Sound and Structural Vibration
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Lecture - 34
Sound Radiation from a Baffled Panel

Good morning and welcome to this next lecture on sound and structural vibration. So, last time, we ended up with this Rayleigh integral.

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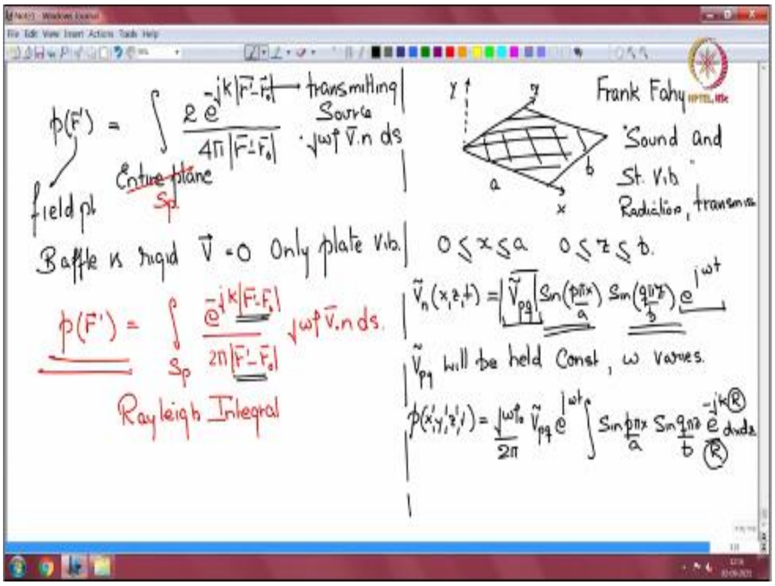

And this time, I said, we will find an expression for sound radiation from a plate set in a rigid baffle. So, my plate sitting in the xz plane; my plate sits in the xz plane and let me say this that this portion I am taking from the book by Frank Fahy and its title is somewhat sound and structural vibration and radiation and transmission or something like that. I am taking directly from this book. So, you can find these derivations there.

So, now, this is my plate; this is my plate sitting in the xz plane; the response is in the y direction. So, the plate extends between $0 \le x \le a$ and between $0 \le z \le b$ and my velocity, panel velocity is given by

$$\tilde{V}_n(x,z,t) = \left|\tilde{V}_{pq}\right| \sin\left(\frac{p\pi x}{a}\right) \sin\left(\frac{q\pi z}{b}\right) e^{j\omega t}.$$

These are the mode shapes of a simply supported plate in the pq^{th} mode and this is the amplitude, and this is the time indicator harmonic forcing sinusoidal.

Now, in this derivation $\tilde{V}_{pq}$ will be held constant even when omega changes, omega various. Normally, if omega varies, the modal amplitude will change with frequency. If you go close to a resonance, it will go up. If you move away from a resonance, it will go down etcetera, but we are not going to do that. In this in this line of thought $\tilde{V}_{pq}$ is going to be held constant.

There is a reason for that it will become clearer a little later. Now, the pressure I want

$$p(x', y', z', t) = \frac{j\omega\rho_0}{2\pi}\tilde{V}_{pq}e^{j\omega t}\int \sin\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi z}{b}\right)\frac{e^{-jkR}}{R}\, dx\, dz.$$

This R carries the scalar distance of $\vec{r} - \vec{r}_0$ numerator and denominator. It is the distance between the source and receiver points. Now, R of course has x and z dependents or will carry x and z.

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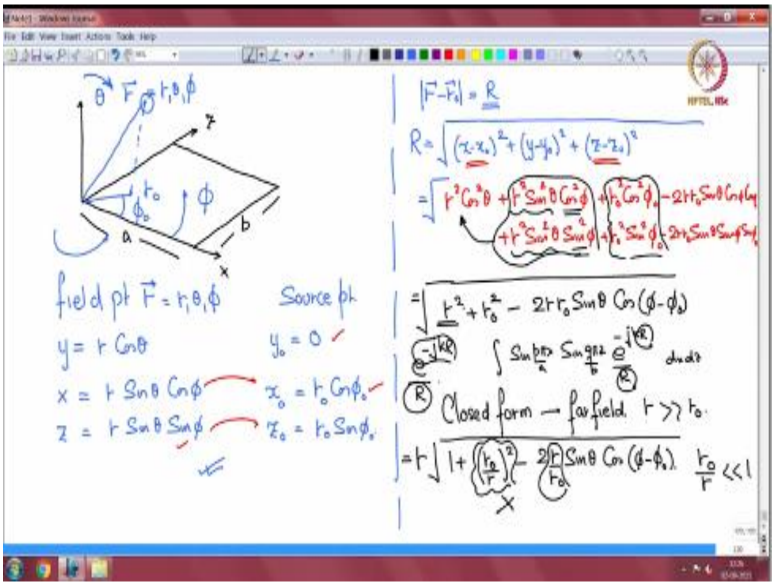

So, let us look at this. We have a panel. So, the field point is that $\vec{r}$ which is given by (r, θ, ϕ) in spherical coordinates and θ is measured from vertical. The θ is measured from vertical; this ϕ is measured from here; ϕ is measured from the x direction and θ is measured from the vertical. ϕ is the azimuthal; θ is the elevation angle.

Now, we need the coordinates in rectangular description. So, my y is equal to $r\ \cos\theta$, the y of this point, y value of this point is $r\ \cos\theta$; the x value is $r\ \sin\theta\cos\phi$; the z is equal to $r\ \sin\theta\sin\phi$. So, this is the receiver. The source point, it is $y_0 = 0, x_0 = r_0\ \cos\phi_0$ and $z_0 = r_0\sin\phi_0$.

So now, we need to formulate this R which is the distance between $r - r_0$, which we will call as R. So, what is R equal to now? It is equal to the $\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$. So, in terms of the values, we have found, this is equal to I will just put a square root symbol to remember it is there. So, what will we get?

We will get from here

$\sqrt{r^2\cos^2\theta + r^2\sin^2\theta\cos^2\phi + {r_0}^2\cos^2\phi_0 + r^2\sin^2\theta\sin^2\phi + {r_0}^2\sin^2\phi_0 - 2rr_0\sin\theta\cos\phi\cos\phi_0 - 2rr_0\sin\theta\sin\phi\sin\phi_0}$

$$R = \sqrt{r^2 + {r_0}^2 - 2rr_0\sin\theta\cos(\phi - \phi_0)}.$$

This is R. So, this is the one this R will sit as e^{-jkR} and R in the Rayleigh integral, now and this is going to be integrated over you know x and z.

So, the x and z will be part of the numerator here and part of the denominator here and that integral is not amenable to close form solution. If we write the integral which has $\sin\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi z}{b}\right)\frac{e^{-jkR}}{R}\,dx\,dz.$ Where R contains x z you know dx dz. This is not amenable to close form solution. So, in order to do find a closed form, in order to find a closed form answer, we do what is called a far field approximation that means my r is much greater than r_0.

r_0 lies on the plate and r is that receiver, so r is much greater than r_0, we do that. So, if we do that, I am sorry, this should be square, if we do that, we take r square outside, so it becomes r; inside it becomes $1 + \left(\frac{r_0}{r}\right)^2 - \frac{2r r_0\sin\theta\cos(\phi-\phi_0)}{r}$, I am sorry, $\frac{r_0}{r}$ is much less than 1.

$$R = r\sqrt{1 + \left(\frac{r_0}{r}\right)^2 - \frac{2r_0\sin\theta\cos(\phi - \phi_0)}{r}}.$$

So, this carries one level of smallness, and this carries the square level of smallness. So, we will drop this term. Then what happens?

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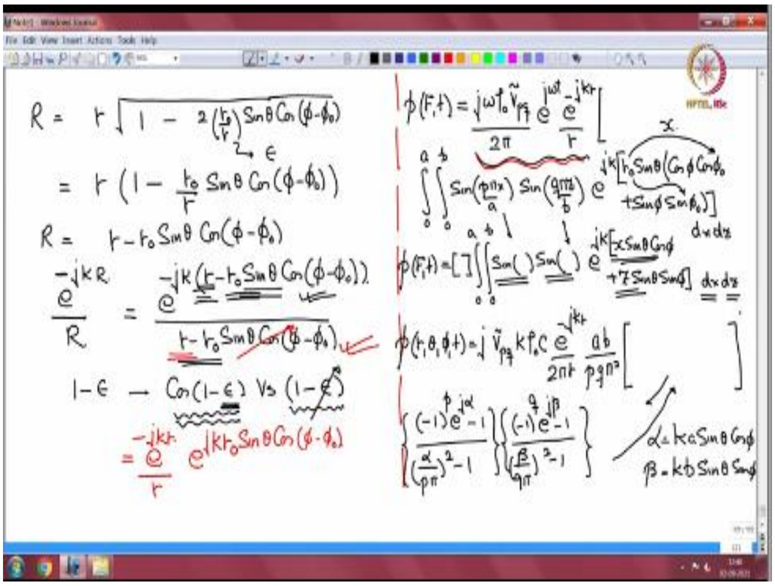

We have

$$R = r\sqrt{1 - 2\left(\frac{r_0}{r}\right)\sin\theta\cos(\phi - \phi_0)}.$$

And if $\frac{r_0}{r}$ is a small quantity like epsilon, then you can expand this and get

$$= r\left(1 - \frac{r_0}{r}\sin\theta\cos(\phi - \phi_0)\right),$$

$$R = r - r_0\sin\theta\cos(\phi - \phi_0).$$

So, now, one more level of approximation. In the integral, this R goes in as $\frac{e^{-jkR}}{R}$.

So, that means what? It becomes

$$\frac{e^{-jkR}}{R} = \frac{e^{-jk(r - r_0\sin\theta\cos(\phi-\phi_0))}}{r - r_0\sin\theta\cos(\phi - \phi_0)}.$$

Now, this is still not amenable; still not amenable to close form because we have $x\ z$ here and $x\ z$ over there. Now, what we do is that it is like if I have an expression 1 – epsilon, then I look at the expression *cos* of 1 – epsilon. It is now suddenly in radians versus I look at the expression 1 – epsilon in just linear terms.

So, this is a nonlinear function of 1 – epsilon whereas here it is just a linear function. So, if epsilon is a small quantity, it is safe to neglect epsilon over here. It is not so. It is not so safe to ignore epsilon over here where it appears as radians. So here in the numerator, even though r is bigger than r_0, I have this expression so, I will not ignore this r_0 term compared to this r term. Whereas in the denominator in comparison to r, this will be a small term I am going to ignore.

So, what will this look like? It looks like

$$= \frac{e^{-jkr}}{r} e^{jkr_0 \sin\theta \cos(\phi - \phi_0)}.$$

In the denominator, we will take it as r that is the approximation. So now, if we put all this together, my solution here pressure at a certain location r including time, let us say, including time, I often forget to put the time part because these are harmonic.

So, they do not hurt us very much.

$$p(\vec{r}, t) = \frac{j\omega\rho_0}{2\pi} \tilde{V}_{pq} e^{j\omega t} \frac{e^{-jkr}}{r} \left[\int_0^a \int_0^b \sin\left(\frac{p\pi x}{a}\right) \sin\left(\frac{q\pi z}{b}\right) e^{jk[r_0 \sin\theta\, (\cos\phi \cos\phi_0 + \sin\phi \sin\phi_0)]}\, dx\, dz \right].$$

However, $r_0 \cos\phi_0$ is x and $r_0 \sin\phi_0$ is z. So, this becomes p some location r and t; this entire term in front here double integral 0 to a, 0 to b, then sin this term, sin this term and e to the power of $jk[x \ \sin\theta \ \cos\phi + z \ \sin\theta \ \sin\phi]\ dx\ dz$. Now, this has to be integrated.

This has x only; this has x only; you have dx. This has z only; the z only here; you have dz. So, you can use the product rule and integrate them separately. So, what is the final result? The final result, I write as pressure at a certain scalar

$$p(r, \theta, \phi, t) = j\tilde{V}_{pq} k \rho_0 c \frac{e^{-jkr}}{2\pi r} \frac{ab}{pq\pi^2} [\ \ ],$$

[ ] term, which I am going to write here, what is that?

$$\left\{ \frac{(-1)^p e^{j\alpha} - 1}{\left(\frac{\alpha}{p\,\pi}\right)^2 - 1} \right\} \left\{ \frac{(-1)^q e^{j\beta} - 1}{\left(\frac{\beta}{q\,\pi}\right)^2 - 1} \right\}.$$

So, this total term enters this box here where α is equal to $k\ a \sin\theta \ \cos\phi$, whereas β is equal to $k\ b \sin\theta \ \sin\phi$.

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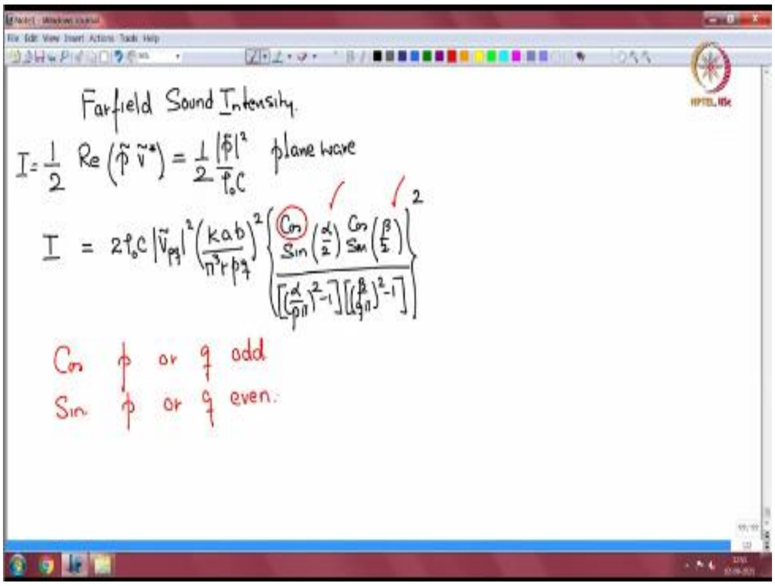

Now, we are interested in the far field sound intensity, sound intensity in the far field. So, those who remember their acoustics, it is given by

$$I = \frac{1}{2}\, Re(\tilde{p}\tilde{v}^*) = \frac{1}{2}\, \frac{|\tilde{p}|^2}{\rho_0 c}.$$

So, there is a plane wave, by this time, the wave become a plane wave. So, now pressure was found earlier.

If we use this expression now, my intensity in the far field is going to be

$$I = 2\rho_0 c\left|\tilde{V}_{pq}\right|^2 \left(\frac{k\,ab}{\pi^3 r\,pq}\right)^2 \left\{\frac{\begin{matrix} cos \\ sin \end{matrix}\left(\frac{\alpha}{2}\right)\begin{matrix} cos \\ sin \end{matrix}\left(\frac{\beta}{2}\right)}{[\left(\frac{\alpha}{p\,\pi}\right)^2 - 1][\left(\frac{\beta}{q\,\pi}\right)^2 - 1]}\right\}^2.$$

Now, here what is this new notation? Here cos will be used when p or q, p comes in here and q comes in here, the cos will be used when p or q is odd.

The sin will be used when p or q is even. We are running out of time so; I will stop here. We will continue in the next class. Thank you.