Sound and Structural Vibration

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Lecture – 32
Coupled Resonance Analysis Using Matrices

Good morning and welcome to this next lecture. We were looking at coupling of modes and the coupling coefficient matrix, its nature and you know how it looks like. So, for $M = 3$ and $N = 2$, it can be written in this form and that can be approximately written in this form and exactly what is it? It is the same 2 terms in front with some extra terms and these extra terms become negligible, as the relative fluid density goes down.

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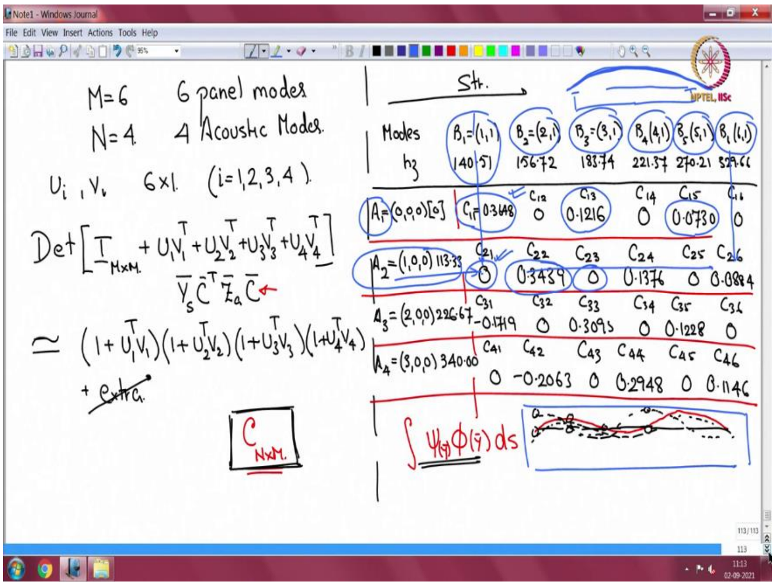

So, now let us see, we will go to the system that Kim and Brennan have used. They use 6 panel modes. That means M is equal to 6, so, 6 panel modes or plate modes and they use 4 acoustic modes. So, N is equal to 4. 4 acoustic modes. Therefore, the U_i and the V_i have dimensions 6 cross 1 and there are 4 such vectors i running from 1, 2, 3 and 4. So, now the matrix, the determinant of that matrix, $[I_{M\times M} + U_1 V_1{}^T + U_2 V_2{}^T + U_3 V_3{}^T + U_4 V_4{}^T]$.

This is how it looks like. This whole term was $\vec{Y}_s \vec{C}^T \vec{Z}_a \vec{C}$. This whole term was like this, so, the determinant approximately now looks like into $(1 + U_1{}^T V_1)(1 + U_2{}^T V_2)(1 + U_3{}^T V_3)(1 + U_4{}^T V_4)$. Approximately actually, there are some extra terms which again become negligible as fluid density to plate density goes down. Now let us look at the values, the numerical values, so, they have considered an aluminium panel with air and etcetera.

So, let us I will write the matrix over here, so we have modes. So, these are structural modes that go this way structure. So, let us see B_1 first mode is $1, 1$. It has a resonance uncoupled resonance of 140.51 these are in hertz, then I get B_2 which is equal to $2, 1$ and a resonance uncoupled 156.72. B_3 is equal to $3, 1$. B_4 is equal to 4, 1. B_5 is equal to 5, 1. B_6 is 6, 1. This has 183.74 hertz, 221.37 hertz and this is 270.21 hertz, and this has 329.66 hertz. These are uncoupled values, then we have acoustic modes here.

So, A_1. It has 3 mode numbers 3 numbers to indicate the mode. The rigid body mode has a 0 hertz frequency. The first you know dynamic mode. Let us say is 1, 0, 0. It has 113.33 hertz, then A_3 the third acoustic mode 2, 0, 0 has a linear increment. Then A_4 has 3, 0, 0 and 340 hertz. So, these are uncoupled values. So, let me; these are uncoupled values. So, now here we will fill in the C coefficient matrix values, we had $C_{N\times M}$.

This C that is coming up again and again is actually $C_{N\times M}$. and what is it to remind ourselves? It is the integral of the acoustic mode and the panel mode on the panel. So, let us say on the panel. The panel mode does this and on the panel, the acoustic mode does this. This is the mean line, so, I want to show that they are orthogonal actually, peak here has a 0 here, 0 here is a peak here. So, 0 here is a peak here, 0 here is a peak here etcetera.

So, one is a cosine, one is a sine. So, if you integrate that you will get a 0, whereas if it happens that the other one is also like this, then you will get a perfect match. So, those are the 2 ends of the spectrum and in between is all in between, you will get various values based on how much match occurs. So, this is a very important matrix. This decides how much match is there between an acoustic mode and a panel mode? and that decides coupling.

So, these values decide now the coupling. So, I will put the values now. So, I have C_{11} here is 0.3648. C_{12} is a 0, then C_{13} is a 0.1216. C_{14} is a 0, C_{15} is a 0.0730, C_{16} is a 0. Then C_{21} is a 0, C_{22} is a 0.3439, C_{23} is a 0, C_{24} is a 0.1376, C_{25} is a 0, C_{26} is 0.0884. Then C_{31} is –0.1719, C_{32} is a 0, C_{33} is 0.3095, C_{34} is a 0. Similarly, C_{36} is a 0, C_{35} is 0.1228. Then C_{41} is a 0, C_{43} is 0, C_{45} is 0, C_{42} is –0.2063, C_{44} is a 0.2948 then C_{46} is a 0.1146.

So, as I said, this matrix is very important, it shows that A_1 can couple to B_1 mode. It shows that A_1 will not couple to B_2. It will couple to B_3. It will not couple to be 4. It will couple to B_5 not coupled to B_6 and the magnitude of the value shows how much match there is on the surface. How much of this match I have spoken of? Is there this value says? So, there is a very good match between A_1 and B_1 that should be because B_1 is this, simply supported mode.

There is no 0 or change of phase across the panel and A_1 is this rigid body acoustic mode which just breeds in and out so, there is a best match over here? I have not normalized them, so, you will find these values are different from Kim and Brennan values. So, similarly A_2 here, the second acoustic mode does not couple with B_1 couples with B_2 does not coupled with B_3 etcetera.

Now, it is also important to know that, let us see, we will come to that later. So, this is how the coupling matrix is an indicator of mode couplings. The 0 value indicates that structure and that acoustic mode do not couple also the smallness of the value indicates that acoustic mode and that structural mode have weak coupling. So, I hope that is clear.

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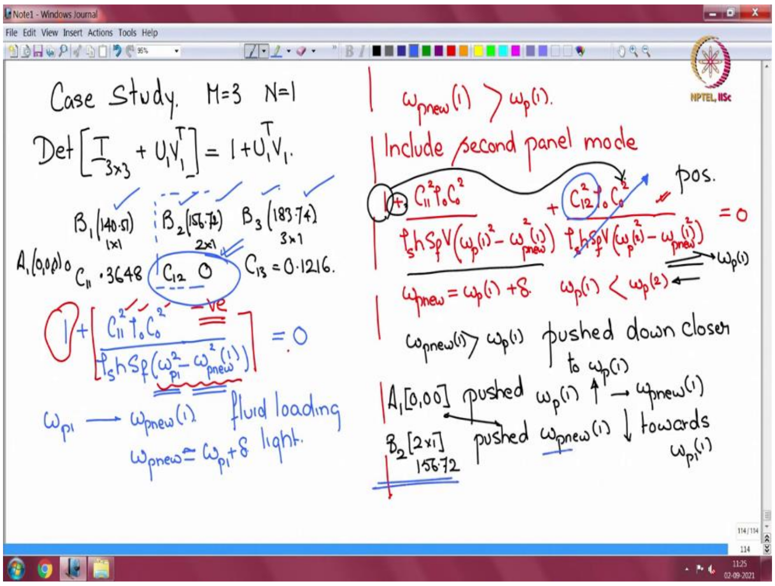

Now, will further examine a case. We will examine a case so, case study one particular case. What is that? We will take a situation where 3 panel modes M is equal to 3 and one acoustic mode? We will examine so we have written down the determinant earlier. So, what did we say that determinant should look like $[I_{3\times3} + U_1 {V_1}^T]$ and that actually is $1 + {U_1}^T V_1$ So, we wrote that, and we wrote it openly also.

Now if we look at the subset of this so, what is that? Let me collect it in red I suppose. 3 panel modes and one acoustic mode. So, this is the relevant subsection of this thing here. This is the portion of the matrix we are interested in or the table we are interested in. So, if we rewrite it, I have B_1, B_2, B_3 and A_1 which is 0 0 0 with 0 hertz, and B_1 has a 140.51 hertz, B_2 has 156.72 hertz, B_3 has 183.74 hertz is a 1, 1 mode, is a 2,1 mode is a 3,1 mode.

And the C values are C_{11} is 0.3648, C_{12} is 0 and C_{13} is equal to 0.1216 for the moment. Let us assume that even B_2 and B_3 were not considered. As I said, is an approximation where you take a model sum and you truncate somewhere, suppose we just truncated at A_1 and B_1. How does the determinant look like? The determinant now looks like $1 + \frac{{C_{11}}^2 \rho_0 {c_0}^2}{\rho_s h S_f V({\omega_{p1}}^2 - {\omega_{pnew}}^2(1))}$. We have to set this to 0 because temporarily we are not accounting for B_2 or B_3. So, C_{12}, C_{13} both are 0. So, this is the approximate, the current state of the determinant that means the effect of B_2, B_3 are not there and see ω is the unknown. So, now I am looking for a correction to the first resonance first panel resonance. The first original panel resonance was at ω_{p1}. I am trying to find a correction to it. Let us call it $\omega_{pnew}(1)$, and fluid loading is light.

Now I am going to assume then that ω_{pnew} will only be a correction, will be a correction to $\omega_{p1} + \delta$ will not be hugely different. But if you see the nature of this determinant value, so, this is a positive quantity added to something must be 0. So, this must be a negative quantity, all values here are positive, except for this which can be negative which means that $\omega_{pnew}(1)$ new must be greater than $\omega_p(1)$.

The correction to one must be greater than $\omega_p(1)$. So, now suppose we include the second panel mode. Then the determinant looks like $1 + \frac{{C_{11}}^2 \rho_0 {c_0}^2}{\rho_s h S_f V({\omega_{p1}}^2 - {\omega_{pnew}}^2(1))} + \frac{{C_{12}}^2 \rho_0 {c_0}^2}{\rho_s h S_f V({\omega_{p2}}^2 - {\omega_{pnew}}^2(1))}$ this should be set to 0 to calculate the resonance value near ω_p.

So, ω, So, now fluid loading is again light, so, we will assume that ω_{pnew} is equal to $\omega_p(1) + \delta$, and $\omega_p(1)$ is of course, less than $\omega_p(2)$. This is panel lower and higher order modes. Now so, this is by default a positive value. This is a positive value, even with new $\omega 1$ and $\omega_p(1)$ is already less than $\omega_p(2)$. So, this is a positive value. So, now 1 is incremented by another positive value.

So, the total is now more positive, so, more positive plus something has to be 0. So, this has to be more negative that means what ω_{pnew} which actually was greater than $\omega_p(1)$ is now pushed down closer to $\omega_p(1)$. So, what does this mean? The A_1 acoustic mode, which is a rigid body mode, each first of all pushed $\omega_p(1)$ to be incremented and then we get $\omega_{pnew}(1)$, So, let me yeah. Let me do it both ways.

So, next we included the influence of B_2, the next mode, which is a 2, 1 mode with the resonances at 156.72. So, what this do? This pushed the $\omega_{pnew}(1)$ caused by the acoustic mode and pushed it down more towards $\omega_p(1)$. I need to make a correction slight correction here. In this table C_{12} is 0. So, if we want to consider the value, then the second mode does not have an influence that is also borne out.

So, the coupling of modes is through fluid, otherwise the earlier modes were orthogonal to each other. These modes were orthogonal to each other. Now, why are they talking to each other or coupling with each other? Because of the fluid and because of this coefficient. So, if we now actually use this event to value the second panel mode has no influence. But hypothetically for some other system, some other value, suppose C_{12} had a value. So, this is what the C_{12} will do.

I mean the second mode window. It will push the $\omega_{pnew}(1)$ down towards $\omega_p(1)$. We shall be borne out by the next line.

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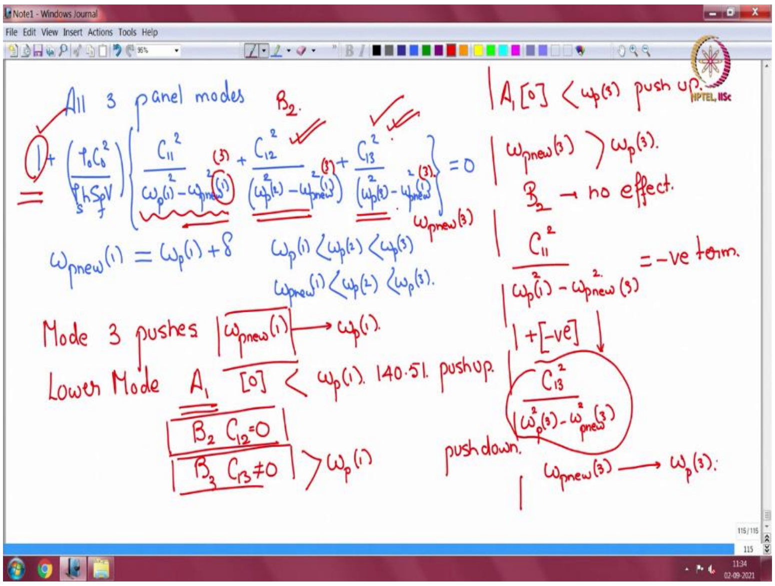

Suppose we consider all 3 modes; all 3 panel modes are considered then my determinant looks like $1+\left(\frac{\rho_0 c_0{}^2}{\rho_s h S_f V}\right)\left\{\frac{C_{11}{}^2}{\omega_{p1}{}^2-\omega_{pnew}{}^2(1)}+\frac{C_{12}{}^2}{\omega_{p2}{}^2-\omega_{pnew}{}^2(1)}+\frac{C_{13}{}^2}{\omega_{p3}{}^2-\omega_{pnew}{}^2(1)}\right\}=0$. Again, $\omega_{pnew}(1)$, I am saying, is close to $\omega_p(1)$ why? Because of like fluid loading that means $\omega_p(1)$ if it is less than $\omega_p(2)$ and that is less than $\omega_p(3)$.

Then $\omega_{pnew}(1)$ will also be less than $\omega_p(2)$ will also be less than $\omega_p(3)$ original. So, what happens is? This term now brings in an additional positive term. This brings in an additional positive term. So, now I have even a further positive term that has to be matched by this and therefore $\omega_{pnew}(1)$, although greater than $\omega_p(1)$ is further closer to $\omega_p(1)$.

So, the mode 3 also pushes, mode 1 down $\omega_{pnew}(1)$ closer towards $\omega_p(1)$. So, that is where the $\omega_{pnew}(1)$ value will lie. That means a lower mode, a lower, resonant mode, acoustic first mode has resonance at 0 which is less than $\omega_p(1)$. $\omega_p(1)$ was at 140.51, so, this will push the mode up push up. A_1 will push it up, whereas if we include the influence of B_2 and B_3 and if we say that C_{12} is 0, then B_2 has no influence.

And B_3 , C_{13} has a non 0 value and its resonance is above $\omega_p(1)$. So, it will push down. Now, if we look at the influence on $\omega_{pnew}(3)$, then this value is $\omega_{pnew}(3)$, $\omega_{pnew}(3)$, $\omega_{pnew}(3)$. So, what the influence of an acoustic mode is that, because acoustic mode is 0 hertz less than $\omega_p(3)$, it will push it up that is seen by 1 plus $C_{13}{}^2$. If we just include the 1 and if we just include the C_{13} term, the one has to be balanced by a negative term and therefore $\omega_{pnew}(3)$ must be bigger than $\omega_p(3)$.

So, first that happens so, $\omega_{pnew}(3)$ becomes bigger than $\omega_p(3)$. Then, what happens if we include the C_{12} term C_{12} is 0. That means mode 2 term C_{12} is 0, so, mode 2 has no influence on mode 3. B_2 will have no influence, no effect, so, you see the power of that C coefficient. Next, we include the first mode. So, what the first mode does is the denominator has what $\frac{C_{11}{}^2}{\omega_{p1}{}^2-\omega_{pnew}{}^2(3)}$.

So, what this is going to do? Is that this is going to take some part of the positivity out. This is going to be a negative term. So, 1 plus a negative term. What happens is the value is now lower, smaller. The smaller positive value so, what should C_{13} term do $\frac{C_{13}{}^2}{\omega_{p3}{}^2-\omega_{pnew}{}^2(3)}$. This has to balance a smaller term.

So, this has to be now bigger, so, $\omega_{pnew}(3)$ moves away distances away from $\omega_p{}^2(3)$ or $\omega_p(3)$. So, that means what $\omega_{pnew}(3)$ is now further bigger than $\omega_p(3)$.

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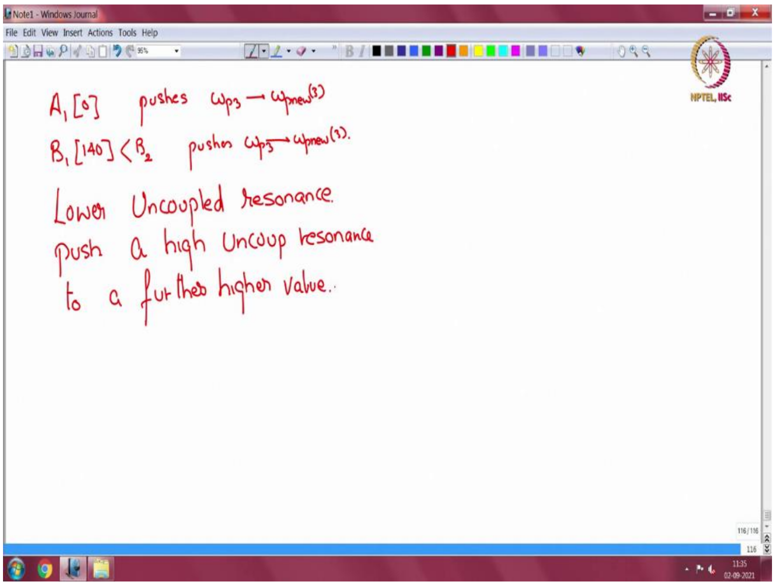

So, the acoustic mode, A_1 with 0 as its frequency pushes $\omega_p(3)$ to $\omega_{pnew}(3)$, the B_1 mode, with the resonance whatever it is, 140 or so. Below B_3 also pushes $\omega_p(3)$ further to $\omega_{pnew}(3)$. So, a lower uncoupled resonance will push a higher uncoupled resonance to a further higher value. Let me stop here, time is running out, we will continue from here. Thank you.