Sound and Structural Vibration

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Lecture - 36
Radiation Efficiency

Good morning and welcome to this next lecture on sound and structural vibration. If you recall last class, we were looking at sound radiation from a single cell.

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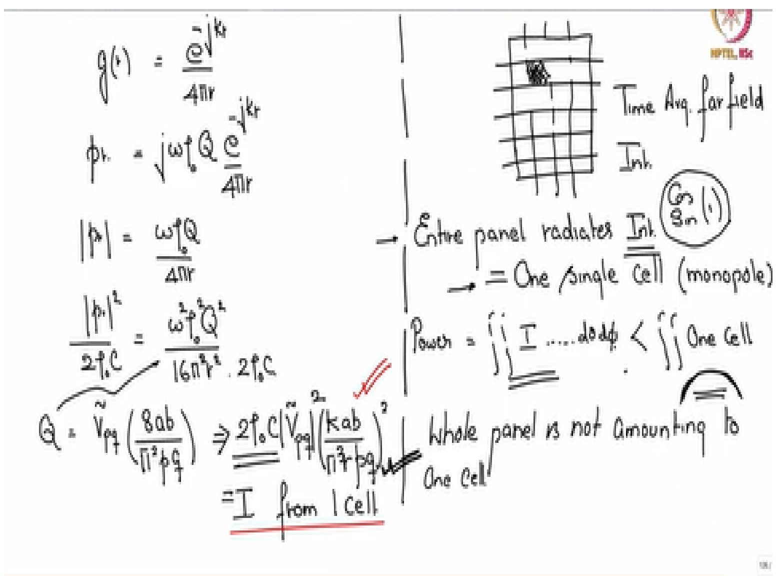

I have a plate or panel vibrating in a mode, a single mode with a certain pq and we are looking at sound radiation from 1 cell out of that and we computed time averaged far field sound intensity. That is this expression from 1 cell. Now I will remind you.

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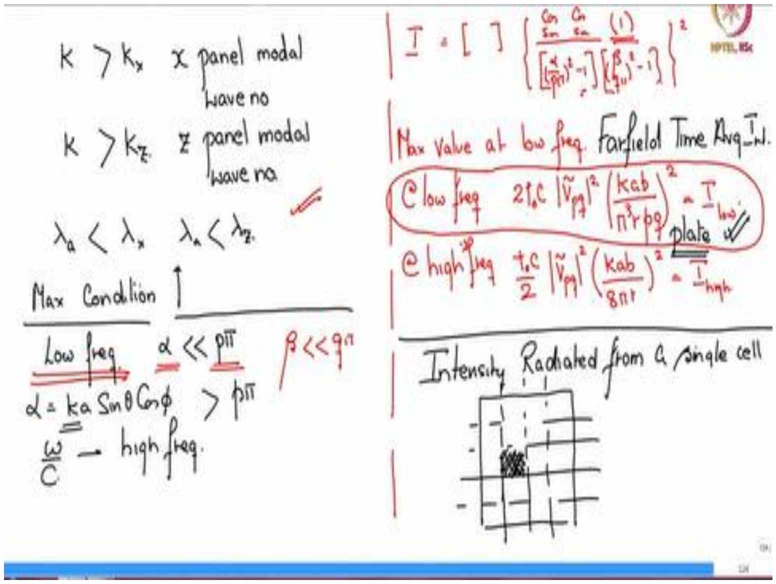

I will show you this picture this is the value over the whole cell at low frequencies. I mean this is the value over the entire plate. This is the sound intensity in the far field from the entire plate at low frequency. So, you will see that both are the same twice $2\rho_0 c|\tilde{V}_{pq}|^2\left(\frac{kab}{\pi^3 rpq}\right)^2$. So, let me go back here $2\rho_0 c|\tilde{V}_{pq}|^2\left(\frac{kab}{\pi^3 rpq}\right)^2$ both are same.

So, what the entire panel has radiates the intensity is equal to what? 1 single cell. Assumed as a monopole. Now which means what? What it means is how do you compute finally the power you compute power by integrating the intensity over angles in the far field. So, you will take your intensity and integrate over $d\theta\ d\phi$. Now this intensity we assumed cos and sin to be 1 in the panel case. We assume the maximum value. But actually, it will vary with angles.

So, that means if we actually compute power from the entire panel it is going to be less than what you get from 1 single cell. Intensity from 1 cell because in the; monopole sense there is no variation in intensity over a hemisphere. For the expression as a radiation from a single cell there is no dependence on theta phi whereas for the entire panel hypothetically, we put cos and sin to be ones. But actually, they contain theta and phi variations. So, we took the maximum value.

So, when we took the maximum value, the intensity was equal to what 1 cell radiates. But in actuality it is not maximum at all angles. So, it will be less than that. So, if you actually compute the power from the entire panel and compute the power from 1 single cell what the 1 entire panel radiates is less than 1 cell. So, the whole panel is not amounting to 1 cell. That is the conclusion.

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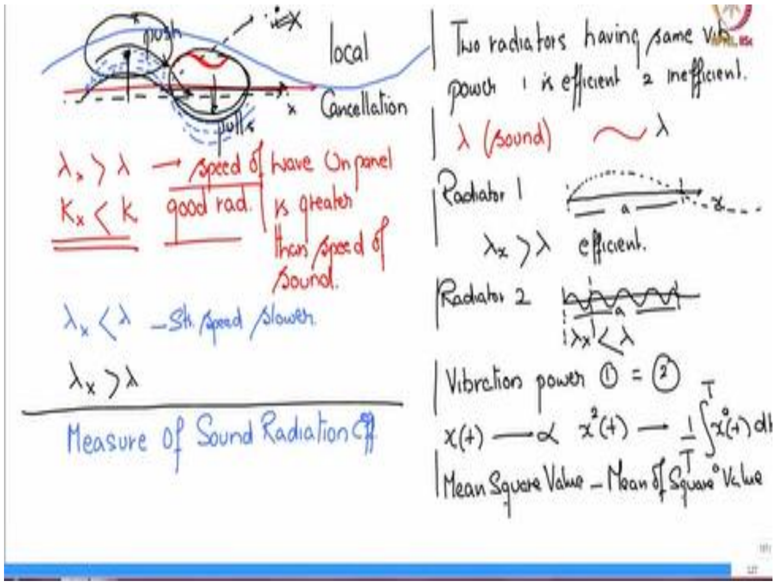

So, what would that mean? So, what that means is this. At the very first cut that is what it means. So, this is a 1 wavelength let us say 1 wavelength on the panel may be in the x direction. And it is radiating sound so, it pushes the fluid here and it pulls the fluid here. And that is how sound is radiated to some distance. And suppose the acoustic wavelength is this much this is the acoustic wavelength. That means what?

λ_x is greater than lambda acoustic which means k_x is less than k which is the condition for good radiation. This gave us the maximum at high frequencies. So, λ_x is greater than lambda which means what? In some sense the speed of the wave on the panel is greater than the speed of sound in the medium. The panel wave moves faster than the sound wave in the medium. That is what this condition means. In the flip side what happens?

Suppose the acoustic wavelength is bigger that means structural wavelength is smaller. Which means what? This time λ_x is smaller than lambda. So, structural speed is slower. Then what happens? As this structure starts to push and it starts to pull it is doing it slowly. It is pushing it slowly in comparison to the speed of the fluid. And so, the fluid will try to find an escape route if you try to push it will try to find the path of least resistance and escape around the panel.

And therefore, at a distance the receiver point will not receive sound. There is a local cancellation. Amidst the high regions and low regions of the panel the fluid escapes. In contrast what is it when λ_x is greater than lambda the structural wave moves very fast. And the fluid does not have time to escape. Fluid is slower in this case. So, the structure pushes and pulls much faster than the fluid can respond. And so, the fluid has to be compressed here.

And the fluid has to be rarefied here. So, that at some receiver point there is sound. So, that is what this means. We need a measure of sound efficiency. What that means is that two radiators having same vibrational power and one is efficient and two second fellow is inefficient. Now how do we say that two radiators with the same vibrational power one is efficient, and one is not efficient?

So, let us say that the wavelength of sound lambda is of this dimension and there is radiator 1. And we just plot the x directional vibration. Let us say this is the x direction. This is the dimension of the plate let us say this is a. So, let us say the vibration is like this. Definitely the λ_x is greater than lambda that is when the radiator is efficient. And let us take radiator 2 and that is also having the dimension a but this is lambda acoustic.

So, let us say its profile looks like this. So, this happens to be the structural wavelength definitely less than lambda. Now it is possible that its vibrational power is equal of 1 and 2 are equal. So, regardless of the vibrational powers being the same the sound radiated is different. Now what is the measure of power? If I have a signal $x(t)$ its instantaneous power is proportional to $x^2(t)$.

So, then from there we can derive a mean square value. Mean square means what? Square and mean of it so, mean of square value. So, mean square value is mean of square value,

$$\frac{1}{T}\int_0^T x^2(t)\, dt.$$

So, that is the mean square value. So, that is the indicator of power.

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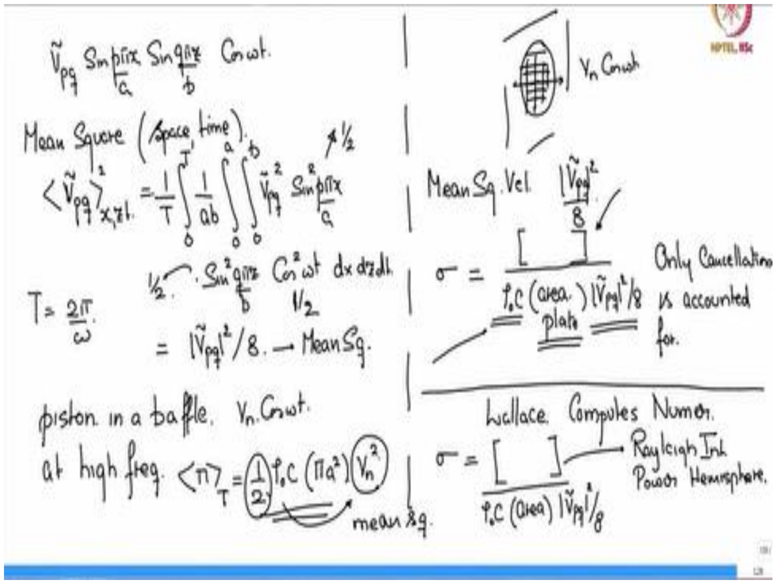

So, suppose I have a signal of this form $\tilde{V}_{pq}\sin\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi z}{b}\right)\cos\omega t$. This is the form of vibration on the panel. So, I will do a mean square value of this, but it will be mean in the sense of space and in the sense of time. So, we will indicate this as a mean square value x, z and t. So, what will that be? That will be

$$\left\langle \tilde{V}_{pq}\right\rangle^2_{x,z,t} = \frac{1}{T}\int_0^T \frac{1}{ab}\int_0^a\int_0^b \tilde{V}_{pq}{}^2 \sin^2\left(\frac{p\pi x}{a}\right)\sin^2\left(\frac{q\pi z}{b}\right)\cos^2\omega t \; dx\, dz\, dt.$$

$T = \frac{2\pi}{\omega}$, panel is vibrating at ω frequency. So, if you compute this this is going to be equal to $\frac{|\tilde{V}_{pq}|^2}{8}$. You get a half from here; you get a half from here and you get a half from here and you get $\frac{|\tilde{V}_{pq}|^2}{8}$. Now that is your mean square velocity.

Now what is a good radiator? So, consider a piston a circular piston in a baffle radiating with velocity $V_n\cos\omega t$ and at high frequency where it is most efficient. The power, the time averaged power is given by

$$\langle \Pi\rangle_T = \frac{1}{2}\,\rho_0 c\,(\pi a^2)V_n{}^2.$$

Because there are no special variations as the piston vibrates. If you have a piston in a baffle vibrating back and forth at $V_n\cos\omega t$ oscillating back and forth it is V_n is uniform over the circular piston.

There are no ups and downs as it happens in the vibrating plate. So, this is now the power. And this is the mean square velocity, mean square velocity only related to time. But now the panel has variations in space also. And therefore, the mean square velocity is $\frac{|\tilde{V}_{pq}|^2}{8}$. So, now suppose we define a measure σ as the power radiated by the panel at ω. Time averaged power vibrating from a vibrating panel as the numerator.

And in the denominator, we put $\rho_0 c$ area. This area is the same as the area of the panel and $\frac{|\tilde{V}_{pq}|^2}{8}$. So, now this gives a measure where only cancellation is accounted for. So, the denominator is a radiator radiating into the same medium, has the same area, has the same mean square velocity. And so does the numerator but it so happens that the numerator has volume velocity cancellations and therefore, it will radiate less.

Now there is a paper by Wallace which you will see in some detail later. And that is what it does it computes numerically the radiation efficiency using this measure above which is equal to the numerically computed power from a rectangular panel mostly with simply supported boundary conditions. And this denominator $\rho_0 c$ area into $\frac{|\tilde{V}_{pq}|^2}{8}$. This is of course computed using the Rayleigh integral and then the power over the hemisphere power computed over the hemisphere.

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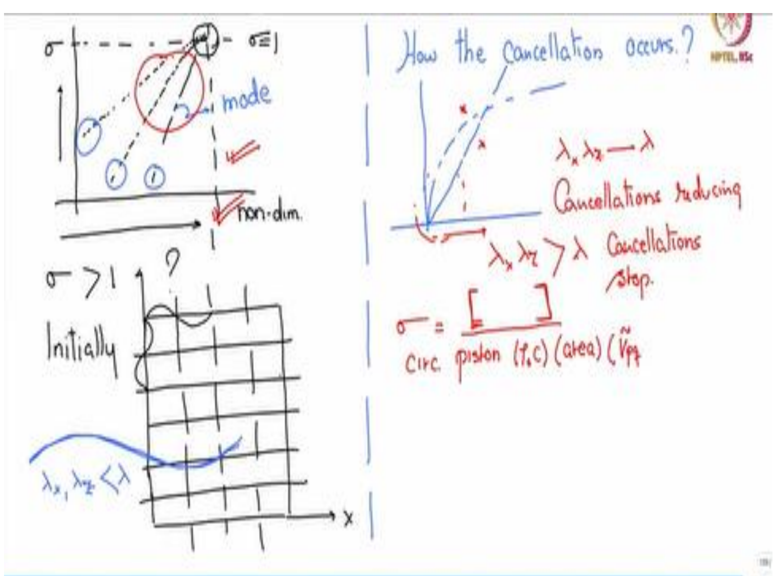

If you do that and we plot there is a non-dimensional frequency here which I will explain a little later. So, let us say that non-dimensional frequency attains the value 1 here and the σ is plotted over here. Then you will get these pictures radiation efficiency is 1, σ equal to 1 here. So, you will get an increment with frequency, and something happens here. You can go above 1 also you get this sort of pictures. So, do not worry about this portion here exactly.

This measure σ can be greater than 1. But do not worry about it for now we will talk about it may be little later. But main feature is that as frequency increases in this direction the radiation efficiency goes up. So, what is happening is that initially if you look at your panel and let us say we are in some particular mode. So, you can kind of see that this is the wavelength in the x direction this is x and that is z let us say.

And this is the kind of wavelength in the z direction etcetera. So, you start off at low frequencies where the wavelength of sound is much bigger and so cancellations will occur. So, λ_x, λ_z the panel mode wavelengths are less than lambda the acoustic wavelength. And therefore, there will be cancellations occurring of volume velocity. So, how the cancellations occur? We will see again a little later. But let us just say that cancellations do occur.

And then so you are somewhere here at low frequencies. So, you are not so efficient. These are modes by the way any particular mode. Now as frequency starts to go up again the same coincidence picture starts to come in. Your acoustic wave number goes like this, and your structural wave number goes like this. And you are somewhere in this region where acoustic wavelength is less, or acoustic wave number is less, or acoustic wavelength is higher.

And as you go up in frequency the wave number behaviour is square root ω and directly proportional to ω here same thing happens. So, somewhere you will have λ_x and λ_z approach lambda. So, that the cancellations are reducing. So, maybe you are somewhere here in this region. And then finally when you cross this non-dimensional frequency of 1, λ_x and λ_z they become greater than lambda and the cancellation stop.

No more cancellation that means now your panel is at its most efficient radiating condition. So, this picture entirely captures that cancellation effect. And therefore, σ is a reasonably good measure. So, it is the sound radiated in the far field by a single mode at a single frequency. The time averaged power radiated divided by the power radiated by a circular piston. And it is most efficient in the same medium having the same area as that of the plate and the same space time means square averaged velocity.

So, we are out of time. So, we will continue from here. And we will look at what is the cancellation nature. We will do that in the next class. Thank you.