Sound and Structure Vibration
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Module No # 04
Lecture No # 18
The Coupled Partial Differential Equation

Good morning and welcome to this next lecture on Sound and Structural Vibration.

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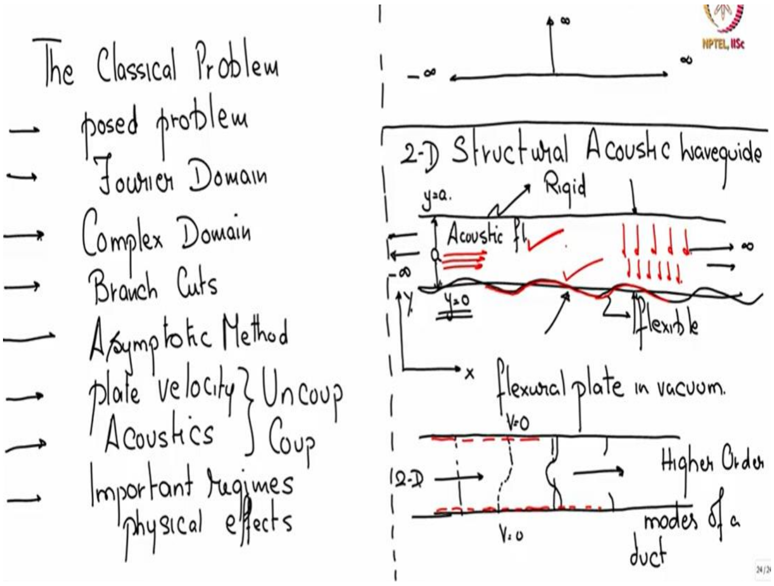

So far in so many lectures we have seen the classical problem as I call it, we posed the problem. We went into the Fourier Domain we decided to do the work in Complex Domain. So, we learnt some roles in the Complex Domain we used branch cuts we learnt about Asymptotics**.** We looked at the plate velocity and the related acoustics uncoupled and coupled.

And we understood the important frequency regimes for different physical effects. I am writing this because this kind of gives you the gamut that is involved typically in sound and structural vibration problems**.** Now the problem we looked at was infinite dimensional the panel was infinite dimensional the sound field half space was infinite dimensional I mean extends to infinity.

So, we are going to look at the next problem now, the next problem is this I will call it a 2-D Structural Acoustic Wave guide**.** What is it the geometry is this I have a 2-D channel of acoustic fluid there is acoustic fluid here and this channel extends to ∞ in both the directions and has a height a**.** Now the top is rigid this surface is rigid and bottom surface now can carry a flexural wave its flexible bottom is flexible.

So, from the earlier problem here one dimension is bounded in this direction it is bounded in the other direction it is ∞. But in this vertical direction is bounded and let me say my coordinate system happens to be x in this direction y in direction here this is my $y = 0$ the bottom is my $y = 0$ top is y = a**.** So, what does what is going to be revealed or what is interesting about this problem.

The fact is that if this flexural plate the bottom plate was placed in vacuum, then it will carry a flexural wave unaffected by any fluid. On the other hand, if there was a rectangular 2-D rigid duct then you will have various waves propagating you will have a plane wave propagating then you will have the next cut on propagating which has $zero$ velocity at the walls.

And then you will have the next level wave cut on propagating and so forth and all higher orders which all start with zero velocity here will be propagating**.** These are higher order Modes of a duct so this comes from acoustics which I am not going to repeat one should know this from acoustics. So uncoupled from the other medium the flexural plate placed in vacuum or this acoustic fluid with rigid walls being rigid they behave uncoupled in isolation**.**

But now we are going to place an acoustic fluid in contact with a flexible plate. So, the fluid will see now a not a rigid boundary now, but it will see a flexible boundary. Similarly, this flexing plate will see acoustic fluid applying pressures to it**.** So, this is a coupled problem that means as the flexural wave propagates the pressures in the fluid will be applied back. They will influence and change the behaviour of the plate.

And the acoustic fluid carrying its own cut on waves plane wave and higher order modes will see the flexing of a nearby boundary and so their behaviour is going to be modified. So, each medium will modify the behaviour of the other, so this is a coupled problem, so this is going to be interesting to look at**.**

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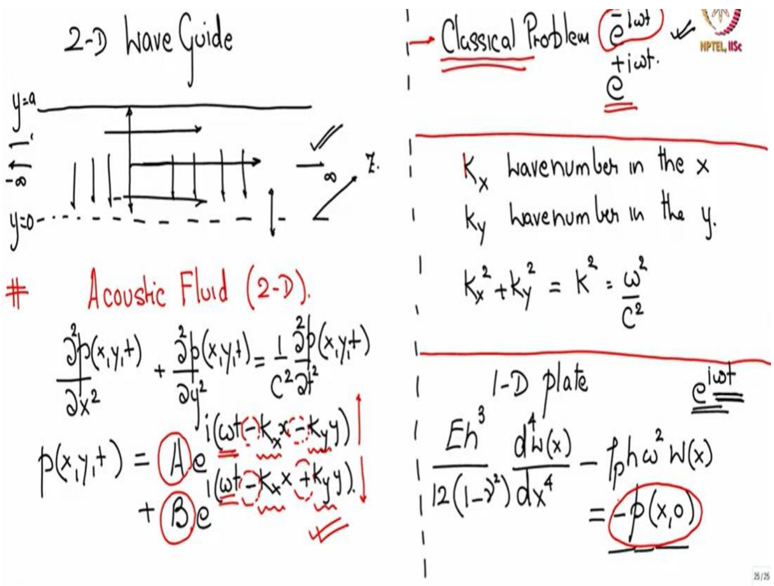

So let use see so again this is my geometry here 2-D wave guide. So top is a rigid plate bottom is a flexible plate my y begins here y is 0 and $y = a$, and they go off to ∞ in both directions. So, we will formulate the equations for this system now the so let us see now the acoustic fluid it is a 2-D domain. And so, the related acoustic wave equation in pressure is,

$$\frac{\partial^2 P(x,y,t)}{\partial x^2} + \frac{\partial^2 P(x,y,t)}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 P(x,y,t)}{\partial t^2}.$$

So, what is the general solution to this? We are looking at harmonic sound fields that was the statement I made right in the beginning. So, if this system is somehow driven at a harmonic single frequency the general solution is this

$$P(x,y,t) = A e^{i(\omega t - k_x x - k_y y)} + B e^{i(\omega t - k_x x + k_y y)}.$$

So, I am looking at waves that are propagating back and forth up and down and propagating in one direction that is adequate.

So, this is not the most general in that sense but that is adequate so far off on the left there is some source which is localized source which disturbs this system. And so now the waves are propagating in the positive x direction and they are propagating up and down in the y direction. So that is this solution.

So, this minus sign here and here says that the waves are moving towards the right and this minus sign says they are moving towards the positive y direction and this plus says the waves are moving in the negative y direction. And one more statement I should make is that the classical problem that we did I use the $e^{-i\omega t}$ convention. So approximately half the world uses this $-i\omega t$ convention and the other half uses the $e^{+i\omega t}$ convention.

So unfortunately, it is confusing for the students but what to do and the reason I use the $-i\omega t$ for the classical problem is the entire literature which is very rich and studies this problem uses this $-i\omega t$**.** So, if I now suddenly change the notation the student will go when here, reads the paper this $+i\omega t$ and that difference makes a lot of difference in the branch cuts.

So, it will cause more confusion than use so I kept it that way. So, if you have looked at that problem the way I presented and then you go look at the literature you will not be confused about the notation. Whereas I myself I am comfortable with $+i\omega t$ so the rest of the problems I will use $+i\omega t$**.** In the long run if you understand what is happening there is no problem you can do it both ways after you understand.

But at the learning stage these are very cumbersome if the notation is changed. So, this problem, I am going to do with $+i\omega t$ convention**.** So, now having said that so, we have this solution for the pressure field and where let us see k_x is the wave number in the x direction and k_y is the wave number in the y direction**.** So that now you should know ${k_x}^2 + {k_y}^2$ should be equal to the acoustic wave number square which is $\frac{\omega^2}{c^2}$ and c is the speed of sound**.**

So that is for now the information about the acoustic fluid and A and B are unknowns. There are more unknowns here but A and B are unknowns, frequency ω is known**.** So, k_x and k_y are still undecided now what about the panel so or what about the plate that is the boundary? The plate it is a, 1D plate as before so it is a 1D plate that is the boundary so what is the equation governing that.

It is

$$\frac{\mathrm{E}h^3}{12(1-\nu^2)}\frac{\mathrm{d}^4\, w(x)}{\mathrm{d}x^4} - \rho_p h\omega^2 \mathrm{w(x)} = -p(x,0).$$

So, the time behaviour has been removed because it is a linear system and every variable vibrates at ω that has been removed. And so now because the y dependence what I mean is the z dependence for the plate in the other direction**.** The z dependence in the other direction is not there so it is just dependent on x so I have written it accordingly, the time has been removed because its harmonic.

The z dependence has been removed because it is the way we have posed it and so the displacement w of the bottom boundary is the sole function of x so this is the equation. And now there is a pressure that is applied on it from the fluid**.** So, there is pressure acting on the plate bottom plate due to the acoustic fluid that is this term here**.**

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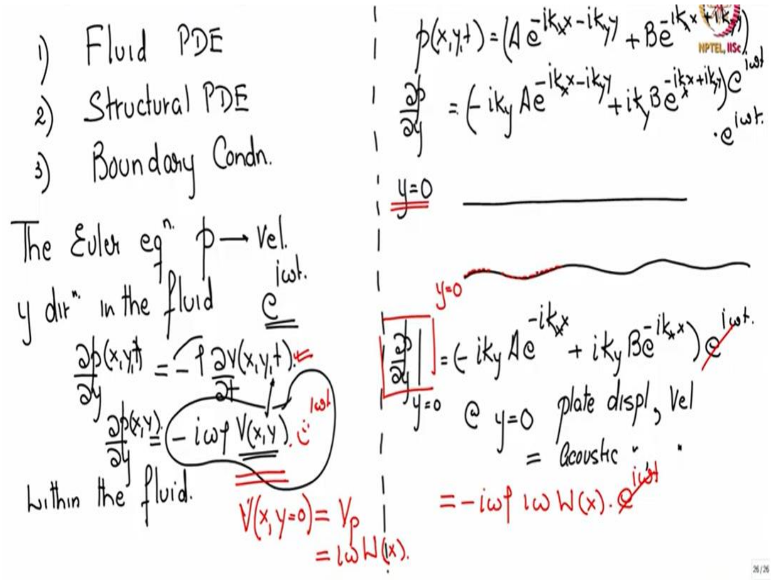

Next now we know that we have two equations the one the fluid equation or the fluid PDE and then we have the structural PDE**.** Now we need to connect the 2 which is a boundary condition**.** Now first of all the Euler equation which is our friend, and we want to relate pressure to velocity is let us say in the y direction in the fluid

$$\frac{\partial P(x,y,t)}{\partial y} = -\rho \frac{\partial V(x,y,t)}{\partial t}.$$

But as I say my time dependence is always this so if I rewrite pressure as some special pressure into $e^{i\omega t}$ and velocity as some special velocity and some this $e^{i\omega t}$ then $e^{i\omega t}$can be removed**.** So now what I have is? This

$$\frac{\partial p(x,y)}{\partial y} = -i\omega\rho v(x,y).$$

So, the upper V is v times $e^{i\omega t}$ so please get used to this otherwise like I have to keep writing too much**.**

$$V(x,y,t) = v(x,y)e^{i\omega t}.$$

So here what we are saying is this v is the coefficient of the time term. So, this is the Euler equation valid within the fluid**.** So, what is next, the next is we will try to find out what my P let me rewrite because its

$$P(x,y,t) = Ae^{i(\omega t - k_x x - k_y y)} + Be^{i(\omega t - k_x x + k_y y)}.$$

So, what is $\frac{\partial P}{\partial y}$? So that is

$$\frac{\partial P}{\partial y} = (-ik_y A e^{-ik_x x\,-\,\mathrm{i}k_y y} + ik_y B e^{-ik_x x\,+\,\mathrm{i}k_y y})e^{i\omega \mathrm{t}}.$$

Now this suppose we evaluate this at $y = 0$ why do we do that we do that because in the wave guide the bottom plate velocity and the acoustic fluid particle velocity that is contiguous with the vibrating plate must be same. So, this is the $y = 0$ position so we evaluate the Euler equation at $y = 0$.

So,

$$\left.\frac{\partial P}{\partial y}\right|_{y=0} = (-ik_y A e^{-ik_x x} + ik_y B e^{-ik_x x})e^{i\omega \mathrm{t}}.$$

Now this is equal to this term over here or this term over here. Now what we say is that at $y = 0$ the panel displacement or plate displacement and velocity are equal to acoustic particle displacement and velocity. So, this term here which is acoustic particle velocity when we write it as acoustic particle velocity at $y = 0$ is the equal to the plate velocity.

Let us call it v_p and the plate velocity = $i\omega w(x)$. So, I have

$$\left.\frac{\partial P}{\partial y}\right|_{y=0} = -i\omega\rho i\omega w(x)e^{i\omega t}$$

So, this is what I have so if I rewrite this what I have is?

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$$\omega^2 \rho w(x) = (-ik_y A + ik_y B)e^{-ik_x x}.$$

So, we are at the end of the time frame so I will stop over here, and I will continue with this portion in the next class thank you.