Sound and Structural Vibration
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Module No # 05
Lecture No # 25
Heavy Fluid Loading

Hello and welcome to this next lecture on sound and structural vibration this is where we stopped last time.

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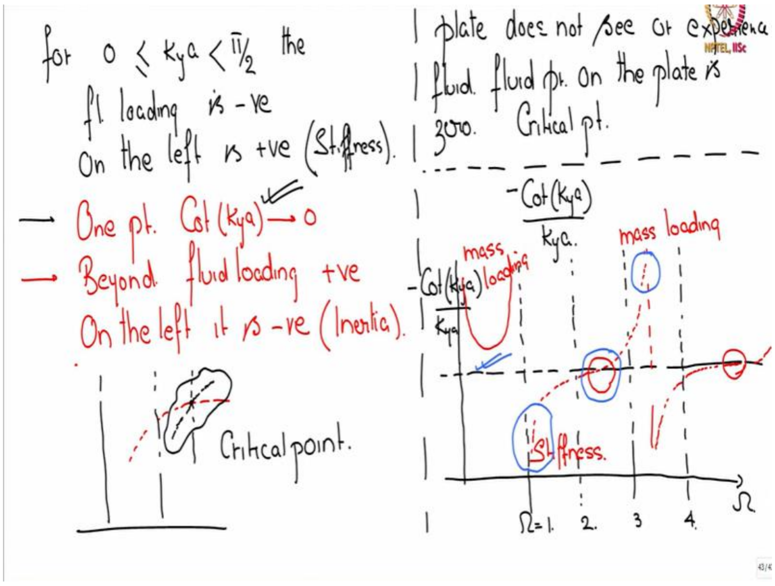

We were looking at the correction term and it has this behavior below coincidence it has a purely inertial loading on the structure and beyond coincidence it has an oscillatory behavior. There are points where it is stiffness like there are points where it is inertia like and there are points where $\cot(k_y a)$ goes to 0. So, it adds no correction and hence uncoupled and coupled cross each other. Say these are critical points I said so let us see what is this? What are these 2 critical points?

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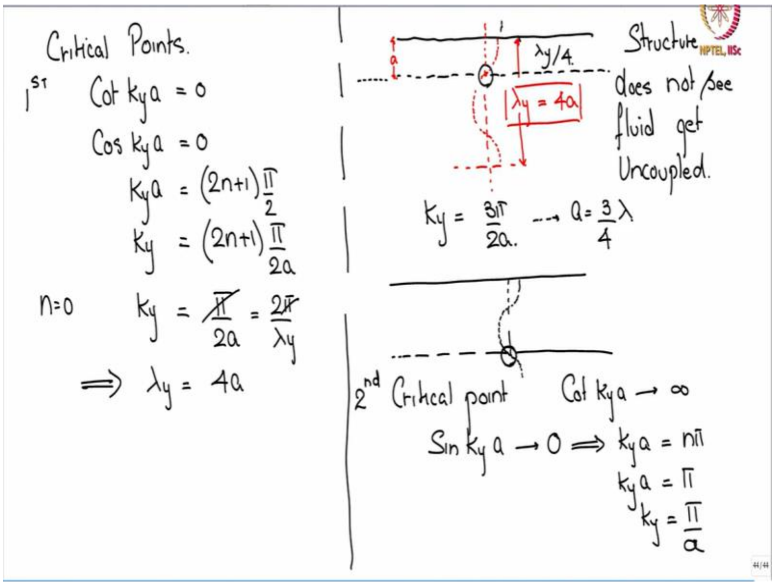

So first critical point first is when $\cot(k_y a) = 0$ which means $\cos(k_y a) = 0$. Which means $k_y a$, is of the form let us say $(2n + 1)\frac{\pi}{2}$ and therefore k_y = $(2n + 1)\,\frac{\pi}{2a}$. So let us take the first fellow which is equal to 0 let us say then k_y = $\frac{\pi}{2a}$, and which is equal to $\frac{2\pi}{\lambda_y}$. This is the relation between λ_y and wave number k_y is the y directional wave number.

So, λ_y is like y directional wavelength so if we now do this? It means what it implies that λ_y $=4a$. So, 4 times of a will picturize this so I have the top surface of my wave guide I have the bottom flexible surface of the plate. Now at the top surface my pressure will have a 0 gradient because velocity is 0 so now let us imagine this is the 0 pressure lines suppose.

Let me take it small else I will take up the whole page so let us say this is my structural acoustic wave like bottom flexible plate. And then the top so this is the mean pressure line 0 pressure line. So, at the top the velocity is 0 so I will have peak velocity now I have to come up with λ this is a. So, this λ is 4 times a how can it be so this is total λ you can see that λ_y one full cycle.

And it is also showing that it is equal to 4 times of a so what does it mean when you enforce this relation such that a, is the fourth of λ the pressure here goes to 0. This is $\lambda_y/4$ so first being λ_y you have to show one wavelength. On top of that it is relationship is with a, is 4 times of a. If you infuse that relationship by that it reaches the bottom flexible plate the pressure is 0.

So that is the critical point so when pressure is 0 the structure will not see the fluid and for brief moment in frequency gets uncoupled. In the presence of the fluid and therefore the 2 curves cross. So now if I go to the next level which is less of us say $k_y = \frac{3\pi}{2a}$ and then start computing my a will be equal to $\frac{3}{4}\lambda$. So how will the picture actually look?

The picture will look like this it will start off in the mean pressure line it will start off and it will start off it will come and go to 0 here like this. Then you have the wavelength full wavelength this is a is $\frac{3}{4}\lambda$ and so forth. So, every time you have a pressure at the flexible plate surface, so the flow plate does not see the fluid and a crossover occurs. So, this is 1 critical point the second critical point is when $\cot k_y a$ goes to ∞.

We have seen those pictures we have seen these region $\cot k_y a$ goes to ∞ so what happens there? So, cot is cosine over sine which means now $\sin k_y a$ goes to 0 which implies that $k_y a$ = nπ. So let us take the first one which is $k_y a = \pi$ or $k_y = \frac{\pi}{a}$.

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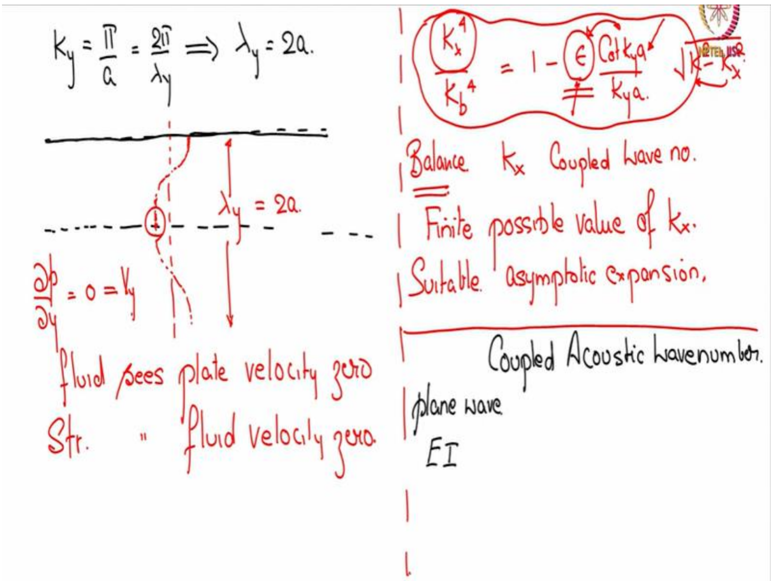

So, $k_y = \frac{\pi}{a}$ is equal to $\frac{2\pi}{\lambda_y}$ implies λ_y is equal to $2a$. So now if I plot it like this is my top rigid this is my bottom lower flexible and so I start of here let me this is a mean line. So, I start off at the top it is a rigid plate is start with pressure gradient 0 so it will start with high pressure then it will come it will have again a peak pressure and then it completes the wavelength so this full wavelength λ_y.

And it is what equal to $2a$ now but what is the net result you can see here $\frac{\partial p}{\partial y} = 0$ which is equal to v_y. The velocity here is 0 the fluid velocity so the fluid see plate velocity 0 and the structure sees fluid velocity as 0. Hence it is region where one is seeing a high impedance from the other. So, if we look at these equation

$$\frac{{k_x}^4}{{k_b}^4} = 1 - \epsilon \frac{\cot k_y a}{k_y a}.$$

If we look at this equation now, we have to balance this equation that means what finite values of k_x we had to find finite possible values of the coupled wave number. And therefore, when this is blowing when this term is blowing up going to ∞ the one that prevents complete loss is this ϵ. ϵ mean balance out the growing term the ϵ which is the small parameter will balance out the blowing up of this cot term so that this can be balance this equation can be balanced.

So, there is a finite possible value of k_x in other words based on ϵ the degree to which $\cot k_y a$ blows up that means $\cot k_y a$exactly does not go to ∞ will not be allowed to go ∞ why because k_y is $\sqrt{k^2 - {k_x}^2}$ so how close k_x will get to k_y will depend on ϵ such that the blowing up is prevented. So, such that equation gets balanced, and you get a finite possible value of k_x you have to find a suitable asymptotic expansion.

So that is what is happening when you do the full coupled calculations, I will summarize this again when we get a chance but right now these are the 2 critical points and how they behave. And how k_x behaves accordingly so now the next step here is we have looked at flexure we have to look at coupled acoustic wave number. We will just look at the plane wave because by now you know how to deal with the physics and the mathematics just look at the plane wave. So, the equation I have is EI or let me change the page because I will need it for arguments.

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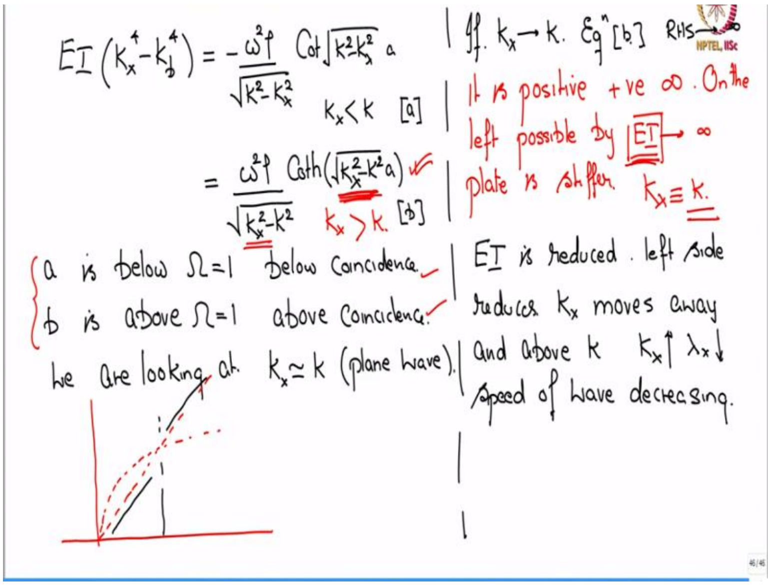

So, the equation I have is

$$EI\left({k_x}^4 - {k_b}^4\right) = -\frac{\omega^2 \rho}{\sqrt{k^2 - {k_x}^2}} \cot \sqrt{k^2 - {k_x}^2} a\ .$$

this is for k_x less than k will call it some equation a. Which is also now equal to

$$= \frac{\omega^2 \rho}{\sqrt{{k_x}^2 - k^2}} \coth\left(\sqrt{{k_x}^2 - k^2 a}\right).$$

We will call this b.

So, you can see that the b equation or the, a equation is for below $\Omega = 1$ below coincidence and b is above coincidence $\Omega = 1$ above coincidence. So, we are looking at k_x closed to k which this the plane wave why do I assume this? I do not assume this using asymptotic wave found this. This was found using mathematics, so I am stating it and now I am looking because we are going to explain the result the mathematical result is already derived so I am explaining the result, so this was found out earlier.

And we are looking at k_x close to the acoustic wave number and how does it look like? Just refresh so this is my acoustic plane wave, and this is my let us say flexure my coupled plane wave looks like this it is below here this is coincidence, and it is above here. So, let us explain that so let us look at if my k_x approaches k then we are looking at equation a, and what happens? Equation a cot is an argument going to 0 so the right-hand side RHS tends to ∞.

We are looking at equation b here so as k_x approaches k any in equation b the right-hand side approaches ∞ and it is positive. So, I am going towards positive ∞ now on the left how is it possible on the left this is possible by EI that is made to tend to ∞ which means the plate is stiffer. So as my plate becomes more and more stiff above coincidence my k_x will start approaching k and that is how both sides will balance.

So here again I mean infinite stiffness cannot be achieved but that is the mathematical limit. So, if you manage EI tending to ∞ the left-hand side will go to ∞ and the right-hand side go to ∞ then my k_x exactly lands on k the plane wave. So, this is above coincidence on the other hand the argument holds if EI reduced that means the stiffness is reduced the plate is becoming more flexible the left side reduces.

And the coupled wave number k_x moves away and above k so moving away means k_x increasing which means λ_x decreasing which means the speed of the wave decreasing.

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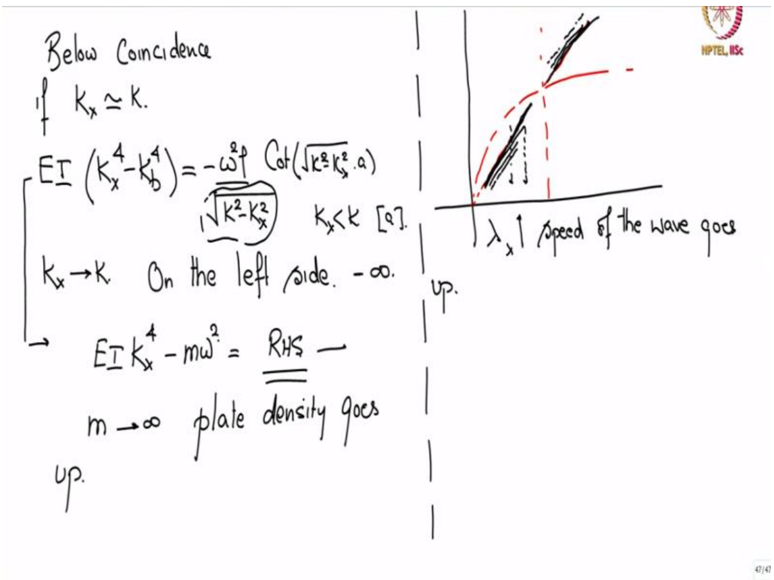

So, this is a story above coincidence now below coincidence again we will keep k_x close to the plane wave k. Then I will need write the equation for clarity

$$EI\left({k_x}^4 - {k_b}^4\right) = -\frac{\omega^2\rho}{\sqrt{k^2 - {k_x}^2}} \cot\sqrt{k^2 - {k_x}^2}a\,.$$

Here k_x is less than k the, a equation below coincidence. Now again if the value approaches ∞ and k_x is approaching k, a cot value approaches ∞ and this approaching 0.

Now looking at the left on the left side I have to approach negative ∞ then what happens? What will do that? If I open this term out here, I have $EI{k_x}^4 - m\omega^2$ is equal to the right-hand side.

This right-hand side is reaching minus infinity and therefore the only way that can be balanced by the left side is if mass tends to infinity. That means the plate density goes up.

Or the fluid if this is the original plane wav this is the original uncoupled flexure and this is coincidence. So here the fluid sees the structure as a stiffness if it is get stiffer and stiffer the wave number comes down and here the structure is infinity stiff. Whereas here the fluid sees the plate as inertia increases, we approach again the uncoupled, so this is infinite inertia of the plate.

As the inertial value drops the wave number starts to drop so it is as though there is infinite inertial loading from the plate and that is when you get the uncoupled plain wave. For any other finite value, the wavenumber drops. So inertial loading drops so the wave number drops and therefore the λ_x goes up, so the speed of the wave goes up. So, I will stop here we are running out of time I will continue from here in the next class thank you.