Sound and Structural Vibration
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Lecture – 35
Derivation of Pressure Response

Good morning and welcome to this next lecture on sound instructional vibration. We started looking at this new topic of a plate set in a rigid baffle in radiating sound.

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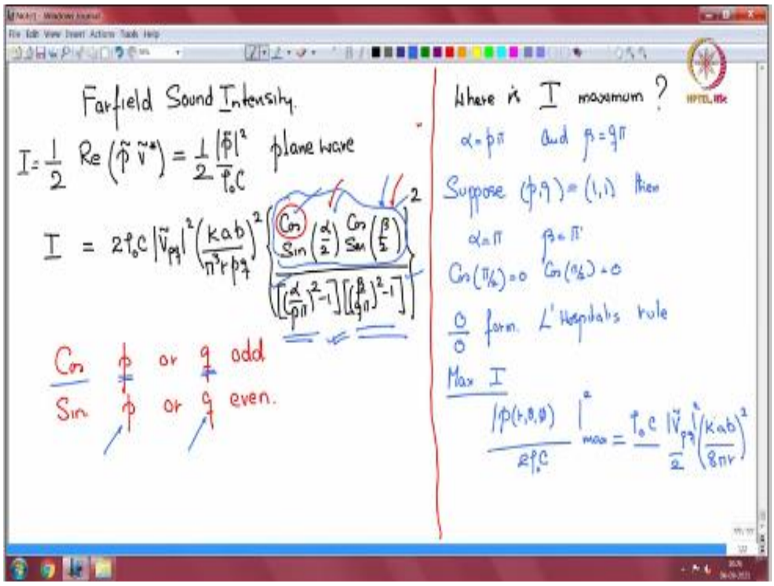

We had reached up till this point so, the intensity, we had computed as given by this expression where this expression indicates that you choose the cosine when p is odd or q is odd, p comes in here, q comes in here and you choose the sin when p is even or q is even so, that is the intensity. Now, this is the far field intensity. We had done a far field approximation on the Rayleigh integral. So, this is far field intensity.

Now, we asked the question where the intensity maximum is or under what conditions is it a maximum, we asked this. And just on the surface of it looking at the expression, it seems like when the denominator goes to 0, it seems like when α is equal to $p\pi$ and β is equal to $q\pi$. But remember suppose we choose the $1, 1$ mode p, q is equal to $1, 1$. Then p is odd and q is odd, then both places I should choose the cosine.

I should choose the cosine here and cosine here and α is now equal to π and β equal to π. Then we will have $\cos\left(\frac{\alpha}{2}\right)$ which is $\frac{\pi}{2}$ and we will have $\cos\left(\frac{\beta}{2}\right)$. Both are equal to 0. So, at α

$= p\pi$, denominator is 0 and numerator is 0. So, we have a 0 by 0 form. So, we have to use LHopital's rule, so, it is doable; it is a little cumbersome but doable. So, I will not do it here.

Just a simple calculus, we will get to the expression. So, let me just write the expression for maximum I, maximum intensity that expression looks like

$$\frac{|p(r,\theta,\phi)|^2}{2\rho_0 c} max = \rho_0 c \frac{\left|\tilde{V}_{pq}\right|^2}{2}\left(\frac{kab}{8\pi r}\right)^2.$$

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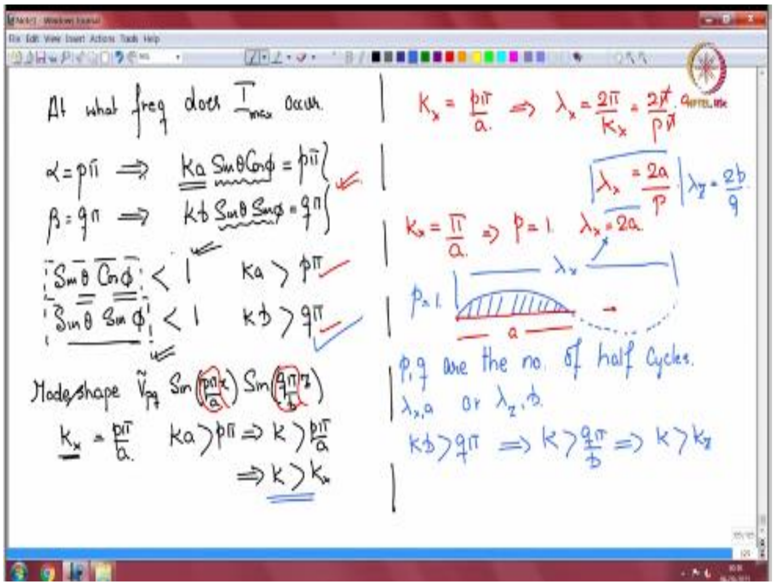

Now, this is time averaged value and therefore, there is no time unit. There is only space which means distance and angles so and frequency. So, at what frequency, does I max occur? At what frequency, does it occur? So, let us see. α is equal to $p\pi$ which implies $ka \sin\theta\ \cos\phi$ is equal to $p\pi$. Now, then β is equal to $q\pi$ which implies $kb \sin\theta\ \sin\phi$ is equal to $q\pi$. Both these are happening. Both conditions are happening.

So, now, what does it mean? $\sin\theta\ \cos\phi$ is usually less than 1 for most angles. Then $\sin\theta\ \sin\phi$ is also less than 1 for most cases. So, that means what? It means that ka multiplied by something that is less than 1 is equal to $p\pi$ which means ka is greater than $p\pi$. Similarly, kb into something that is usually less than 1 is equal to $q\pi$ which means kb is greater than $q\pi$.

So, we are insisting on both conditions being satisfied that is important because somebody could say that no, no $\sin\theta\ \cos\phi$ can be 1 at some place yeah, it can be 1 at some place but they at that place $\sin\theta\ \sin\phi$ will be 0. Somebody could say that this statement is not true.

Yeah, that is correct, it is equal to 1 in some places but when this is equal to 1, this is equal to 0. So, we are insisting on both conditions being satisfied.

So, those will happen for some odd angles, not peak values like 0 or π or $\frac{\pi}{2}$ etcetera. So, essentially now what that means is ka must be greater than $p\pi$ and kb must be greater than $q\pi$. Now, let us see. We have a mode shape. What was my mode shape? $\tilde{V}_{pq} \sin\left(\frac{p\pi x}{a}\right) \sin\left(\frac{q\pi z}{b}\right)$. so, which means what my k in the x direction is equal to $\frac{p\pi}{a}$.

So, ka greater than $p\pi$ implies k greater than $\frac{\pi}{a}$ implies k greater than k_x the modal wave number. So, these are the model wave numbers. So, now, let us see if the modal wave number is $\frac{p\pi}{a}$ that implies that my modal wavelength is $\frac{2\pi}{k_x}$ which is $\frac{2\pi a}{p\pi}$ which is equal to $\frac{2a}{p}$. So, λ_x is equal to $\frac{2a}{p}$. So, suppose my k_x happens to be $\frac{\pi}{a}$ which means p is equal to 1.

Then λ_x is equal to $2a$. And how does the shape on the panel look like? So, this is my panel length along x direction and p is 1 which would imply that is; how it looks like; this is p is 1. Why is that? Because λ_x is $2a$. So, if I extend the wavelength into the imaginary distance in x, so, this happens to be my λ_x. So, λ_x is $2a$, so, which means p or q are the number of half cycles.

There is one half cycle on the panel that is why p is equal to 1. So, that is how λ_x and a are related. λ_x and a or λ_z and b are related. So, λ_x is $2a$ by p and λ_z is $2b$ by q. So, now getting back to our criterion. The maximum now occurs when k is greater than k_x, similarly, when kb is equal to greater than $q\pi$ from here which implies that k is greater than $\frac{q\pi}{b}$ which implies k is greater than k_z. So, again what are we heading towards?

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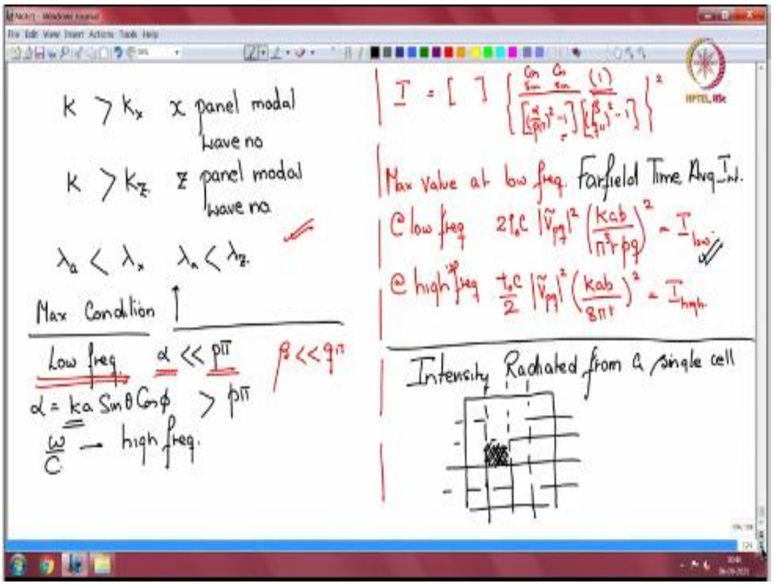

We are saying that k the acoustic wave number should be greater than k_x, the x panel modal wave number and k should be greater than k_z which means z direction k_z , z direction panel modal wavenumber. So, this is indicating the local coincidence. This is not the true full coincidence. There is not a coincidence wave number, but better to see it in terms of lambda.

So, what this saying is that lambda acoustic should be less than lambda x modal wavelength and lambda a should be less than lambda z the modal wavelength in the z direction so, that is the maximum condition. So, this is the maximum condition. You have covered the maximum. What is the condition for maxima? In case, a single mode is there in the plate and it is you know radiating sound.

Now the flip side, what about low frequency which means α is much less than $p\pi$. I suddenly mentioned low frequency. Let me just say this that α carries $ka \sin\theta \cos\phi$ and we said this has to be greater than $p\pi$ for good radiation. So, k is ω by c. So, greater than means, it has to be certain high frequency. So, we have seen that good radiation is achieved when frequency is high that is the portion, we have just covered.

Now, we are seeing low frequency that means α is much less than $p\pi$ and β is much less than $q\pi$. What happens there? Here, in this case, it is very simple because in the intensity, you have many expressions in front, and you have this cos sin terms and you have $\left(\frac{\alpha}{p\,\pi}\right)^2 - 1$, $\left(\frac{\beta}{q\,\pi}\right)^2 - 1$ etcetera. So, we are examining low values of α, β. So, these are smaller than and there is a whole square also.

So, these are much smaller than 1 so, 1 dominates in the denominator and I am looking at the maximum value, again the maximum value at low frequencies, so for the cosines and sins, I will take value 1 as the maximum value. So, then what is the intensity? The intensity at low frequencies now is given by $2\rho_0 c\left|\tilde{V}_{pq}\right|^2\left(\frac{kab}{\pi^3 rpq}\right)^2$. This is intensity at low frequency.

Now, we will just compare at high frequency. High frequency, we will just compare $\frac{\rho_0 c}{2}\left|\tilde{V}_{pq}\right|^2\left(\frac{kab}{8\pi r}\right)^2$. Now, we have to dig into this and understand it a little bit more. So, what we do; let us look at so, this is radiation on the entire panel intensity, far field intensity, far field time average intensity, far field time averaged intensity at low frequency.

So, now we will try to compare it with the intensity radiated from a single cell that means what we have this panel or plate is vibrating in some mode, these are the modal of cycle regions plus, minus etcetera. So, we are going to look at radiation from 1 cell. So, let us see now.

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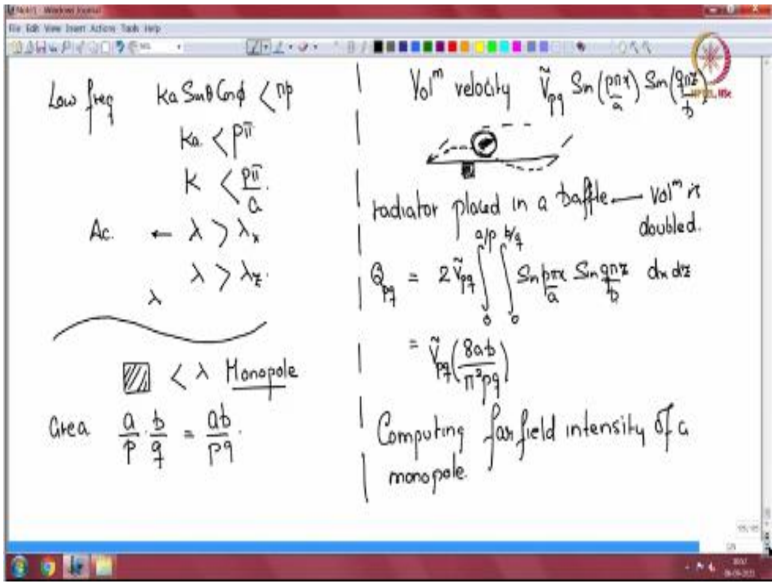

Now, just remind you low frequency which means $ka\sin\theta\ \cos\phi$ is less than $p\pi$ or ka is less than $p\pi$ which implies k is less than $\frac{p\pi}{a}$ or lambda is greater than lambda x, lambda acoustic wavelength is greater than lambda x the panel modal wavelength. Similarly, lambda is greater than lambda z the panel modal wavelength in the z direction.

So, now, if the lambda is bigger, let us say this is lambda and it is bigger than the entire wavelength in the x and z direction of the panel which means the single cell dimensions are both directions or much less than acoustic wavelength. So, we will consider this a monopole that is what I was driving at. So, now, what is the area? What is the area of the single cell? Area is $\frac{a}{p}\,\frac{b}{q}$ that is $\frac{ab}{pq}$.

And what is the volume velocity? So, we are going to compute intensity from a monopole. So, we need volume velocity. When the whole panel is vibrating as $\tilde{V}_{pq}\sin\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi z}{b}\right)$. So, if I look at 1 direction, the whole panel is doing something like this and somewhere I have a small cell sitting in the 2D panel. Somewhere there is a small cell sitting.

So, I have to compute the volume velocity from that single cell and I will mention this that we have a radiator placed in a baffle and therefore, the baffle effect is to double the volume velocity. So, I will straightaway assume this volume velocity gets doubled, this I am hoping you move from acoustics. So, my volume velocity now, which I will let us say Q_{pq} is given by

$$2\tilde{V}_{pq}\int_0^{a/p}\int_0^{b/q}\sin\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi z}{b}\right)\,dx\,dz.$$

If you compute this, we will get $\tilde{V}_{pq}\left(\frac{8ab}{\pi^2 pq}\right)$ that is the volume velocity. So, let us do this calculation now of computing far field intensity, intensity of a monopole.

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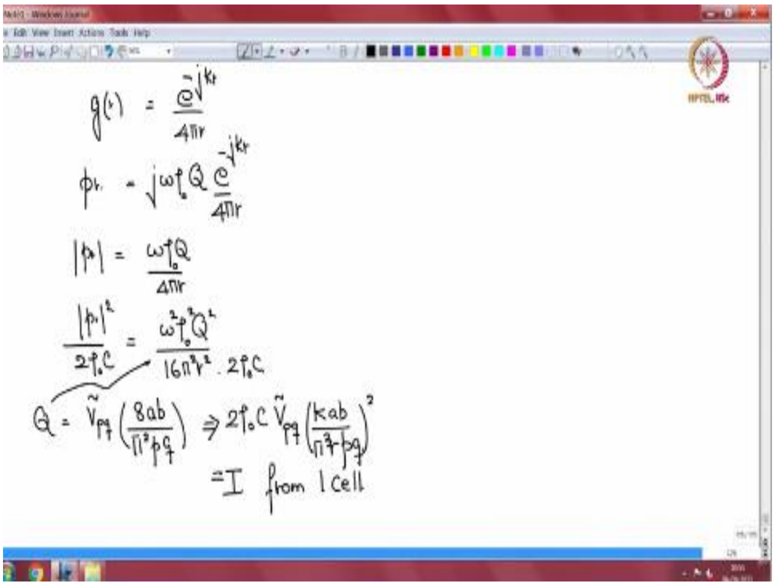

So, if you recall the green function was given by $\frac{e^{-jkr}}{4\pi r}$ and now, to convert the green function to pressure units, I need $j\omega\rho_0 Q\frac{e^{-jkr}}{4\pi r}$. So, now, in order to convert this pressure to intensity, I need first the magnitude of pressure which is $\frac{\omega\rho_0 Q}{4\pi r}$, then I need magnitude pressure square by $2\rho_0 c$ which is $\frac{\omega^2{\rho_0}^2Q^2}{16\pi^2r^2\ 2\rho_0 c}$.

Now, Q, we just found out was $\tilde{V}_{pq}\ \left(\frac{8ab}{\pi^2pq}\right)$. So, now if I substitute this Q into this expression and do all the cancellations etcetera, I will get $2\rho_0 c\tilde{V}_{pq}\left(\frac{kab}{\pi^3rpq}\right)^2$. This is intensity from a single cell. So, let me stop here. The time is running out. We will start from here next class. Thank you.