Sound and Structural Vibration
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Lecture - 30
Derivation of Vibro-Acoustic response Continued

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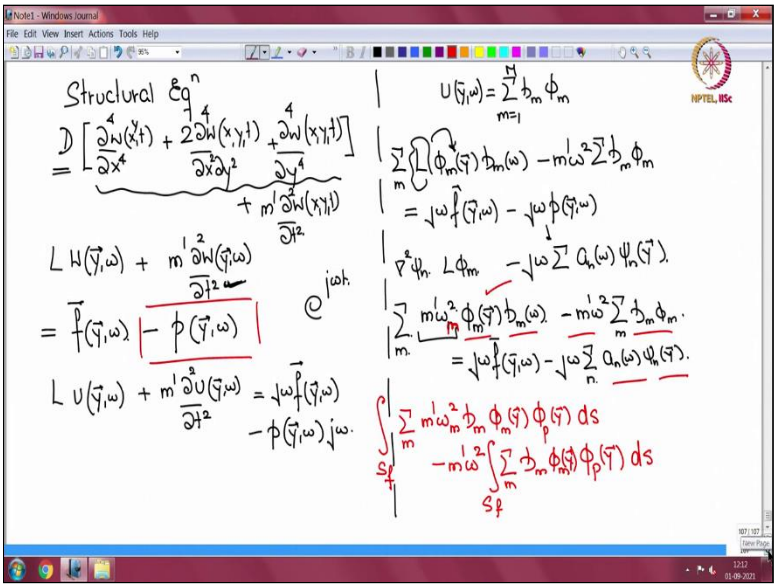

Welcome to this next lecture on sound and structural vibration. So, we ended up here last time we are looking at the structural equation the structural equation is currently here this is the stiffness term the operator has resulted in L, $\omega_m{}^2\phi_m(\vec{y})b_m(\omega)$ this is the inertial term $-m'\omega^2 b_m\phi_m$ and the pressure from the cavity is $a_n(\omega)\psi_n(\vec{y})$ with this minus sign.

Now what happens to this we have to multiply this

$$\int\sum_{m=1}^{M} m'\omega_m{}^2 b_m\,\phi_m(\vec{y})\,\phi_p(\vec{y})\,dS - m'\omega^2\int\sum_{m=1}^{M} b_m\,\phi_m(\vec{y})\,\phi_p(\vec{y})\,dS.$$

Then we will do the forcing terms on the other side oh yeah force term.

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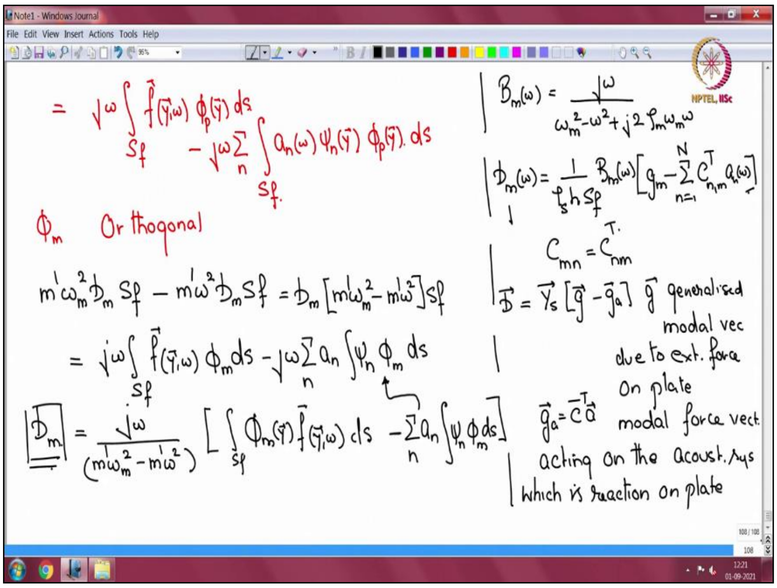

And the force terms will be equal to

$$j\omega \int \vec{f}(\vec{y},\omega)\, \phi_p(\vec{y})\, dS - j\omega \sum_{n=1}^{N} \int \psi_n(\vec{y})\, \phi_p(\vec{y})\ a_n(\omega)\, dS.$$

Now again the ϕ's are orthogonal the ϕ's are orthogonal ϕ_m's are orthogonal. So, what happens is that out of the sum one of one fellow is chosen. So, we get

$$m'{\omega_m}^2 b_m S_f - m'\omega^2 b_m S_f = b_m[m'{\omega_m}^2 - m'\omega^2]S_f,$$

and the right side gives me equal to $j\omega \int \vec{f}(\vec{y},\omega)\phi_m dS - j\omega \sum_{n=1}^{N} a_n \int \psi_n \phi_m\, dS.$ So, what happens here, so, if we isolate b_m I get

$$b_m = \frac{j\omega}{(m'{\omega_m}^2 - m'\omega^2)} \left[\int \phi_m(\vec{y})\vec{f}(\vec{y},\omega)\, dS - \sum_{n=1}^{N} a_n \int \psi_n \phi_m\, dS\right].$$

So, let us remember. So, we can write if we take my

$$B_m(\omega) = \frac{j\omega}{{\omega_m}^2 - \omega^2 + j2\zeta_m \omega_m \omega}.$$

Then my

$$b_m(\omega) = \frac{1}{\rho_s h S_f} B_m(\omega) \left[g_m - \sum_{n=1}^{N} {C_{n,m}}^T\, a_n(\omega)\right].$$

We know that C_{mn} is equal to ${C_{nm}}^T$. So, if we do a compact notation of this $\vec{b} = \vec{Y}_s[\vec{g} - \overrightarrow{g_a}]$ where $\vec{g}$ is the generalized model vector due to two external force on the plate and $\overrightarrow{g_a}$ is $\vec{C}^T\vec{a}$, a C is n, m C transpose is m, n this is the modal force acting on the acoustic space model force vector acting on the acoustic space system which is now a reaction force which is a reaction force on the plate.

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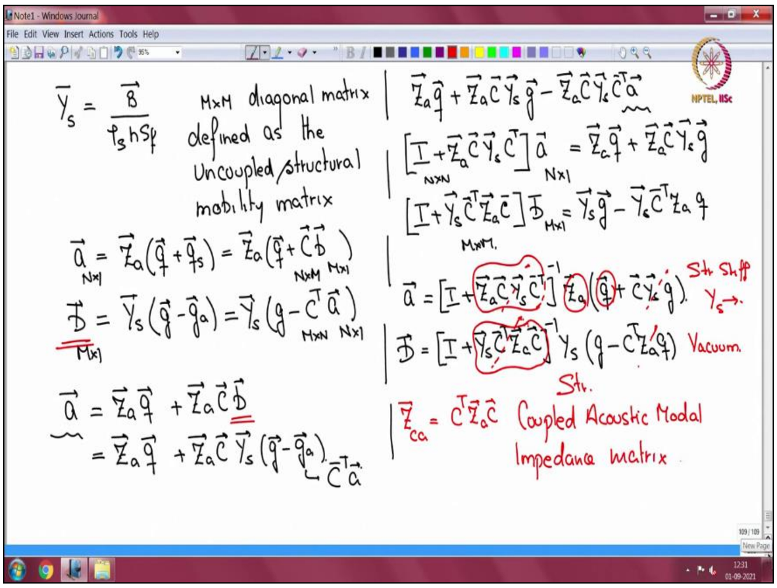

And $\vec{Y}_s$ is $\frac{\vec{B}}{\rho_s h s_f}$. It is an $M \times M$ diagonal matrix defined as the uncoupled structural mobility matrix. So, if we now write both equations in a matrix form $\vec{a}$ the acoustic side unknown is

$$\vec{a}_{N\times 1} = \vec{Z}_a(\vec{q} + \vec{q}_s) = \vec{Z}_a\left(\vec{q} + \vec{C}_{N\times M}\vec{b}_{M\times 1}\right)$$

and the $\vec{b}$ the plate side unknown

$$\vec{b}_{M\times 1} = \vec{Y}_s(\vec{g} - \vec{g}_a) = \vec{Y}_s(\vec{g} - C^T{}_{M\times N}\vec{a}_{N\times 1}).$$

Now what do we do, we have

$$\vec{a} = \vec{Z}_a\vec{q} + \vec{Z}_a\vec{C}\vec{b},$$

$$\vec{a} = \vec{Z}_a\vec{q} + \vec{Z}_a\vec{C}\vec{Y}_s(\vec{g} - \vec{g}_a),$$

$$\vec{a} = \vec{Z}_a\vec{q} + \vec{Z}_a\vec{C}\vec{Y}_s\vec{g} - \vec{Z}_a\vec{C}\vec{Y}_s\vec{C}^T\vec{a}.$$

So, now we have $\vec{a}$ on both sides we have $\vec{a}$ here $\vec{a}$ here. So, I will collect. So, I get

$$\left[I + \vec{Z}_a\vec{C}\vec{Y}_s\vec{C}^T\right]\vec{a}_{N\times 1} = \vec{Z}_a\vec{q} + \vec{Z}_a\vec{C}\vec{Y}_s\vec{g}.$$

Similarly, I am not going to repeat the whole thing similarly we get an

$$\left[I + \vec{Y}_s\vec{C}^T\vec{Z}_a\vec{C}\right]\vec{b}_{M\times 1} = \vec{Y}_s\vec{g} - \vec{Y}_s\vec{C}^T\vec{Z}_a\vec{q}.$$

$\left[I + \vec{Z}_a\vec{C}\vec{Y}_s\vec{C}^T\right]$ is a square matrix N cross N and $\left[I + \vec{Y}_s\vec{C}^T\vec{Z}_a\vec{C}\right]$ is a square matrix M cross M and they can be inverted. So, if we invert, we get

$$\vec{a} = \left[I + \vec{Z}_a \vec{C} \vec{Y}_s \vec{C}^T\right]^{-1} \vec{Z}_a \left(\vec{q} + \vec{C} \vec{Y}_s \vec{g}\right),$$

and $\vec{b}$ given by

$$\vec{b} = \left[I + \vec{Z}_a \vec{C} \vec{Y}_s \vec{C}^T\right]^{-1} \vec{Y}_s \left(\vec{g} - \vec{C}^T \vec{Z}_a \vec{q}\right).$$

So, you can check that if structure becomes stiff structure is stiff which means $\vec{Y}_s$ will go to zero then it's purely acoustics this term will go to zero this term will go to zero its purely acoustical forcings that decides $\vec{a}$. Similarly, if there is vacuum in the cavity the cavity then $\vec{Z}_a$ will go to zero this will go to zero now this purely structural excitation. So, these are the coupling matrices these are what couple structure and acoustics.

So, let us say here that $\vec{Z}_{ca}$ we will denote it as $C^T \vec{Z}_a \vec{C}$ which is the coupled acoustic model impedance matrix.

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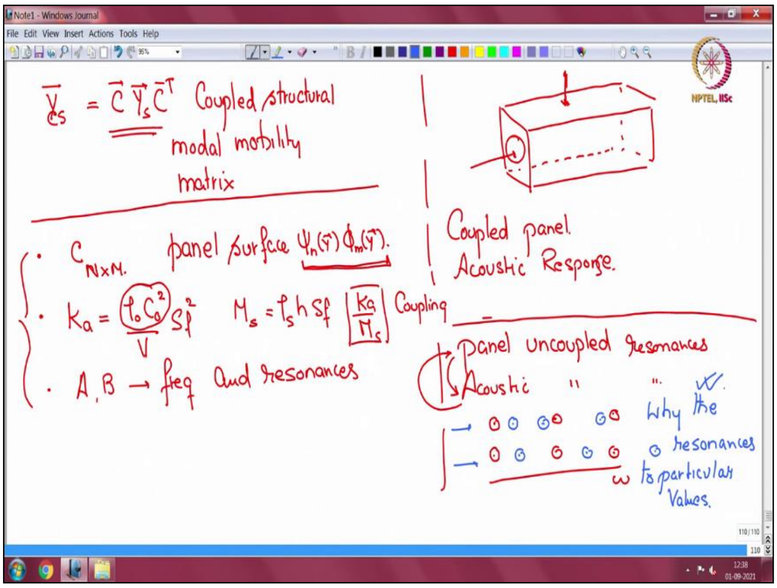

And $\vec{Y}_{cs}$ is $\vec{C}\vec{Y}_s\vec{C}^T$ is the coupled structural modal mobility matrix. Now what decides coupling of course these are the coupling matrices that decide coupling but more than that within the details we have a matrix C which is N cross M. This couples on the panel surface the acoustic mode to the panel mode is a very important matrix.

The coefficients of these are very important if it is 0 some element is 0 that means these two modes do not interact with each other. This is on the surface it is very important the other are these factors there is a factor K_a which will come up which is $\frac{\rho_0 {c_0}^2}{V} S_f$ and you get another

M_s factor composed of $\rho_s h S_f$ then $\frac{K_a}{M_s}$ also decide coupling. If it goes down that means fluid is very light.

Then the fluid does not see the structure the structure will not see the fluid in addition. Of course, the A and B they carry the frequency and resonances these matrices also decide coupling. So, these are the three factors that decide coupling. Now what we will do is that we will take a will take a particular setup. So, there is a rectangular box is what the paper does. So, I am just doing what the paper does will take a rectangular box of a certain dimension then we will force it using a speaker over here a volume velocity source.

And we will force it the other walls are assumed rigid the bottom plates are assumed rigid. I mean all the other five are assumed rigid only this is assumed to be flexible, and this is forced using a point force. Then we will compute of course the coupled response the coupled panel and acoustic response and I will just show you the answer there is no computation that I am going to do.

I will just show you the paper and how what the response is but beyond that what we will do is that you have panel resonances you have panel uncoupled resonances. And you have acoustic cavity uncoupled resonances that means hardware resonances. Now the panel influences the acoustic resonances the acoustics influences the panel resonances. So, if the panel resonances are like this in frequency.

Let us say there is a frequency axis and panel resonances are here uncoupled due to acoustics cavity they shift a little bit one way, or the other first resonance may come here the second resonance may come here and so forth they shift left or right. The same with the acoustic cavity resonances they may originally be again in some again here or whatever are acoustic resonances but now the panel comes in panel flexibility come in they may shift somewhat here what here or here start shifting.

So, we will give an answer to why they shift to where they shift why the resonances shift to the values that they shift to particular values you mean the coupled resonances the uncoupled resonances have shifted to new places because of coupling. So, why they shift we will give an answer to that. So, time is up I will close the lecture here we will continue from the next class, thanks.