Sound and Structural Vibration
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Module No # 04
Lecture No # 17
The 2-D Structural-Acoustic Waveguide

Good morning and welcome to this next lecture on sound and structural vibration so we were looking at far field or we were going to look at far field.

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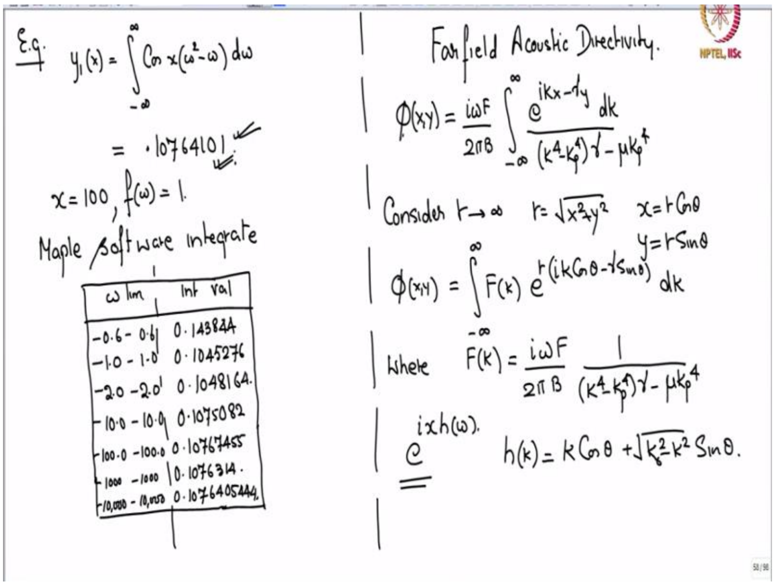

Acoustic directivity and I said the integral we were interested in is

$$\phi(x, y) = \frac{i\omega F}{2\pi B} \int_{-\infty}^{\infty} \frac{e^{ikx-\gamma y}\, dk}{(k^4 - {k_p}^4)\gamma - \mu {k_p}^4}.$$

So, we will consider r tending to ∞ and what is r? r = $\sqrt{x^2 + y^2}$ where x is $r\cos\theta$ and y is $r\sin\theta$. Now

$$\phi(x, y) = \int_{-\infty}^{\infty} F(k)\, e^{r(ik\ \cos\theta - \gamma\sin\theta)}\ \ dk\ ,$$

where

$$F(k) = \frac{i\omega F}{2\pi B} \frac{1}{(k^4 - {k_p}^4)\gamma - \mu {k_p}^4}.$$

So, this is the integral now we have seen that in this method of stationary phase we have this function $e^{ixh(\omega)}$ time we saw. So that function for us here is

$$h(k) = k\cos\theta \; + \sqrt{{k_0}^2 - k^2}\,\sin\theta\,.$$

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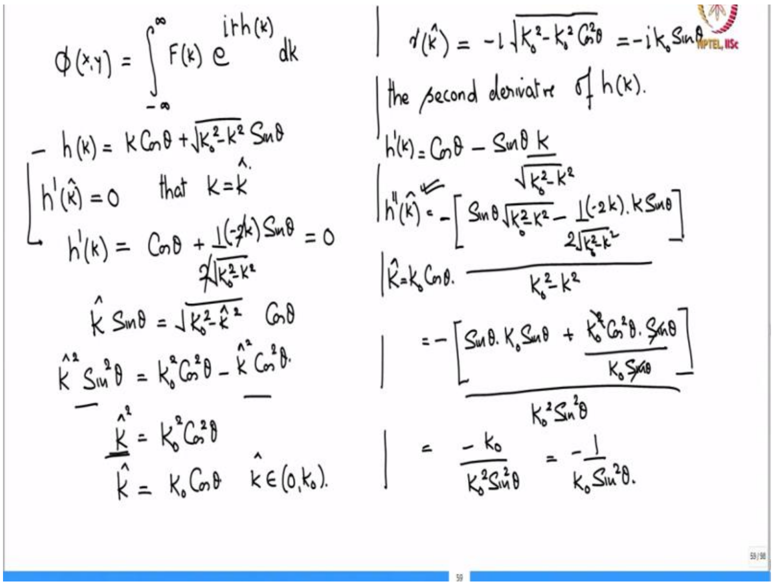

So let us see so my $\phi(x,y)$ written in the stationary phase nomenclature

$$\phi(x,y) = \int_{-\infty}^{\infty} F(k)\, e^{irh(k)}\; dk.$$

Where let me just repeat $h(k) \;=\; k\cos\theta \;+ \sqrt{{k_0}^2 - k^2}\,\sin\theta$. So now we need to find a stationary phase point that means a point where $h'(k) = 0$. We need to find that k value will call it $\hat{k}$ where $h'(\hat{k})$ is 0.

So, $h'(k) \;= \cos\theta + \frac{(-2k)\sin\theta}{2\sqrt{{k_0}^2-k^2}} = 0$. So that means my $\hat{k}\;\sin\theta = \sqrt{{k_0}^2 - \hat{k}^2}\;\cos\theta$. We square both sides if we square both sides. We get

$$\hat{k}^2\sin^2\theta = {k_0}^2\cos^2\theta\; - \hat{k}^2\cos^2\theta.$$

So, if we combine this and this, we get $\hat{k}^2 = {k_0}^2\,\cos^2\theta$. or $\hat{k} = k_0\;\cos\theta$ that means $\hat{k}$ is within 0 and k_0. So, this has 2 roots one positive going one negative going we are considering the positive going root. So, what is $\gamma(\hat{k})$ now

$$\gamma(\hat{k}) = -i\sqrt{{k_0}^2 - {k_0}^2\;\cos^2\theta}\;\; = \; -ik_0\sin\theta.$$

Now the second derivative of $h(k)$ so let us see

$$h'(k) = \cos\theta - \sin\theta \frac{k}{\sqrt{{k_0}^2 - k^2}},$$

$$h''(k) = -\frac{\left[\sin\theta \sqrt{{k_0}^2 - k^2} - \frac{1(-2k)\, k \sin\theta}{2\sqrt{{k_0}^2 - k^2}}\right]}{{k_0}^2 - k^2}.$$

I am evaluating at $\hat{k} = k_0 \cos\theta$, so

$$h''(\hat{k}) = \frac{-\left[\sin\theta\, k_0 \sin\theta + \frac{{k_0}^2 \cos^2\theta \sin\theta}{k_0 \sin\theta}\right]}{{k_0}^2 \sin^2\theta}$$

$$= \frac{-k_0}{{k_0}^2 \sin^2\theta},$$

$$= \frac{-1}{k_0 \sin^2\theta}.$$

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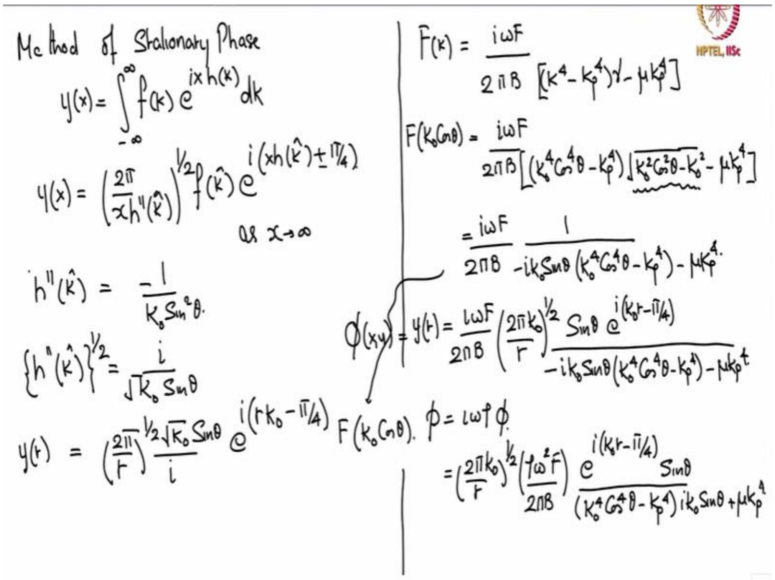

So, the formula let us see if we had

$$y(x) = \int_{-\infty}^{\infty} f(k)\, e^{ixh(k)}\, dk,$$

$$y(x) = \left(\frac{2\pi}{x\left|h''(\hat{k})\right|}\right)^{1/2} f\left(\hat{k}\right) e^{i(xh(\hat{k}) \pm \pi/4)}\ as\ x \longrightarrow \infty.$$

So, in this case here my $h''(\hat{k})$ turned out to be $\frac{-1}{k_0 \sin^2\theta}$.

So,

$$\left\{h''(\hat{k})\right\}^{1/2} = \frac{i}{\sqrt{k_0}\ \sin\theta}.$$

so, what I have is if I plug my expression in

$$y(r) \ = \ \left(\frac{2\pi}{r}\right)^{1/2} \frac{\sqrt{k_0}\ \sin\theta}{i}\ e^{i(rk_0-\pi/4)}F(k_0\cos\theta).$$

So, if we rewrite this now

$$F(k) = \frac{i\omega F}{2\pi B\left[(k^4 - {k_p}^4)\gamma - \mu {k_p}^4\right]}.$$

So,

$$F(k_0\cos\theta) = \frac{i\omega F}{2\pi B\left[({k_0}^4\cos^4\theta\ - {k_p}^4)\sqrt{{k_0}^2\cos^2\theta - {k_0}^2}\ - \mu {k_p}^4\right]},$$

$$= \frac{i\omega F}{2\pi B}\frac{1}{-ik_0\sin\theta\left({k_0}^4\cos^4\theta - {k_p}^4\right) - \mu {k_p}^4}.$$

So, if this is plugged back in here what is the total expression y at r going to be equal to.

Or let me write it as

$$\phi(x,y) = y(r) \ = \frac{i\omega F}{2\pi B}\left(\frac{2\pi k_0}{r}\right)^{1/2}\frac{\sin\theta\ e^{i(k_0r-\pi/4)}}{-ik_0\sin\theta\ \left({k_0}^4\cos^4\theta - {k_p}^4\right) - \mu {k_p}^4}.$$

This is actually $\phi(x,y)$ so from here if we have to make a pressure out of it then just multiply by $i\omega\rho$ times ϕ.

$$p = i\omega\rho\phi.$$

So, if we do that my pressure is equal to

$$= \left(\frac{2\pi k_0}{r}\right)^{1/2}\left(\frac{\rho\omega^2 F}{2\pi B}\right)\frac{e^{i(k_0r-\pi/4)}\sin\theta}{\left({k_0}^4\cos^4\theta - {k_p}^4\right)ik_0\sin\theta + \mu {k_p}^4}.$$

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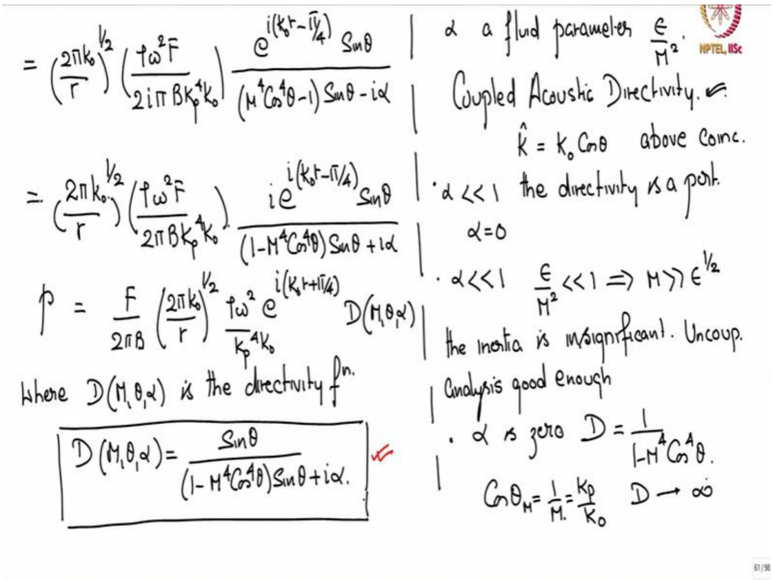

We will further write this as

$$p = \left(\frac{2\pi k_0}{r}\right)^{1/2}\left(\frac{\rho\omega^2 F}{2i\pi B {k_p}^4 k_0}\right)\frac{e^{i(k_0 r-\pi/4)}\sin\theta}{(M^4\cos^4\theta-1\,)\sin\theta-i\alpha}.$$

$$= \left(\frac{2\pi k_0}{r}\right)^{1/2}\left(\frac{\rho\omega^2 F}{2\pi B {k_p}^4 k_0}\right)\frac{ie^{i(k_0 r-\pi/4)}\sin\theta}{(1-M^4\cos^4\theta\,)\sin\theta+i\alpha}.$$

Now we write

$$p = \frac{F}{2\pi B}\left(\frac{2\pi k_0}{r}\right)^{1/2}\frac{\rho\omega^2}{{k_p}^4 k_0}\;e^{i(rk_0+\pi/4)}D(M,\theta,\alpha)\,.$$

where $D(M,\theta,\alpha)$ is the directivity function given by

$$D(M,\theta,\alpha) = \frac{\sin\theta}{(1-M^4\cos^4\theta\,)\sin\theta+i\alpha}.$$

So α is also a fluid loading parameter which turns out to be $\frac{\epsilon}{M^2}$.

So, in that sense α carries frequency in it so now we will examine this directivity function to discuss the coupled acoustic directivity. We should note that we are talking $\hat{k} = k_0\cos\theta$ which means. We are above coincidence so below coincidence there is no sound there is no wave number that can generate sound. We are talking about coincidence now if you look at this factor which is called directivity for α much less than 1 the directivity is a perturbation to $\alpha = 0$.

α much less than 1 means, $\frac{\epsilon}{M^2}$ much less than 1 which implies M much greater than $\varepsilon^{1/2}$ which we have seen. In this range the fluid loading or inertial loading is insignificant that means uncoupled analysis is good enough. Now when α is 0 we get the directivity D is given by $\frac{1}{1-M^4\cos^4\theta}$ and M is greater than 1.

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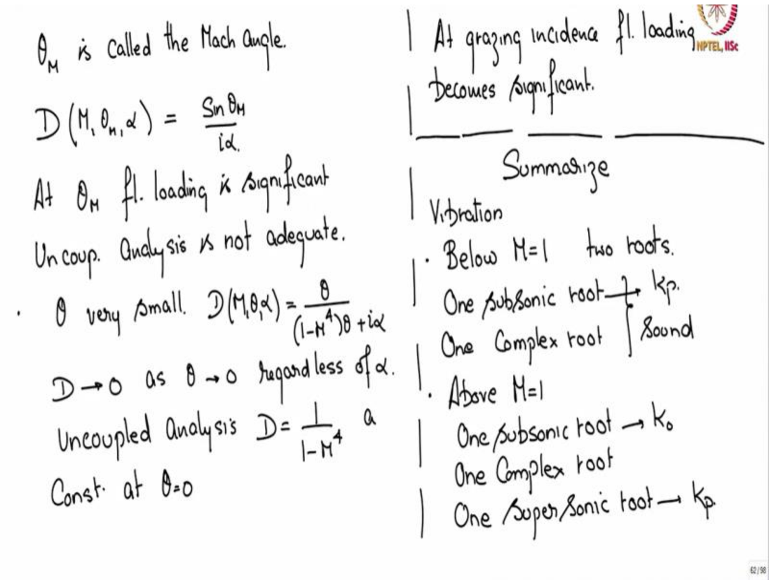

And therefore, there is an angle $\cos(\theta_M) = \frac{1}{M} = \frac{k_p}{k_0}$ where D tends to ∞ this angle θ_M is called the Mach angle. Now at the Mach angle if now we look at the D factor with fluid loading then becomes finite so there is a drastic change. So, at Mach angle fluid loading is significant which means uncoupled analysis is not good enough not adequate you have to do coupled analysis. Next for θ very small that means close to grazing you can see that

$$D(M, \theta, \alpha) = \frac{\theta}{(1 - M^4)\,\theta + i\alpha}.$$

So D tends to 0 as θ tends to 0 regardless of α whereas in the uncoupled analysis D = $\frac{1}{1-M^4}$, a constant at θ = 0. Which means at θ = 0 at grazing incidence again fluid loading significant. So that is as far as coupled fluid directivity.

Now let me just summarize this here summarize the whole situation in vibration part below M = 1 we get 2 roots one is a subsonic root one is a complex root neither produces sound. Then above coincidence we have one subsonic root so let us say this subsonic root is near k_p. We get one subsonic root that is near k_0 and we get again that one complex root then we get one supersonic root which is near k_p that is called the leaky pole.

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Called the leaky that responsible for sound.

- B_1 B_2 Branch line integrals.

$$\underline{\underline{e^{ik_0x}}} \quad > O(x^{-3/2}).$$

- Sound already discussed.

That is responsible for sound other than that from the branch line integrals the B_1 and B_2 branch line integrals, we get a vibration contribution which has this as the dominant propagator, and it decays as $x^{-3/2}$. And the sound part we have seen just now already discussed so with this I close this topic of this classical problem. So, there is a lot of new and deep material in it I hope it is useful thank you.