Sound and Structural Vibration
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Lecture – 3
The Concept of Coincidence Frequency

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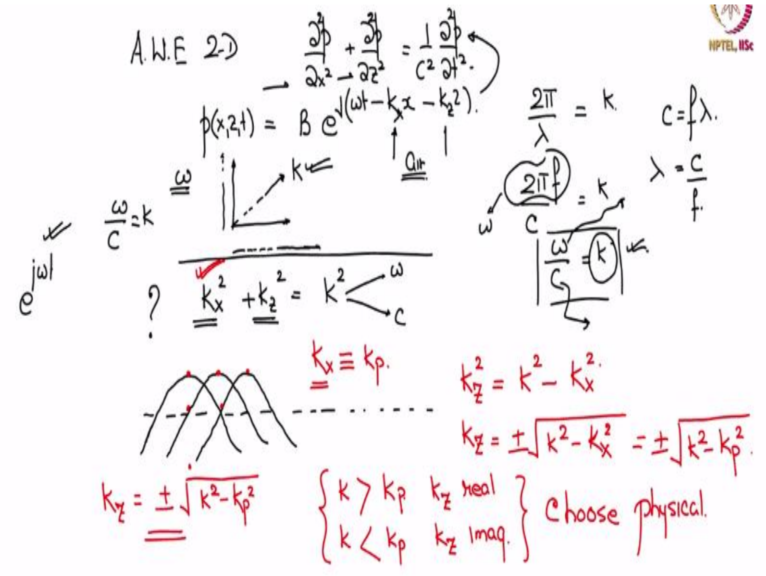

Welcome to you all. So, let us look at k_x. We said that the plate wave number is k_p. So, let us say the plate was not vibrating, but now it vibrates, so there is a pulse on the plate like this and therefore a point in the medium air got lifted up here and then the wave moves towards the right. So, the wave has moved like this to the right and so another point got lifted up because the medium is connected.

Medium is contiguous to your vibrating structure and then this point moves here right and then at the next instant let us say the wave in the plate moves here. So, another point got lifted here whereas this point came down here and this point came down here and therefore particles of air that were contiguous with my structure they move along with the structure and therefore we can say that my k_x is bound to be equal to k_p, k_p is the plate wave number, it is known to us.

So, now k_x is known to us. So what happens? I have now k_z the only unknown which is equal to $k^2 - {k_x}^2$ and therefore k_z is equal to $\pm\sqrt{k^2 - {k_x}^2}$, but k_x is k_p. So, this is equal to $\pm\sqrt{k^2 - {k_p}^2}$. So, let me write it here once more

$$k_z = \pm\sqrt{k^2 - {k_p}^2}.$$

Now, the relationship between k and k_p becomes important.

If $k > k_p$, then I have a real quantity, k_z is a real quantity whereas if k happens to be $< k_p$, then k_z will be an imaginary quantity. So, we will have to examine these two cases and in addition we have to choose the sign because there is a plus and minus, we have to choose the sign based on some physical considerations. So, let us look at that.

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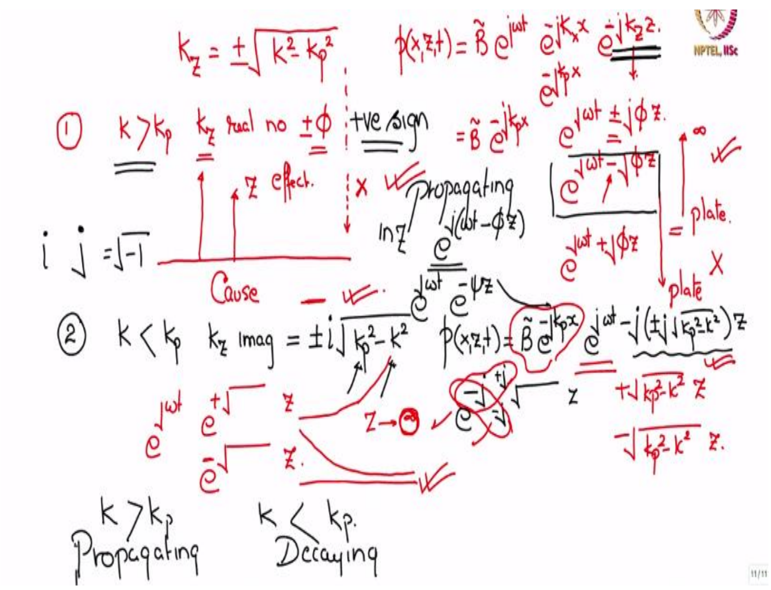

My $k_z = \pm\sqrt{k^2 - {k_p}^2}$ and my sound wave solution $\mathrm{p(x, z, t)}$ in the 2D half space was what $\tilde{B}\, e^{j\omega t} e^{-jk_x x} e^{-jk_z z}$, so the $k_x = k_p$,

$$\mathrm{p(x, z, t)} = \tilde{B}\, e^{j\omega t} e^{-jk_p x} e^{-jk_z z}.$$

Now k_z has these possibilities. So, first let us see. If $k > k_p$, then k_z is the real number.

Let the positive value be ϕ, then it is going to be $\pm\phi$. Then the sound field description is $\tilde{B}\, e^{-jk_p x}$, I put it separately $e^{j\omega t \pm j\phi z}$ because k_z became ϕ, $\pm\phi$. So, I will have plus or minus

$j\phi z$. Here is the thing, the sound field on the plate is exciting what is happening in the z direction. So, this is a cause and this is the effect.

So, any sound field should look like it is moving in the positives z direction. It should not look like a sound wave is coming from infinity towards the plate because not possible, there is no source at infinity. So, if you look at the z direction it should appear that the sound field is moving from the plate towards infinity. Now, $e^{j\omega t - j\phi z}$ this descriptor says that the sound field is moving towards ∞ from the plate.

I will discuss this in the next class maybe, but here please take it this minus sign indicates that the plate is generating the sound and the sound field is moving towards ∞ which is physically possible, whereas the other descriptor if I choose the other sign $+ j\phi z$ what this says is that there is a sound field which is coming from ∞ towards the plate and there is no source at ∞, we are describing the problem.

So, this is not possible. So, when k happens to be $> k_p$, we have to choose the positive sign for k_z. Then it becomes $- jk_z z$. When we choose the positive sign for k_z it becomes $- jk_z z$ and then that descriptor works well with the causality. On the other hand, the second case when k happens to be $< k_p$, k_z is an imaginary number. So, how does it look like that is equal to $\pm i\sqrt{{k_p}^2 - k^2}$.

And therefore how does the pressure description look like

$$p(x, z, t) = \tilde{B}\, e^{-jk_p x} e^{j\omega\, t - j\left(\pm\, j\sqrt{{k_p}^2 - k^2}\right) z},$$

and thus so please note that i and j are used interchangeably, so do not get confused, both mean the same same, square root of –1, I have got used to both, so I use j somewhere and i somewhere, they are the same.

So, if you examine this part there are two j's coming in, right. So I have e^{-j} and I can choose a $+j$ or I can choose a $-j$ and then a square root that is a real number now because I have switched k_p with k, $k_p > k$ and then z. So, this $-j$ into j gives me plus. If I choose it this way, if I choose this combination a positive imaginary, so j square is negative I get a plus, I get a $+\sqrt{{k_p}^2 - k^2}$z.

Whereas if I choose the negative imaginary and this negative goes with that negative, then j square gives me negative, I get a $-\sqrt{{k_p}^2 - k^2}$z. So, I have two choices, what is it? I have $e^{j\omega\, t}$ which is fine, I have either e to the power plus a real number into z or e to the power minus a real number into z. So this $e^{j\omega\, t}$ and this which is there in the exponent I am putting this out for a moment because it is there.

Now this as z goes to ∞ blows up, this description as z tends to ∞ blows up whereas this as z tends to ∞ decays to 0. So, in linear systems for a finite amount of power that is given I cannot have an infinite sound field at ∞, it is not possible. I cannot put so much power into the system without breaking or doing some damage. We have only finite energies, mind you ∞ is bigger than any number you think, any number you think ∞ is bigger.

So, you cannot have large sound field there. So, the only logical physically possible situation is a decaying sound field. So, I need to choose the negative sign here, negative imaginary. So, in this case I have to choose the positive and k greater than k_p, here I have to choose the negative when k less than k_p. Here there is a propagating sound field, this is a propagating sound field in the z direction, why it is of the form e to the power j omega t minus some real number z, it is a propagating sound field.

So, the $e^{j\phi}$ is oscillatory. Just as $e^{j\omega\, t}$ is oscillatory $e^{j\phi z}$ is oscillaroty. Whereas here I have $e^{j\omega\, t}$, but I have $e^{-\phi z}$ or psi z let us say, $-\psi z$. So, $-\psi z$ decays with z. So the behaviour is radically different. Whether k happens to be greater than k_p or k happens to be less than k_p, the behaviour is radically different.

Here there is a propagating sound field, here this is a decaying sound field. So, it is a 0 1 situation, $k\ > k_p$ you have propogating, $k < k_p$ you have decaying.

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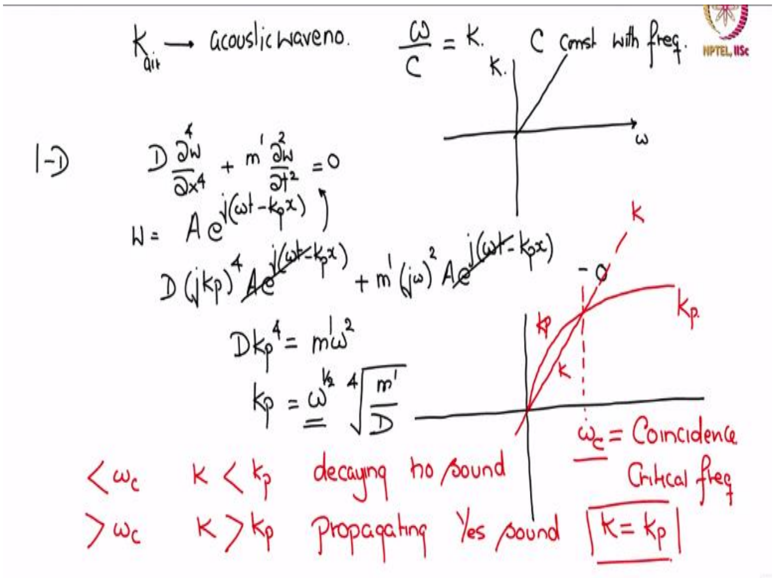

So, let us see now k the acoustic wave number we said was given by $\frac{\omega}{c}$ is equal to k and c is a constant, c is more or less constant with frequency. The speed of sound in air is more or less constant with frequency, so $\frac{\omega}{c}$ is k, so that means what? If I plot k as a function of frequency it will be a straight line. This is my omega axis, this is my k axis. So that is the behavior for air.

Now, we said the 1-D plate was given by

$$\mathrm{D}\frac{\partial^4 w}{\partial x^4} + m'\frac{\partial^2 w}{\partial t^2} = 0,$$

this is the equation of motion and we also said w was given by $\mathrm{A}e^{j(\omega t - k_p x)}$. So, if we substitute this back what do I get? I get

$$\mathrm{D}(j\omega)^4\mathrm{A}e^{j(\omega t - k_p x)} + m'(j\omega)^2\mathrm{A}e^{j(\omega t - k_p x)} = 0.$$

So, this part is common, I can take it out, so I am left with the other. So, j to the fourth is 1, so

$$\mathrm{D}k_p^4 = m'\omega^2.$$

So, k_p if I want becomes $\omega^{1/2}\sqrt[4]{\frac{m'}{D}}$. So, whereas k moved as a linear function of ω, k_p moves as a kind of a parabola. So, if I plot both on the same axis, so this is my acoustic wave number and this is my plate wave number.

Acoustic wave number goes up. So, now what there is a region where this is k_p, this is k, this is k, this is k_p. You see both the inequalities have come up right. Here, let us call this a certain ω_c. So, below ω_c I have $k < k_p$ and from earlier description decaying sound field or no sound field whereas greater than ω_c where k happens to be greater than k_p propagating sound field, yes there is sound.

So, this frequency now is called the coincidence frequency or some people call it critical frequency where the acoustic wave number is equal to the plate wave number.

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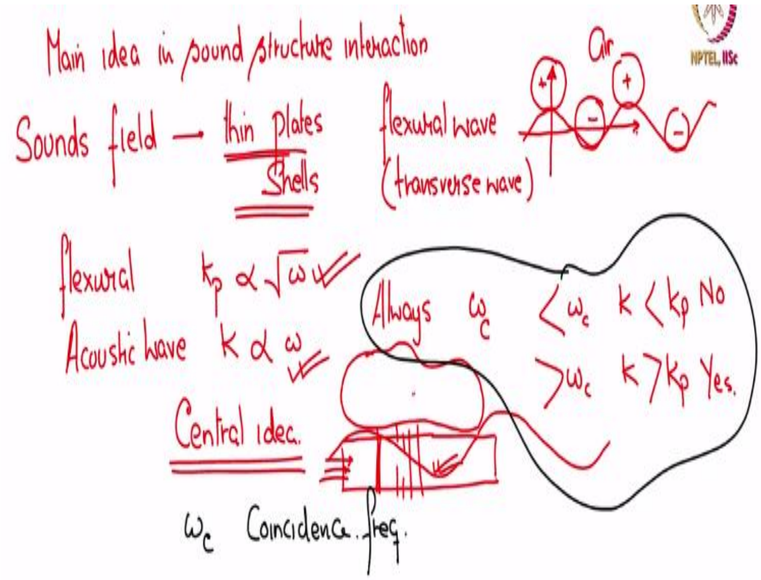

So, I said that the main idea in structural acoustics in sound structure interaction this is the main idea. Sound fields are largely generated by thin structures, thin plates, thin shells. That means what? The type of wave they carry is a flexural wave, a transverse wave. That means you have this sort of a behaviour, the particles move this way, the particles of the plate move this way whereas the wave moves that way.

It is a transverse wave that is how this medium can be compressed. This region is compressed, this region is rarefied, this region is compressed, this region is rarefied. So, let me say plus for compressed, negative for rarefied. So, flexural waves are the ones that generate sound fields and flexural waves are efficiently present in thin plates and thin shells and a flexural wave has this dependence on frequency.

The k_p is typically proportional to the $\sqrt{\omega}$ whereas the acoustic wave is proportional directly to ω. So, there is always a coincidence frequency such that below coincidence k happens to be less than k_p and above coincidence k happens to be greater than k_p, so here there is no sound, here there is yes sound. This is the central idea in sound structure interaction.

This decides many other issues. As opposed to a flexural wave, what wave could you have? You could have a longitudinal wave the one I kind of showed you in a spring, you can have a longitudinal wave. There is this bar or rod and there is a longitudinal wave. There are regions of compression, there are regions of rarefaction, but if there is an acoustic medium in the neighbourhood, there is no efficient transmission of this longitudinal movement to air.

The only ways if the flexural wave gets excited that is when you have the neighbouring medium being compressed and rarefied. So, a flexural wave, a transverse wave interacts with the sound medium in the neighbourhood and the flexural wave has this behaviour, this $\sqrt{\omega}$ relation whereas the acoustic wave has an ω relation. So, this always happens and this is crucial to sound structure interaction.

And therefore the coincidence frequency is a very important idea in sound structure interaction. So, let me close the lecture here for today. We will continue with this in the next class. Thank you.