Sound and Structural Vibration
Prof. Venkata Sonti
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Module No # 05
Lecture No # 24
Critical Points

So welcome to this next lecture on sound and structural vibration we have been looking at the structural acoustic wave guide.

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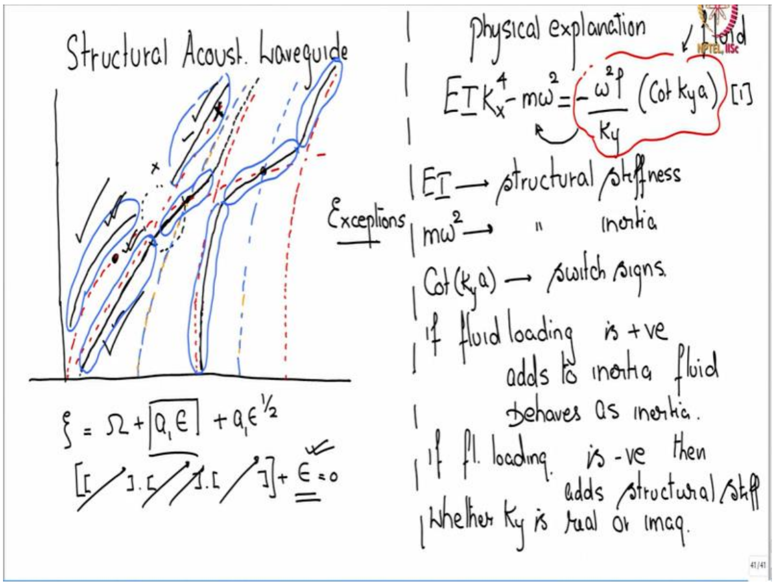

Last time we looked at the software called Maple let me show you what now the picture should look like. The rigid cut-on calculation did not work out hopefully I will show it to you sometime. So, I will keep the uncoupled waves dotted in light so that is the plane wave this is my flexural wave. Now this is my first rigid duct cut-on this is my second rigid duct cut-on and so forth. Now we saw that the coupled plane wave was below here a coupled plane wave was above here.

And in this region, it blew up I told you that will happen the reason is what we did this expansion $\xi = \Omega + a_1\epsilon$. So, this was valid for that region and this region not valid for that region. That means this correction should have some other form I will tell you that form it should have this form $+a_1\epsilon^{1/2}$ that is why it did not work out.

Now the flexural wave how did it look the coupled flexural wave looks like this here and below what happened. There was the point it crossed below it was lower above it was higher now let me draw in some light color here let may be that is good. There are these points there are these light lines what is that point it is a critical point? What is that? Let me draw these so there are these lines.

Now what essentially happens is my coupled wave starts here it remains close to the plane wave here transitions slowly into the flexural coupled wave continuously. Now I am on to the coupled flexural wave line which crosses at a single point the uncoupled flexural wave and then it goes beyond and then it starts to follow the next rigid duct. I told you that these black lines can never cross the other dotted lines because there are three terms here in a product and they have to be small enough to balance my ϵ.

If they exactly go to 0 at anywhere at any point, then there is no way to balance this and therefore if I am any one of these dotted lines then I am exactly 0. If I am here, I am exactly on the acoustic plane wave. So, second term will go to 0 if I am exactly here, I am on the uncoupled flexure. So, first term will go to 0 so this entire term will go to 0 then ϵ will be left hanging so no uncoupled line I mean no coupled line can exactly cross the uncoupled line except there are exceptions.

And these are this exception points so what happens to the rigid coupled curve which I said did not work out. So, it follows the rigid coupled line I mean the uncoupled rigid line comes up becomes the coupled flexure again so remains below crosses the uncoupled flexure goes above and it starts to follow the next rigid uncoupled line and this picture repeats. So, if I have to again pick let us say blue this is the coupled acoustic plane wave this is the acoustic coupled plane wave.

This is the coupled flexure this is the coupled flexure this is the coupled rigid duct cut-on first fellow, and this is the coupled rigid duct cut-on second fellow and so forth. Now the naming is also appropriate if I coupled line is close to an uncoupled line, we will call it the appropriate coupled line. So, this black line is close to flexure so we will call it coupled flexure. So, we will call it uncoupled flexure and other is coupled flexure.

So that is the pictures now let us look at a physical explanation we look at the physical explanation why this happens? That means why the coupled flexure here is above why the coupled plane wave here is above why the coupled plane wave here is below and so forth. We will see the physical explanation for that. So, in order to do that let us look at the original equation

$$EI{k_x}^4 - m\omega^2 = -\frac{\omega^2\rho}{k_y}(\cot k_y a).$$

We will give this a number 1 now we will say EI is the structural stiffness and $m\omega^2$ is the structural inertia. Now please note that $\cot k_y a$ because it is composed of cosine and sine it will switch signs it will have positive regions and negative regions in frequency. So, it has a minus here so let us say this entire term including the minus entire term including the minus.

If that term will call it fluid loading term if the fluid loading term that means along with the sign is positive. Then it comes to the left side when it transferred to the left side it becomes negative and it adds where it adds to inertia it adds to structural inertia that means the fluid behaves as inertia. So, the structure sees it as inertial loading whereas if the entire fluid loading term along with the sign is negative. Then it adds to the structural stiffness so whether k_y is real or imaginary does not matter.

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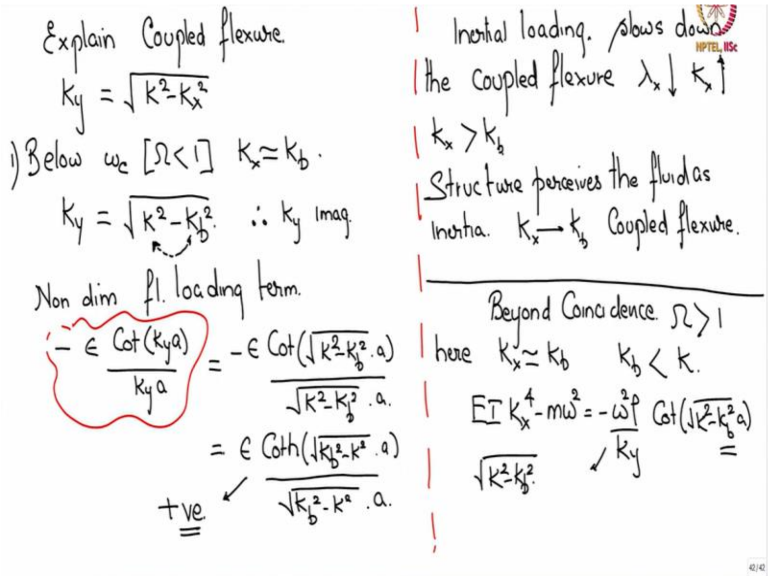

Now we will see what that means so we will try to explain the coupled flexure try to explain the coupled reflection so what is k_y? k_y is equal to $\sqrt{k^2 - {k_x}^2}$ and it is part of the equation is part of equation 1. So let us say we are looking for below coincidence or below ω_c or Ω is less than 1 and k_x is approximately near k_b flexure. Or we will call it k_p I forget what I have been using.

I think I have been using k_b so let us stick with k_b and therefore k_y also will have $\sqrt{k^2 - {k_b}^2}$. And because k_x is close to k_b the relationship between k and k_b also holds. So below coincidence k happens to be below k_b and therefore k_y is imaginary so now what happens? We have if we look at the non-dimensional form of the fluid loading term which is $-\epsilon \frac{\cot(k_y a)}{k_y a}$.

That term tends out let me write it once more

$$-\epsilon \frac{\cot(k_y a)}{k_y a} = -\epsilon \frac{\cot\left(\sqrt{k^2 - {k_b}^2 a}\right)}{\sqrt{k^2 - {k_b}^2 a}},$$

$$= \epsilon \frac{\coth\left(\sqrt{{k_b}^2 - k^2 a}\right)}{\sqrt{{k_b}^2 - k^2 a}}.$$

So, note that I have switched k and k_b positions and this is a positive term.

So, this entire term along with the sign is a positive term on the right so what we will do? If it is positive on the right, it is negative on the left that means it is an inertial loading it is an inertial loading. And so, what it does is actually slows down the uncoupled flexure uncoupled what I mean is? As an inertial influence on the flexure and slows down the coupled flexure that means the coupled λ_x is smaller remember λ_x is distance covered in one cycle.

So, if it slower λ_x is smaller and therefore k_x happens to be bigger and so k_x becomes bigger than k_b. So, the structure perceives the fluid as inertia and now since k_x is a modification to k_b we will call it coupled flexure that is the nomenclature coupled flexure that is influence of both structure and fluid. But because k_x is close to k_b we will call it coupled flexure now what about beyond coincidence that means ω greater than one.

Here again I am looking for k_x close to k_b but k_b is less than k and therefore you can see in the equation in the original equation which is

$$EI{k_x}^4 - m\omega^2 = -\frac{\omega^2 \rho}{k_y} \cot\left(\sqrt{k^2 - {k_b}^2 a}\right).$$

I will open this square root of $k^2 - {k_b}^2$ and k_b is less than k already. So here also it is the same term square root of $k^2 - {k_b}^2$, k_b is less than k. So therefore, the cot term will change signs the cot term will change signs so how does it do that?

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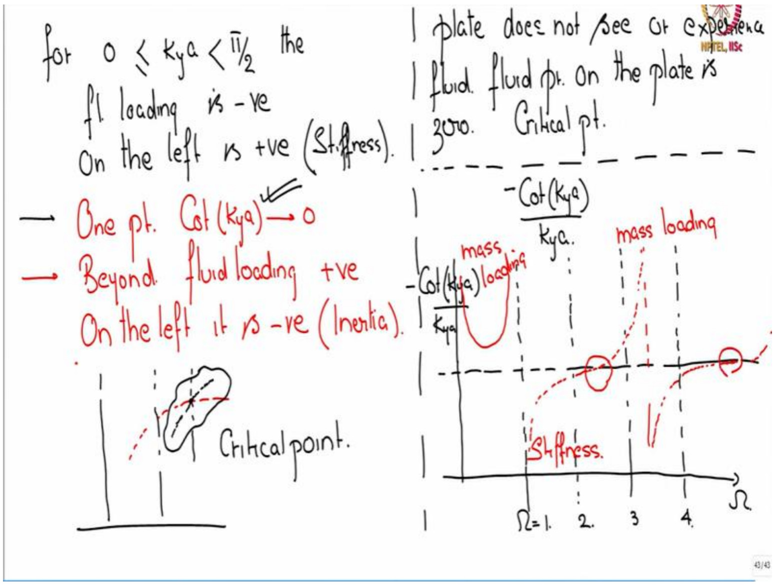

So, for $0 \leq k_y a < \frac{\pi}{2}$ the correction the fluid loading term along with the sign is negative hence what happens? When it is transferred to the left-hand side it is positive on the left it is positive and therefore it adds as stiffness. Then this is important there is one point where cot goes to 0, $\cot(k_y a)$ goes to 0. And then beyond the fluid loading term which is sine switches sine and becomes positive along with the sign is positive which means on the left it is negative.

And therefore, it behaves as inertia so then what happens so if we look at that picture in the localized picture here my coupled my original flexure is moving like this and some where I am beyond coincidence and somewhere. The $\cot(k_y a)$ is 0 beyond it is positive and therefore it becomes inertial before it is negative. So, it becomes stiffness, so this is now my coupled flexure beyond coincidence.

So, there is this one point which is where $\cot(k_y a)$goes to 0 it is a critical point I told you earlier that coupled and uncoupled curves cannot cross but this is one exceptional point where this crossing occurs that means the coupled and uncoupled are equal. That means what the correction is 0 that means what the correction is 0. So even by the language what that means is that structure or plate does not see the fluid does not see or experience the fluid pressure on the plate is zero and hence the coupled wave behaves like the uncoupled wave now we look at that critical point that is we look at what happens at that critical point in a few minutes. But before that this whole term which is $-\frac{\cot(k_y a)}{k_y a}$ which is part of correction term or main contributor is how does it behave in frequency? So, this is my I am going to plot it with respect to non-dimensional frequency.

So, this is Ω = 1 then Ω = 2 then Ω = 3 Ω = 4 this is Ω axis below Ω so let us a 0 value let us say this is a 0 value let me plot it in red so below $\Omega = 1$ this term is totally positive. We have shown you that it is a hyperbolic chord, so it behaves like this, so it makes it inertial. So, this is region is mass the fluid loads the structure as a mass but just beyond it starts to behave as stiffness become very light.

Somewhere here there is a cross over and at value of π , Ω equal to π this thing blows up and this region is mass loading again and this region is stiffness. And this repeats now somewhere here again there is a stiffness part it becomes light loading crosses over and again at 2π. It starts to blow up so this is reflected in the correction of the flexural coupled wave at the time is running out. So, I will explain to you why this crossover point is there and what happens here in the next class we will stop here thank you.