Sound and Structural Vibration
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Module No # 04
Lecture No # 19
Derivation of the Coupled Dispersion Equation

Good morning and welcome to this next lecture on sound and structural vibration we have been looking at a couple structural acoustic waveguide behavior. And we started deriving the equations for the acoustic fluid for the bottom boundary plate and the continuity boundary condition.

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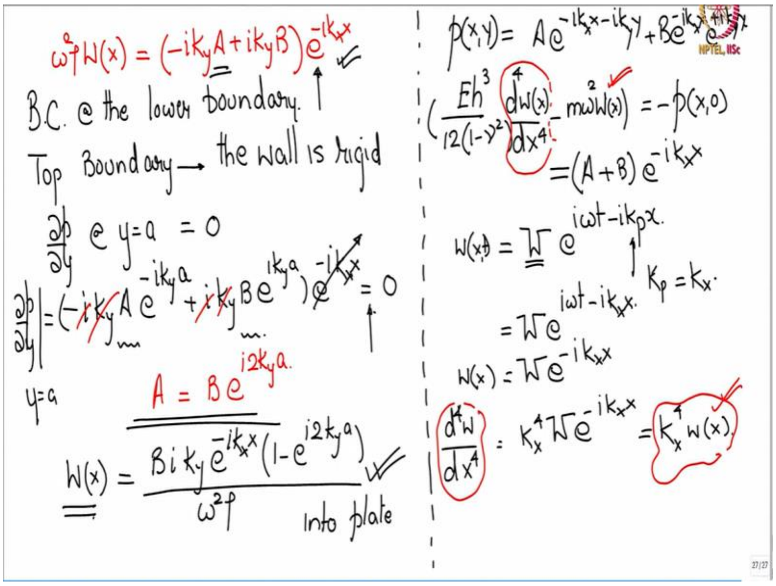

So, we stopped over here I will continue with this derivation. So now so this is the condition at the bottom part this is the boundary condition at the bottom edge bottom surface of lower boundary let me call it lower boundary. What about at the top boundary? At the top boundary the wall is rigid so what we will have $\frac{\partial p}{\partial y}$ at $y = a$ will be 0 this from acoustics. So, what does this give me?

$$\left.\frac{\partial p}{\partial y}\right|_{y=a} = (-ik_y A e^{-ik_y a} + ik_y B e^{ik_y a})e^{-ik_x x} = 0.$$

So, this is equation equated to 0 so $e^{-ik_x x}$ can be removed and therefore we get a relationship between A and B.

$$A = B\, e^{i2k_y a}.$$

So, if we replace the A here using this condition over here what do we get?

$$w(x) = \frac{Bik_y e^{-ik_x x}\left(1 - e^{i2k_y a}\right)}{\omega^2 \rho}.$$

Now this is $w(x)$ now what did we have? So, what is remaining now? We had to bring this $w(x)$ into the plate we have to utilize the plate equation.

So, plate equation let me just remind you that the pressure is $p(x, y)$ if I do not write the time term is

$$p(x, y) = A\mathrm{e}^{-ik_x x\, -\, \mathrm{i}k_y y} + B\mathrm{e}^{-ik_x x\, +\, \mathrm{i}k_y y}.$$

And there is the panel equation which is

$$\frac{\mathrm{E}h^3}{12(1-\nu^2)} \frac{\mathrm{d}^4\, w(x)}{\mathrm{d}x^4} - m\omega^2 \mathrm{w(x)} = -p(x, 0),$$

$$\frac{\mathrm{E}h^3}{12(1-\nu^2)} \frac{\mathrm{d}^4\, w(x)}{\mathrm{d}x^4} - m\omega^2 \mathrm{w(x)} = (A+B)e^{-ik_x x}.$$

And then $w(x,t)$ is what? $w(x,t)$ is some let us say $W\, e^{i\omega t - ik_p x}$. So, the displacement $w(x,t)$ is some W amplitude $e^{i\omega t - ik_p x}$ now for the first classical problem we know that this $k_p x$ must be equal to $k_x x$. This k_p must be equal to k_x so this is equal to let us keep the time $W\, e^{-ik_x x}$.

So now if I decide to drop the time term my $w(x)$ is equal to this $W\, e^{-ik_x x}$ and

$$\frac{\mathrm{d}^4\, w(x)}{\mathrm{d}x^4} = {k_x}^4\, W e^{-ik_x x} = {k_x}^4 w(x).$$

So, $\frac{\mathrm{d}^4\, w(x)}{\mathrm{d}x^4}$ term is replaced with ${k_x}^4 w(x)$.

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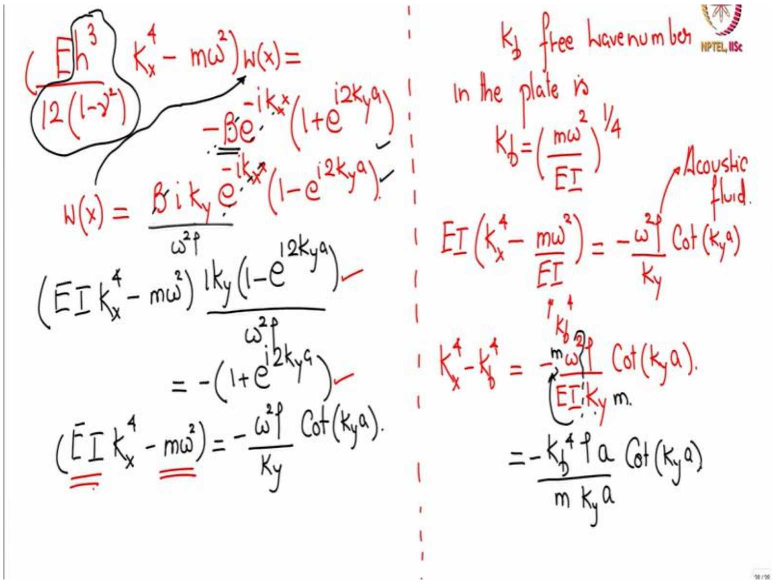

So, what do we have? We have

$$\left(\frac{\mathrm{E}h^3}{12(1-\nu^2)}{k_x}^4 - m\omega^2\right)w(x) = -Be^{-ik_x\mathrm{x}}\left(1+e^{i2k_ya}\right).$$

So now $w(x)$ was also known so what was $w(x)$? Let me remind you $w(x)$ we found out was $\frac{Bik_ye^{-ik_x\mathrm{x}}\left(1-e^{i2k_ya}\right)}{\omega^2\rho}$.

So, this will be substituted here, and you have B on the left you have B on the right so B disappears so what happens now? So, $\frac{h^3}{12(1-\nu^2)}$ portion I will club it as I and not keep repeating it so I write

$$\left(EI{k_x}^4 - m\omega^2\right)\frac{ik_y\left(1-e^{i2k_ya}\right)}{\omega^2\rho} = -\left(1+e^{i2k_ya}\right).$$

So, you can see that B got cancelled you can see that B got cancelled you can see $e^{-ik_x\mathrm{x}}$ also cancelled. So just this ik_y survives and these exponent terms survive.

So, if we try to make it even brief or smaller

$$\left(EI{k_x}^4 - m\omega^2\right) = -\frac{\omega^2\rho}{k_y}\cot\left(k_ya\right).$$

So, I can rearrange this in a few ways for my advantage so one is this now from the classical problem I will just use it k_b which is the free wave number in the plate $k_b = \left(\frac{m\omega^2}{EI}\right)^{1/4}$.

This is the free in vacuo flexural wave the number in the plate. So if I divide by EI what do I get? Let us see so I get

$$EI\left({k_x}^4 - \frac{m\omega^2}{EI}\right) = -\frac{\omega^2\rho}{k_y}\cot\left(k_ya\right).$$

ρ is the acoustic fluid density. Now if I take EI to the other side I get

$$k_x{}^4 - k_b{}^4 = -\frac{\omega^2 \rho}{EIk_y} \cot(k_y a).$$

So, if I multiple by m and divided by m let us say then this term here is $k_b{}^4$. So, what do I get? I get I multiply by one extra a and therefore I divided by one extra a. So how does it look?

$$k_x{}^4 - k_b{}^4 = -\frac{m\omega^2 \rho a}{mEIk_y a} \cot(k_y a),$$

$$k_x{}^4 - k_b{}^4 = -\frac{k_b{}^4 \rho a}{mk_y a} \cot(k_y a).$$

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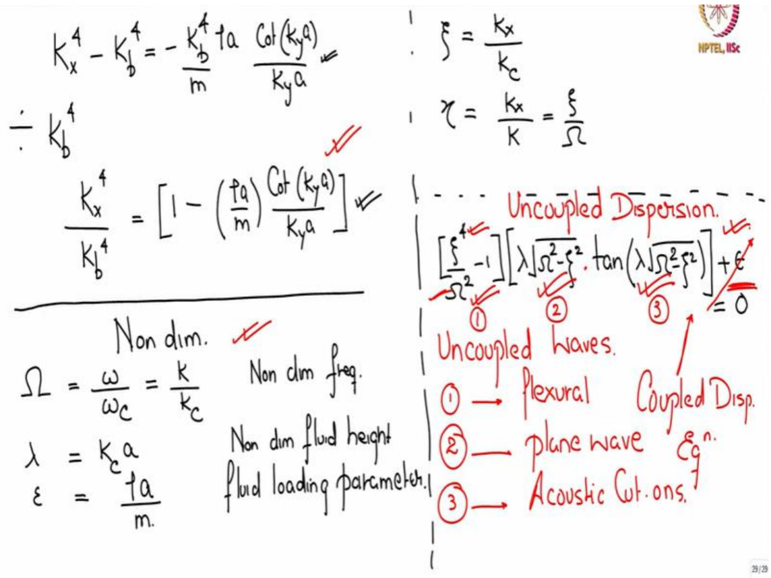

It looks likes like this

$$k_x{}^4 - k_b{}^4 = -\frac{k_b{}^4 \rho a}{m} \frac{\cot(k_y a)}{k_y a}.$$

Now we can divide by $k_b{}^4$. So, then what do we have I can write this equation as

$$\frac{k_x{}^4}{k_b{}^4} = \left[1 - \left(\frac{\rho a}{m}\right) \frac{\cot(k_y a)}{k_y a}\right],$$

so, this is one level now we will do a non dimensionalization so how do we do that?

We have we will bring in a non-dimensional Ω which is $\frac{\omega}{\omega_c}$ which is wave number acoustic by the coincidence wave number.

$$\Omega = \frac{\omega}{\omega_c} = \frac{k}{k_c}.$$

So, this will be the non-dimensional frequency then we will bring a λ we will call it $k_c a$. So, this is a non-dimensional fluid height will bring in an ϵ which is fluid density height by the mass per unit are of the panel which is the fluid loading parameter is very important parameter.

$$\epsilon = \frac{\rho a}{m}.$$

Then this ξ I think is the coupled wave number divided by the wave number at coincidence$\left(\xi = \frac{k_x}{k_c}\right)$ then we have χ which is the coupled wave number to the acoustic wave number which is also ξ over Ω.

$$\chi = \frac{k_x}{k} = \frac{\xi}{\Omega}.$$

So, these are the non-dimensional parameters if we use these non-dimensional parameters. And we non-dimensionalize this equation over here we will get this equation

$$\left[\frac{\xi^4}{\Omega^2} - 1\right]\left[\lambda\sqrt{\Omega^2 - \xi^2}\tan\left(\lambda\sqrt{\Omega^2 - \xi^2}\right)\right] + \epsilon = 0.$$

I will show you how this happens it is very simple nothing to it and in a way if you had acoustics if you should be able to do this, but I will substitute this non-dimensional parameters in this equation and get you to see that this happens over here. I will just say a few words and stop because time is running out. So, ϵ is the fluid loading parameter that means the parameter that says that this structure is contact with the fluid influenced by the fluid or fluid is in contact with the structure influenced by the structure.

So, if ϵ is set to 0 that means there is no coupling this is an equation which is an uncoupled dispersion equation. So, when you said this to 0 when it is happening here what do you expect? You expect to see uncoupled waves so you can see that over here either this term has to be 0 or this term has to be 0 or this tan term has to be 0 only then the right is to be 0. If you said this let us, call it ($\left[\frac{\xi^4}{\Omega^2} - 1\right]$) 1 , this ($\lambda\sqrt{\Omega^2 - \xi^2}$) is 2 and this ($\tan\left(\lambda\sqrt{\Omega^2 - \xi^2}\right)$) is 3 if you set 1 to 0 you will see the flexural wave uncoupled flexural wave in the plate.

You can already see the square on the omega and fourth power on the wave number this flexural wave. If you set 2 to 0 you will get the plane wave you will get the plane wave in the sound field and if you set 3 to 0 you will get the acoustic cut ons. So, by setting ϵ to 0 you see the uncoupled waves and therefore when you have an ϵ what would be the coupled waves, so this equation tells you that this is now with non-zero ϵ the coupled dispersion equation. So, we will see more about this in the next class thank you.