Sound and Structural Vibration
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Lecture - 26
Summary of the Rectangular Waveguide

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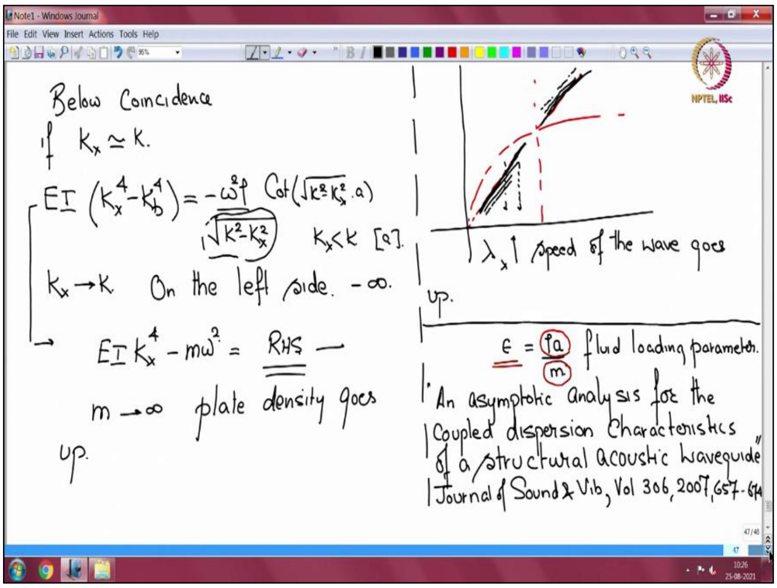

Good morning and welcome to this next lecture on sound and structural vibration. We have been looking at a structural acoustic wave guide and we have gone quite a bit into the into solving the problem we were looking at the physics. So, at the outset there is one point I would like to make this factor the fluid loading parameter ϵ is the coupling parameter between the structure and the fluid if ϵ is set to 0 as we have seen we get the uncoupled roots.

So, it is the ratio of the fluid density to the structural density and if the medium is like air a light medium and the waveguide is made of let us say steel or aluminum then ϵ will be a small quantity in which case the structure is loaded lightly and also the fluid sees more or less rigid structure. Whereas on the other hand if the fluid density goes up let us say it is water and there is some light medium for the wall of the waveguide then ϵ can go up and the situation reverses.

So, ϵ is the coupling parameter I did not mention this explicitly. There is one more point I would like to make this entire derivation on this structural acoustic waveguide can be found in this paper I am giving the title and asymptotic analysis for the coupled dispersion characteristics of a structural acoustic waveguide this is a paper published in the Journal of

Sound and Vibration a volume 306 the year is 2007 and pages 657 to 674. So, there are more details in there than I have presented over here.

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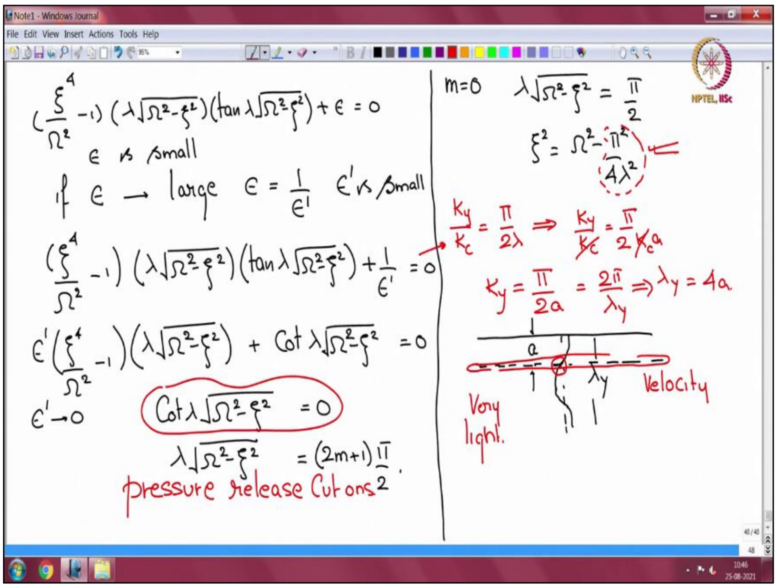

Now continuing the equation, we have is this

$$\left(\frac{\xi^4}{\Omega^2}-1\right)\lambda\sqrt{\Omega^2-\xi^2}\ \tan\lambda\sqrt{\Omega^2-\xi^2}+\epsilon=0.$$

So, this is the equation we use if ϵ is small. However, if ϵ is large then we have to change. The asymptotic method works for small values the technique works like that.

So, if ϵ becomes large then what we do is we write ϵ as $\frac{1}{\epsilon'}$ ok where ϵ' is a small parameter. So, if we do that we will have

$$\left(\frac{\xi^4}{\Omega^2}-1\right)\lambda\sqrt{\Omega^2-\xi^2}\ \tan\lambda\sqrt{\Omega^2-\xi^2}+\frac{1}{\epsilon'}=0\,,$$

$$\epsilon'\left(\frac{\xi^4}{\Omega^2}-1\right)\lambda\sqrt{\Omega^2-\xi^2}+\cot\lambda\sqrt{\Omega^2-\xi^2}=0.$$

Now ϵ' is a small parameter. So, now what is the basic uncoupled solution if ϵ' is sent to 0 and my solutions are for $\cot\lambda\sqrt{\Omega^2-\xi^2}=0$.That means the cosine is equal to 0 and therefore $\lambda\sqrt{\Omega^2-\xi^2}=(2m+1)\frac{\pi}{2}$. Now let us say if m is 0 then I have $\lambda\sqrt{\Omega^2-\xi^2}=\frac{\pi}{2}$ which means my $\xi^2=\Omega^2-\frac{\pi^2}{4\lambda^2}$.

These are non-dimensional this is the non-dimensional y wavenumber and therefore I will write it as $\frac{k_y}{k_c}=\frac{\pi}{2\lambda}$. The square root of $\frac{\pi^2}{4\lambda^2}$ is $\frac{\pi}{2\lambda}$ which is the non-dimensional y wavenumber and this is the k_c was the denominator for non dimensionalizing. So, now what is this which implies $\frac{k_y}{k_c} = \frac{\pi}{2k_c a}$. So, this goes with that. So, k_y is equal to $\frac{\pi}{2a}$ which is $\frac{2\pi}{\lambda_y}$ which means λ_y is equal to $4a$ we have seen this if you recall.

So, now let us see it again briefly. So, I have this waveguide top is rigid bottom lower is this vibrating plate. So, top the pressure will be maximum. So, I will come up this way is one quarter is the next quarter this is the next quarter and then the last quarter. So, this is λ_y and this is a and that is how λ_y is $4a$ which means again I have 0 pressure here. So, the fluid column sees a medium beyond which is very light here.

There is no gravity. So, do not worry about it ok. So, the fluid sees something very light and therefore the pressure is 0 if the fluid saw something stiff and solid you will have high pressure. So, pressure is 0 and there will be displacement velocity is actually high. So, those are the uncoupled solutions here and so these are called the pressure release cut-ons. So, this is called the pressure release boundary condition where pressure is 0 at the boundary of a fluid it is called the pressure release condition zero pressure conditions.

So, these are these pressure release cuttings ok and one of the critical points was intersection of this with the flexural wave we have seen that ok. Now let me go back to the software Maple which we saw last time, and I will again show you the demo of Maple and we will get back here.

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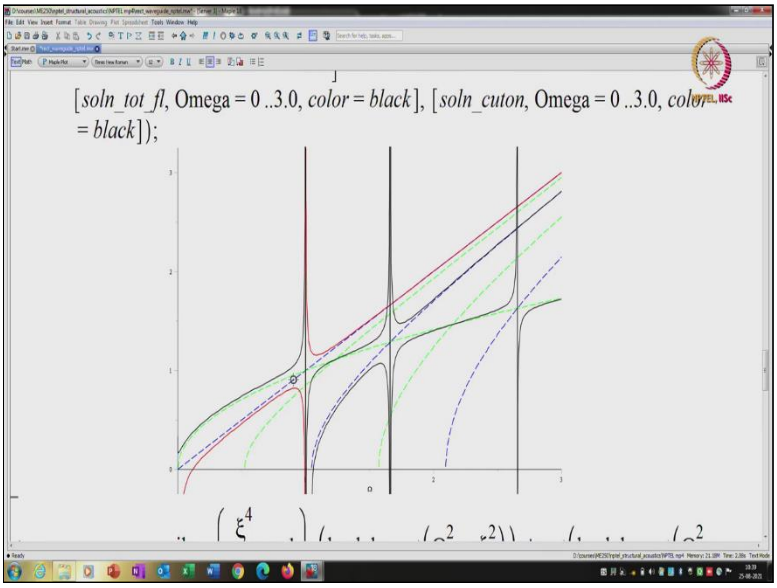

Last time I had shown you the Maple software. So, let us start again. So, start this is the basic equation that is how it looks like first we will do the plane wave solution. So, plane wave solution plus a_1 into ϵ then that trial is substituted into the main equation a Taylor series is made. Then I convert the Taylor series into a polynomial and then find the coefficient to ϵ to the power one.

So, let us do that okay then we have to solve for it that is the solution to the correction and then the final total answer similarly for the flexural wave okay let us run it okay something changed here. So, I will see these equation trial series equation trial 2 epsilon, 2 okay then equation 2 yeah it should work now. So, let us start here trial 2 okay then solve it okay then add as a solution.

Then third one this is the rigid cut-on okay then solve the equation then now we plot it. So, that is the plot I was trying to get all three of them last time. So, let us see here the blue line dashed is the plane wave this blue line is the first rigid cut-on this blue line is the second rigid cut-on the this first green line is the uncoupled flexure this is the pressure release we just spoke about this is the second pressure release we just spoke about okay.

So, the coupled flexure is just above below coincidence then here it blows up okay and that blowing up is because of the choice of our expansion not because the actual wave blows up okay then the this is the couple next coupled flexure it passes through the pressure release ok we have discussed the physics that is where 0 pressure occurs. So, just beyond the wave goes up and just below the wave is below.

So, you have stiffness effect here inertial effect here again this is the next critical point where the fluid loading actually goes to infinity and. So, our expansion fails which means we have to find another expansion and it will work out okay. And so, the coupled flexure repeats between every two rigid cations. So, again it passes through the next pressure release above that is higher below that it is lower and so forth okay.

Now the plane wave the plane wave correction is below the original below coincidence this is the coincidence omega equal to 1 and the correction is above coincidence. So, the plate is looking like a stiffness here and like inertia here ok. As inertia becomes more and more, we approach the plane wave original as inertia drops the wavenumber drops. So, that the inertial effect having reduced the wave speed increases whereas here it is the other way okay.

As stiffness drops the wavenumber goes up okay. So, that λ goes down ok and it for infinite stiffness we are back to the original uncoupled plane wave. Now this is the rigid duct first rigid duct cut-on. So, it starts off with this black line okay and it blows up here because again of our expansion and it transitions to the coupled flexure okay actually it will be do a very smooth transition.

So, the coupled rigid duct comes transits becomes the coupled flexural wave passes through the pressure release point goes forward and it latches on to the next rigid duct cut-on that is how it will happen okay as it is doing over here okay. Now I will also, so these are now values of small ϵ. So, I will show you what happens for large ϵ value. So, I will comment this picture ok.

Now I am doing a pressure release solution here let me start over for this case these are unimportant, but I need to restart the program I will do the calculation but not plot it ok.

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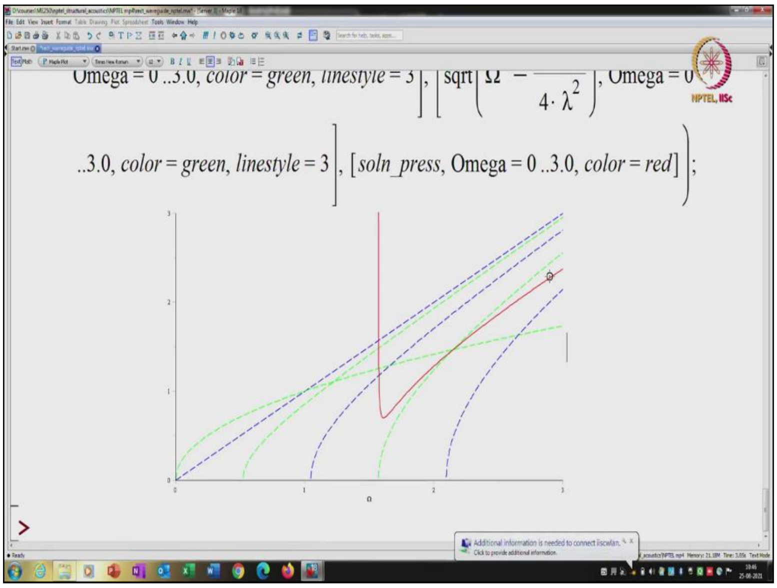

I have chosen two the second pressure release. So, we will see that here and that is how it looks like ok. So, so this is my second pressure release, and this is the coupled pressure release curve ok. So, it this is a frequency where the particular expansion blows up. So, we do not worry about that otherwise it will be a smooth curve. So, it is above below this crossing frequency one critical frequency it goes through that point which is the crossing point between uncoupled flexure and that pressure release and then it transits above ok close to the pressure release curve ok.

So, if I have to show on this graph itself for small values my curve will be close to the rigid duct for small values of ϵ my curve will be close to the rigid duct okay and somewhere it will transition to coupled flexure cross through this point rise above follow the next rigid duct cut on. As my ϵ value goes up okay, I have chosen an epsilon value which is 1 over 0.05 which is like 20 ok this is ϵ' mind you from my derivation ok.

So, ϵ' is a small quantity in order to find the pressure release correction. So, as ϵ starts to become bigger and bigger this curve will shift towards the pressure release that is what has happened okay, and it will straight away pass through the critical point and crossover okay. So, that is the transition from small ϵ to large ϵ ok. So, let me get back to my derivation.

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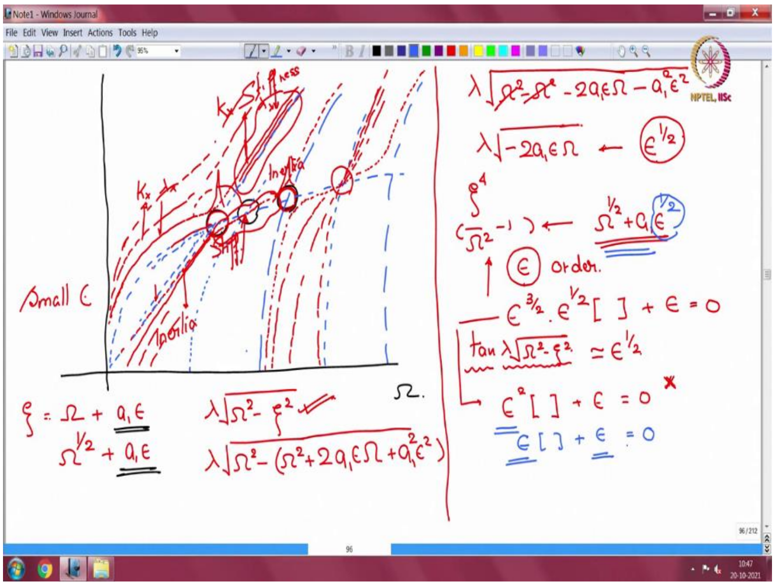

So, now I will summarize what has happened. So, we have this picture over here this is Ω let us see I will put all uncoupled curves as blue okay and light. So, this is my uncoupled plane wave this is my uncoupled flexure this is my uncoupled flexure then let us say this is my uncoupled pressure release and this is my first rigid duct cut-on this is my next pressure release, and this is my next rigid duct cut-on okay.

On top of this let me do the coupled structure coupled structure is above here okay. And at this point there is a transition that is going to happen a smooth transition the coupled structure will smoothly transition become the coupled plane wave. Below we have the coupled plane wave it will come up ok. And as we get close to uncoupled structure it will smoothly transition to coupled structure pass through the pressure release go beyond start following the next rigid duct cut-on ok.

Next this uncoupled rigid duct cut-on starts as a couple rigid duct cut-on smoothly transitions to a coupled flexural wave passes through the pressure release point crosses over and becomes or gets asymptotic to the next rejected cut-off and so forth ok. So, this is the picture for small ϵ ok. So, here the structure sees the fluid as inertia and therefore wave number higher and therefore λ lower ok.

In this region the structure does not see the fluid at this transition pressure release point. Here it sees it as inertia here it sees it as stiffness ok. Here the original plane wave sees the structure as inertia ok. Less and less inertia the wavenumber will drop high inertia you get back your uncoupled plane wave in here the fluid sees the structure as stiffness ok. Less and less stiffness the wavenumber rises k_x goes up and λ_x comes down.

So, that wave speed goes down and high stiffness you approach the plane wave closer and closer at almost infinite stiffness you reach the plane wave back ok. And here let us say for larger and larger values of ϵ this curve will do this for further larger values of ϵ it will do this ok. Finally, it will become a correction to the pressure release cut-on become a correction to the pressure release cut-on ok.

And as ϵ becomes larger and larger this wave will start moving up this way will start moving up ok. Now just a point I want to make you must have seen that at the first corrections these there is a blow up the value blows up at these values at these critical points ok. So, let us see for Ω for a plane wave we took a correction $a_1\epsilon$ for the flexure we took $\Omega^{1/2} + a_1\epsilon$ ok.

Now I will show you if I have $\lambda\sqrt{\Omega^2 - \xi^2}$ let us say. So, ξ is here $\xi = \Omega + a_1\epsilon$. So, ξ^2 ok $\lambda\sqrt{\Omega^2 - (\Omega^2 + 2a_1\epsilon\Omega + {a_1}^2\epsilon^2)^2}$ ok. So, if I cancel of etcetera. So, what I get

$$\lambda\sqrt{\Omega^2 - \Omega^2 - 2a_1\epsilon\Omega - {a_1}^2\epsilon^2}.$$

So, if this cancels out, I have the leading term as $\lambda\sqrt{-2a_1\epsilon\Omega}$ ok. Now when I approach a critical point ok there is a second root also that is approaching there is a flexural root that is also approaching just as the plane wave is approaching ok. So, from the flexural root side also I have $\left(\frac{\xi^4}{\Omega^2} - 1\right)$. So, in this I am going to substitute my $\Omega^{1/2} + a_1\epsilon$ correction ok.

So, what that does is that I will have a correction term after cancellation which is of ϵ order okay. So, here I have a correction of $\epsilon^{1/2}$ order ok. So, this ϵ order correction and that $\epsilon^{1/2}$ correction will give me an $\epsilon^{3/2}$ order correction there is also a tan term and just as this term gave me $\epsilon^{1/2}$ this will be $\epsilon^{1/2}$ and tan of that is also $\epsilon^{1/2}$.

So, this $\epsilon^{3/2}$ and another $\epsilon^{1/2}$ and some terms plus ϵ should balance but this has now become ϵ^2 and we are left with $\epsilon$to be balanced. So, it cannot be balanced and. So, what now that means is that this expansion that we have ok should be of $\epsilon^{1/2}$ order ok then what happens is every correction term is half to the power half then this will become ϵ and it can balance this $\epsilon$from the fluid loading term.

So, what essentially means is that wherever you have two uncoupled routes meeting you have a plane wave meeting a flexural ok. So, there is 2 roots are meeting you need to have half order expansions. So, that the half order corrections from each of the roots multiply they become epsilon and then that epsilon can be balanced by this epsilon. Time is up I will close here we will continue with this topic in the next lecture, thank you.