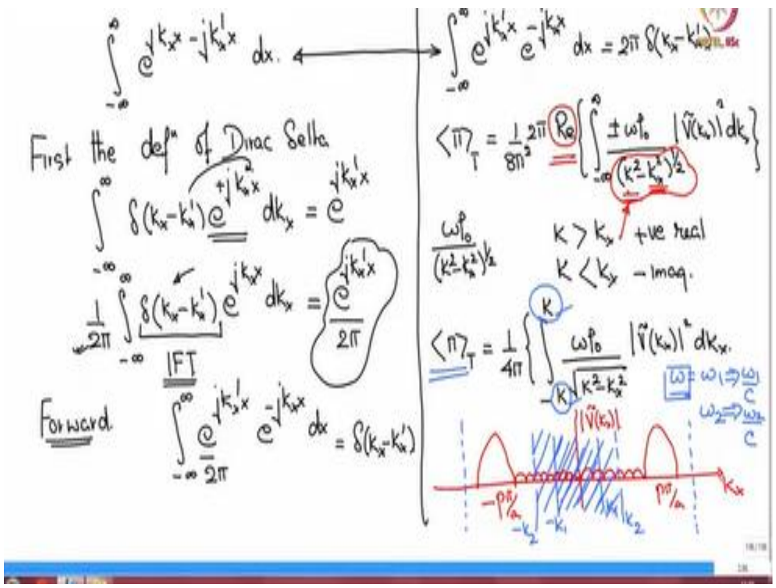

So, what is that? That we have to $\int_{-\infty}^{\infty} e^{jk_x x - jk_x{'} x}\, dx$. So, let us see now. First the definition of a delta function of a Dirac delta function. So, if we have

$$\int_{-\infty}^{\infty} \delta(k_x - k_x{'}) e^{+jk_x x}\, dk_x = e^{jk_x{'} x}.$$

Now what I do?

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(k_x - k_x{'}) e^{+jk_x x}\, dk_x = \frac{e^{jk_x{'} x}}{2\pi}.$$

This is now the inverse Fourier transform of this function. This is the inverse Fourier transform by definition earlier of this function. So, that gives me including the Dirac definition this divided by 2π which is this.

So, now if I look at the forward transform. The forward transform of this now should give me delta back. So, the forward transform which is

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{jk_x{}'x - jk_x x}\, dx = \delta(k_x - k_x{}').$$

$$\int_{-\infty}^{\infty} e^{jk_x{}'x - jk_x x}\, dx = 2\pi\, \delta(k_x - k_x{}').$$

So, this is what we wanted. So, in the earlier integral in the integral we had previous page here. For this part of the integral, we replace it by twice by $\delta(k_x - k_x{}')$ and we are left with two integrals one on k_x one on $k_x{}'$. But this delta will force both of them to be together. So, they will collapse to one integral. So, they will collapse to one integral that means all will be k_x.

So, what we will have as a resultant is that we will have this power time average power per unit width given by

$$\langle\pi\rangle_T = \frac{1}{8\pi^2} 2\pi\, Re\left\{\int_{-\infty}^{\infty} \frac{\pm\omega\rho_0}{\left(k^2 - k_x{}^2\right)^{1/2}}\, \left|\tilde{V}(k_x)\right|^2 dk_x\right\}.$$

Now with this expression we had said before that $k^2 - k_x{}^2$ if k is greater than k_x then we have a real value we will take the positive real root.

If k happens to be less than k_x, we will take the negative imaginary root. So, as long as k is bigger this expression is real. So, this real will choose as long as k is bigger than k_x. The moment k_x exceeds k its imaginary and the real will not choose it. So, k_x cannot exceed k in this integral if it has to survive. So, we will get with change of limits $\frac{1}{4\pi}$ real is removed. Now

$$\langle\pi\rangle_T = \frac{1}{4\pi}\left\{\int_{-k}^{k} \frac{\omega\rho_0}{\left(k^2 - k_x{}^2\right)^{1/2}} \left|\tilde{V}(k_x)\right|^2 dk_x\right\}.$$

So, if I have to bring in the picture that I drew earlier we had $\tilde{V}(k_x)$ shown like this. All these little bumps and a peak at + $\frac{p\pi}{a}$ this is k_x axis and then again bumps here and + $\frac{p\pi}{a}$, $-\frac{p\pi}{a}$ and this was magnitude of $\tilde{V}(k_x)$. So, now comes the ω so far ω did not come in the picture. Now comes the ω so I can choose ω = ω_1 then that would give me k_1 which is ω_1 by c so now ω_1 decides k_1.

So, let us say k_1 is he happens to be here k_1. So, we now integrate based on ω this k value gets decided that is the only way k value gets decided. So, now k = k_1 and the range of integration is - k_1 to k_1. So, at this frequency only this range of wave numbers need to be integrated and power is computed. But suppose the same panel same mode is vibrated at a higher frequency another ω_2 which means the k_2 now is ω_2 by c.

So, if I draw the k_2 line k_2 is here and - k_2 is here then this becomes k_2 and k_2 . Now I get more wave numbers within the range. So, the power will increase in the limit. I could excite the same mode at a further higher frequency where k is such that it crosses this plus and minus $\frac{p\pi}{a}$ and then I would get the maximum power possible radiated.

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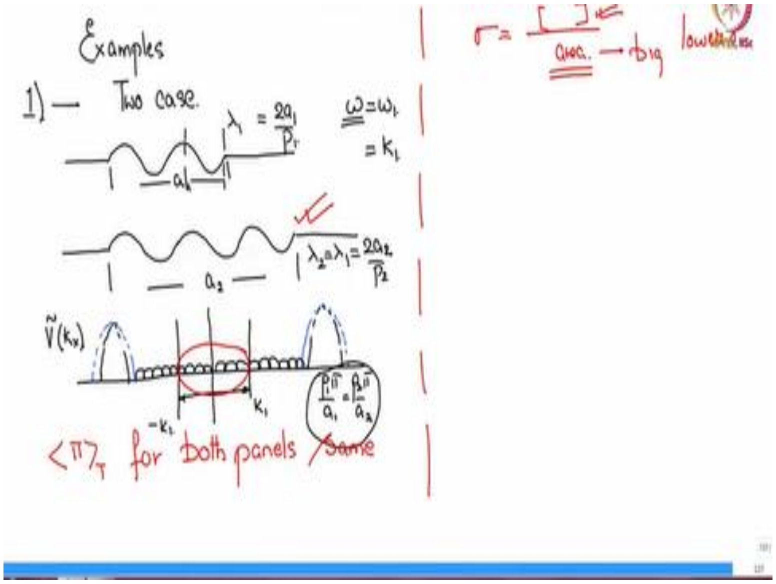

So, let us now just for churning our head look at two cases examples. An example 1 has two different problems, within example one there are two cases possible. So, one is a plate which carries a certain wavelength is he said in a baffle as before it carries a wavelength λ_1 given by $\frac{2a_1}{p_1}$. It is case that is first case within that. The next is a panel now which carries the same wavelength, but it is longer so let us say this one is $a_1$and this one is a_2.

So, this is also λ_1 actually λ_2 which is equal to λ_1 but it is $\frac{2a_2}{p_2}$. The number of half cycles is different. So, if you compute the $\tilde{V}(k_x)$ for both these cases, they look very similar. In the main lobe where it is $\frac{p\pi}{a}$ or you know $\frac{p_1\pi}{a_1}$ or $\frac{p_2\pi}{a_2}$ the shorter one may have wider picture, so same here. Why does picture for shorter one and thinner picture for longer one? But otherwise below the picture for both looks very similar.

So, if I excite both the panels at certain $\omega = \omega_1$which leads to a k_1 and it is less than these wave number values. That means below coincidence of that mode then $k_1$line can be put here k_1 and this will be - k_1 line and the integration limits will extend between - k_1 and k_1. And for both panels because this range looks similar this range looks similar. You will get the same power; power is same for both panels.

Now what about radiation efficiency? Radiation efficiency σ one of the terms in the denominator is the area of the panel. So, now numerator the actual power from panels is same for both. But in one case area is bigger. So, this case area is bigger so it will have lower radiation efficiency. So, we have runoff of time. The second example I will take in the next class. Thank you.