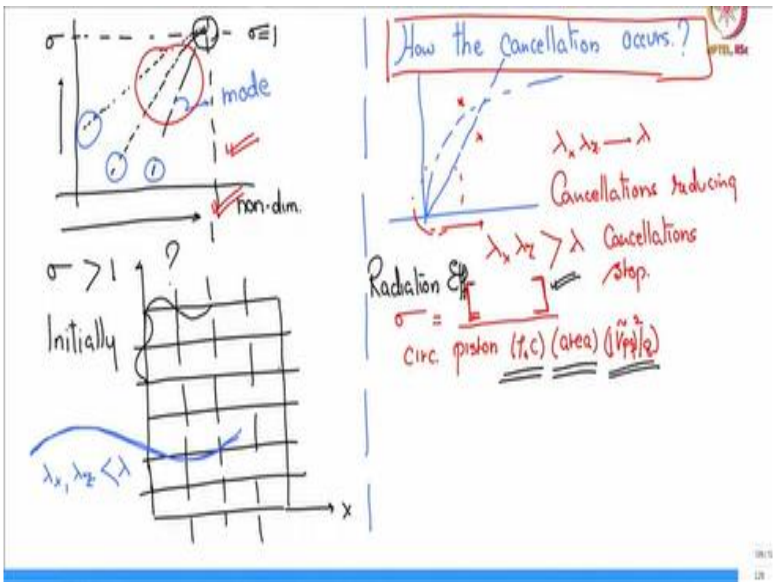
Sound and Structural Vibration
Prof. Venkata R. Sonti
Department of Mechanical Engineering
Indian Institute of Science, Bengaluru

Lecture - 37
Physics of Volume Velocity Cancellation

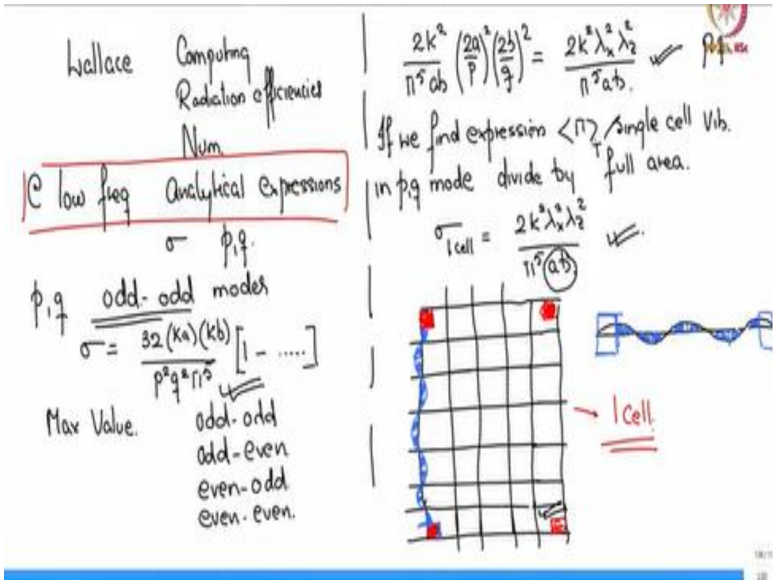
Good morning and welcome to this next lecture on sound and structural vibration. In the last class we reached this point where we were looking at this factor radiation efficiency.

(Refer Slide Time: 00:35)



And we defined it in this manner the sound radiated by a panel in a certain medium at a certain frequency with a certain amplitude divided by the sound power radiated by a circular piston in the same medium with the same area as the panel and same mean square space time averaged velocity.

(Refer Slide Time: 01:22)



Now we have been talking about cancellation, so what how does this cancellation occur? So, we will look at that. So, I refer you back to the paper by Wallace, I will give you the title later maybe. So, Wallace in addition to computing radiation efficiencies numerically provides at very low frequency analytical expressions for radiation efficiency for various types of p and q values. So, now let us take p odd and q also odd they are odd that means $1, 3, 5$ etcetera odd-odd modes.

And at low frequencies he gives the expression $\frac{32(ka)(kb)}{p^2q^2\pi^5}$; this is the dominant term and there are other terms smaller terms. And this turns out to be the maximum value amongst what the odd-odd modes that means p is odd q is odd the odd-even modes p is odd q is even or even-odd modes p is even q is odd and finally even-even modes. Amongst all these modes this odd-odd mode gives this radiation efficiency which is maximum.

Now this can be this expression the dominant term can be written also as $\frac{2k^2}{\pi^5 ab}\left(\frac{2a}{p}\right)^2\left(\frac{2b}{q}\right)^2$ and if you recognize this is $\frac{2k^2{\lambda_x}^2{\lambda_z}^2}{\pi^5 ab}$. Now if we compute if we find the expression of power time averaged power from a single cell let me put T for time hours from a single cell vibrating in the p, q th mode.

So, the panel is vibrating in the pq mode and we are looking at one single cell. We compute the power, and we divide by the full area. So, what we get is we get σ let me call it 1 cell given by

$$\sigma_{1cell} = \frac{2k^2{\lambda_x}^2{\lambda_z}^2}{\pi^5 ab}.$$

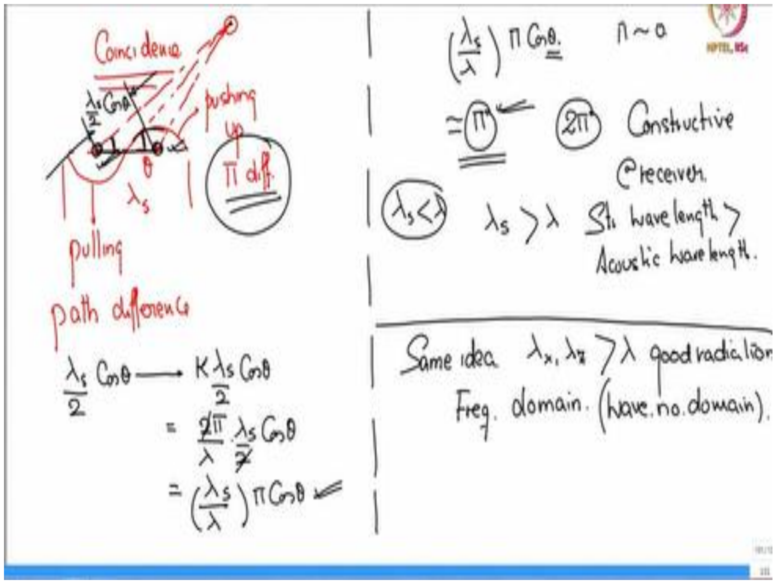
Mind you these are low frequencies; these analytical expressions or low frequency should not forget low frequency. Now if we look at the picture of this thing, so let us first look at this here.

So, the entire panel radiation efficiency at low frequencies is this when p and q are odd. And the radiation efficiency of a single panel but divided by the whole panel area is this. So, what it means is that the entire panel is equal to 1 cell. So, how does this happen so if we look at this edge let us say so now, we have this $1, 2, 3, 4$ half cycle half and we need one more because is odd we need a odd number of half cycles so we need this.

This makes it odd number of half cycles $1, 2, 3, 4, 5, 6$ is also not correct. So, we will let us see we will add one more. Now we have odd half cycle in both directions; both are odd-odd. So, now if this region this positive region cancels with this positive region, this region cancels with that region and this cancels with it and this cancels with this, then I have a half of the last cell un-cancelled and similarly if I look at in the y direction, I mean z direction these are half cycles.

So, this half cycle cancels with this and so forth half here remains un-cancelled. So, I have the panel portion here that is un-cancelled. Let me put it in red. I have this part here un-cancelled and so that is how these four quarters amount to one cell. So, that is how your cancellation is happening.

(Refer Slide Time: 11:09)



So, now to further elaborate this. So, we will just reinforce the idea of coincidence, so if we have let us say a mode in the panel with a certain structural wavelength and this is my structural lambda let us say. And these are the centres of the half cycle, and I am looking at a point here where I am trying to compute my power radiated and it is very far off. So, that these angles are more or less the same.

This angle is the angle theta let us say and these are more or less the same it is so far off. Now so this is pulling down this is pushing up, so there is already a π difference, π phase difference. Now what is the path difference? To this remote point here the path difference is this perpendicular distance this is the path difference from if we collect the volume velocity source to be at the centre here and this volume velocity to be the centre here this is approximately the path difference between the travel to this receiver point.

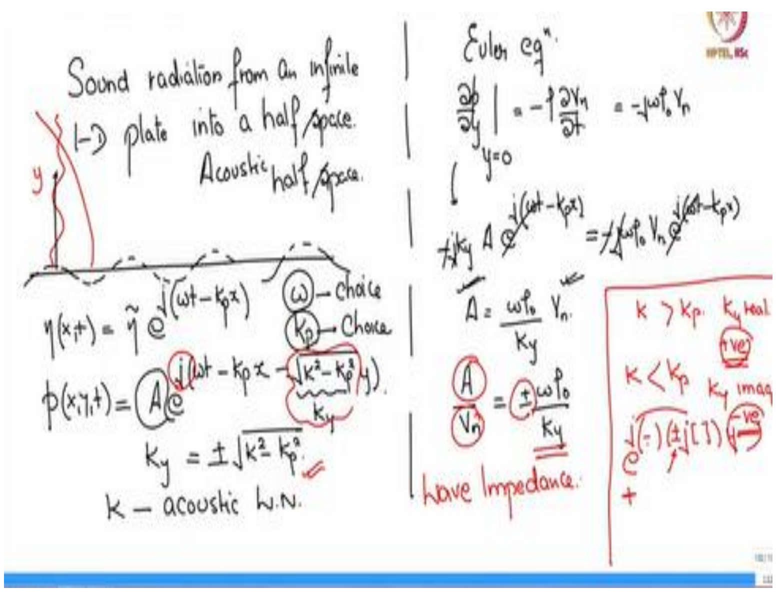
So, this distance is if this angle is theta approximately and this happens to be lambda structure by 2, so this had this is lambda structure by 2 $\cos\theta$, so lambda structure by 2 $\cos\theta$ is the path difference. This path difference amounts to what phase difference it is just multiplied by k because k the acoustic wave number is phase difference for unit distance, so λ_s by 2 $\cos\theta$.

Now if you write k in terms of lambda acoustic what is it its 2π by lambda acoustic into lambda s by 2 $\cos\theta$. So, this 2 goes and I have lambda structure by lambda acoustic into $\pi\cos\theta$. This is achieved purely by path difference, there is already a π difference. So, this expression $\frac{\lambda_s}{\lambda}\,\pi\cos\theta$ or all θ that you can think of. So, then this will be very close to π and it will be close to 0 also based on θ values.

And it has to be approximately π only then you have this π added to the π difference from sign difference there is a sign difference here that is the π difference here. So, this π and this π added together will give you something like 2π that is when you have constructive interference at the receiver. So, in order for that to happen if λ_s is less than λ you will always be less than π so this is a schematic.

So, for a chance of a constructive interference to happen λ_s has to be greater than λ that means structural wavelength has to be greater than acoustic wavelength. Now we will look at this idea from a wave number or frequency domain point of view. The same idea that λ_x and λ_z have to be greater than λ for good radiation we look at in the frequency domain or it is called the wave number domain.

(Refer Slide Time: 18:12)



So, in order to do that I need one small result which I will derive now here a similar idea we have seen before. So, let us see sound radiation from an infinite 1D plate into a half space. I had used this schematic earlier to demonstrate coincidence, but we will do a little bit more now. So, I have this infinite 1D plate which is carrying a flexural wave so what is that description let us say

$$\eta(x,t) = \tilde{\eta} e^{j(\omega t - k_p x)}.$$

For this case ω is my choice we can choose, and we will choose k_p also and this is acoustic half space. So, now if I compute the pressure in the half space due to this panel vibration, I will get

$$p(x,y,t) = Ae^{j\left(\omega t - k_p x - \sqrt{k^2 - k_p{}^2} y\right)}.$$

This is the y wave number k_y is happens to be of this form $\pm\sqrt{k^2 - k_p{}^2}$ where k is again my acoustic wave number.

So, we will try to find this A here that is the idea. So, how to find the A? Now for harmonic situation our friend is the Euler equation which bails us out every time. So, my $\left.\frac{\partial p}{\partial y}\right|_{y=0} = -\rho \frac{\partial V_n}{\partial t} = -j\omega\rho_0 V_n$. So, what is $\frac{\partial p}{\partial y}$? So, $\frac{\partial p}{\partial y}$ is for the moment let us call it $-jk_y Ae^{j(\omega t - k_p x)}$ we are doing it at y equal to 0 that is equal to $-j\omega\rho_0 V_n e^{j(\omega t - k_p x)}$.

$$-jk_y Ae^{j(\omega t - k_p x)} = -j\omega\rho_0 V_n e^{j(\omega t - k_p x)}.$$

$$A = \frac{\omega\rho_0}{k_y} V_n.$$

So, now this is equal to a pressure amplitude, this is my velocity, so if I divide A by V_n let us say then I get $\frac{\omega\rho_0}{k_y}$ and k_y carries a plus minus sign, so I will put a plus minus here.

$$\frac{A}{V_n} = \pm\frac{\omega\rho_0}{k_y}.$$

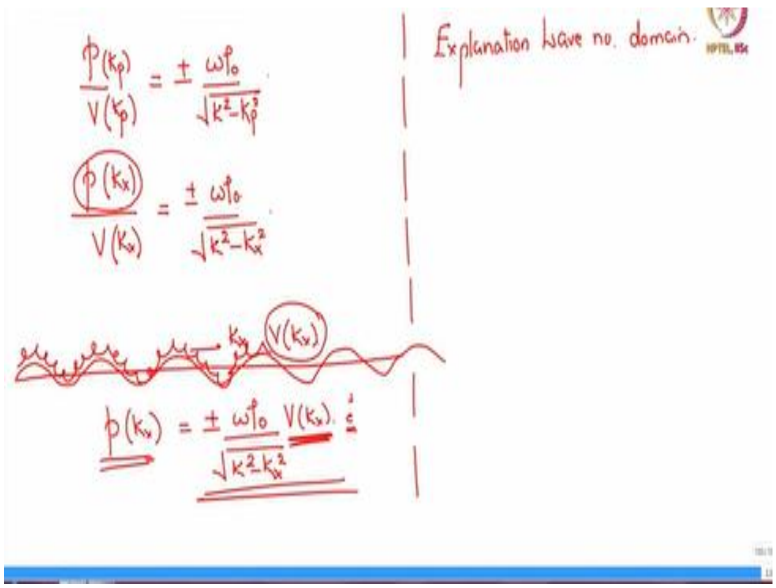
Now why does k_y carry a plus minus sign if see I have chosen k_p and I have chosen ω. So, it can happen. Let me do it here. It can happen that k is greater than k_p in which case it is going to be a real value for k_y, k_y will be real value.

If k_y is a real value, then I would like the sound propagated away in the y direction, so k_y must be real and positive so I have to choose one sign however k could be less than k_p in which case k_y can be imaginary. If k_y is imaginary and I have a j over here and I will have j times something over here, so I will have e to the power j over here then a minus then a plus or minus j times something into y.

So, j square gives me -1 so I already have j square gives me -1 and this gives me another minus, so I had to get a plus. Now here if I choose positive sign this means in the y direction, I have an increasing sound pressure till infinity that cannot happen. So, I need to choose negative, so in case of imaginary I have to choose negative, in case of real I will choose positive, that is this plus minus sign over here for k_y.

So, now this is pressure, this is velocity there is a ratio those this can be considered a kind of an impedance we will call it the wave impedance.

(Refer Slide Time: 27:15)



So, if we I mean change page here let me write it, so I have the pressure I will write it as pressure. Now and I will write it as velocity and pressure. What radiated at single wave number velocity traveling on the plate? At this wave number is equal to $\pm\frac{\omega\rho_0}{\sqrt{k^2-{k_p}^2}}$. So, what does it say? So, I will replace k_p by k_x because of a reason that is coming up.

So, I will say that pressure radiated at a certain wave number that is moving on a panel divided by velocity at the same wave number that is moving on the panel is equal to $\pm\frac{\omega\rho_0}{\sqrt{k^2-{k_x}^2}}$. So, that means what if I have an infinite panel there is a wave moving with a certain velocity amplitude this wave number is k_x and the related velocity is $V(k_x)$.

Then I immediately know this is the amplitude then I immediately know the pressure related to that which is given by pressure related to that wave number is $\pm\frac{\omega\rho_0}{\sqrt{k^2-{k_x}^2}}V(k_x)$. So, there will be the related propagators $e^{j\omega t}$etcetera on both sides but this is the amplitude. So, single wave number moves on the panel with this velocity amplitude what is the pressure on the surface.

This describes the pressure right on the surface what is that pressure that is equal to $\omega\rho_0$ with a plus minus sign divided by $\sqrt{k^2 - {k_x}^2}$, so this result is important for us. Now what we will do is we will look at the explanation in the wave number domain for whatever we found so far. The cancellation effect we spoke of we will describe, or we will try to understand it using Fourier transform domain. Now we are running out of time so I will stop here and will continue from the next lecture.