Sound and Structure Vibration
Prof. Venkata Sonti
Department of Mechanical Engineering
Indian Institute of Science - Bangalore

Module No # 04
Lecture No # 16
The Coupled Acoustic Field and Stationary Phase

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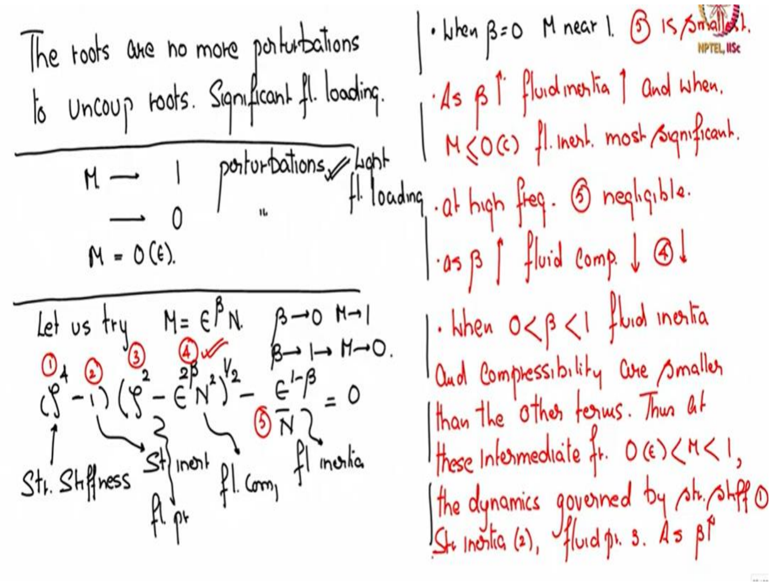

Good morning and welcome to this next lecture we were looking at frequency regimes M being my non dimensional frequency we were trying to see if the roots close to 1. But just less than 1 was still perturbations to the original roots similarly, we try to see for frequency values very close to 0 if the roots were perturbations. For values of M close to 1 we found the roots were perturbations to the uncoupled roots so we will call it still the light fluid loading regime.

But that was not the case when M was order ϵ we could not balance the coupled dispersion equation. So let us try this $M = \epsilon^{\beta} N$, I am putting N but notionally $N = 1$. So, if β moves towards 0, M moves close to 1 if β moves towards any positive value 1 or any higher value then M starts to move towards 0. So let us substitute this if we do that, we have

$$(\zeta^4 - 1)(\zeta^2 - \epsilon^{2\beta} N^2)^{1/2} - \frac{\varepsilon^{1-\beta}}{N} = 0.$$

If you recall we had derived the physical significance of each of these terms this (ζ^4) had come from structural stiffness, this (1) had come from structural inertia, this (ζ^2) had come from fluid pressure this ($\epsilon^{2\beta} N^2$) from fluid compressibility and this ($\frac{\varepsilon^{1-\beta}}{N}$) was denoting fluid inertia.

Now you can see that when β = 0 that means M is just near 1 less than 1. Let us number this just number it $1, 2, 3, 4, 5$. So 5 is the smallest inertia is the smallest fluid inertia is the smallest. Next as β increases that means M is starting to come down fluid inertia starts to dominate I will denote it start going up. And when M is less than equal to order ϵ fluid inertia is most significant at high frequencies, 5 is negligible**.**

Now as β goes up fluid compressibility goes down that means 4 goes down, term four comes down which is seen from here. When $0 < \beta < 1$ fluid inertia and compressibility are smaller than the other terms**.** Thus, at these intermediate frequencies that is $O(\varepsilon) < M < 1$ the dynamics is governed or dominated by structural stiffness which is 1 structural inertia which is 2 and fluid pressure is 3**.**

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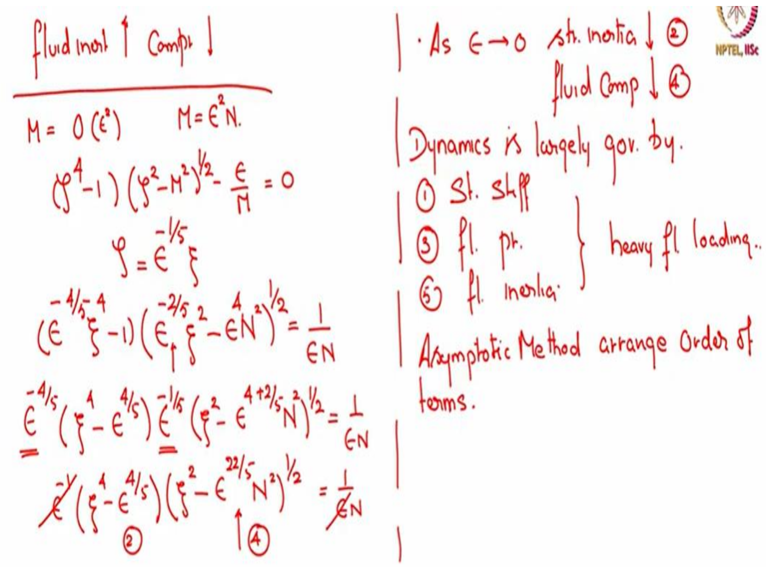

And within this range as β goes up. The fluid inertia starts to dominate, and compressibility comes down. So let us try a further closer range that is M is equal to order ϵ^2 we will try this range. So, we will make this M as $\epsilon^2 N$ again where N is an order 1 quantity. So, we have our friendly coupled dispersion equation $(\zeta^4 - 1)\sqrt{\zeta^2 - M^2} - \frac{\epsilon}{M} = 0$.

In addition, from some prior knowledge I will replace ζ by $\epsilon^{-1/5}\xi$ this is after some trial and error this balance or idea has come. So, if we now do this what do I get? I get

$$(\epsilon^{-4/5}\xi^4 - 1)(\epsilon^{-2/5}\xi^2 - \epsilon^4 N^2)^{1/2} = \frac{1}{\epsilon N},$$

$$\epsilon^{-4/5}(\xi^4 - \epsilon^{4/5})\epsilon^{-1/5}(\xi^2 - \epsilon^{4+2/5}N^2)^{1/2} = \frac{1}{\epsilon N},$$

$$\epsilon^{-1}(\xi^4 - \epsilon^{4/5})\,(\xi^2 - \epsilon^{22/5}N^2)^{1/2} = \frac{1}{\epsilon N}\,.$$

Now as ϵ tends to 0 you can see that structural inertia loses its dominance, so this is structural inertia term 2 which is term 2. And then fluid compressibility also starts to lose this is the term 4. And the dynamics is largely governed by one which is structural stiffness by term 3 which is fluid pressure and 5 which is fluid inertia. So, this region is heavy fluid loading.

Now so this gives you an idea of how to you know use asymptotic how to use asymptotic or the asymptotic method to play with order of the terms. Use an asymptotic method to arrange order of the terms in order to examine physical influences. So, this sort of what shall I say game can be played again and again to get finer and finer ideas about relative importance of these 5 terms.

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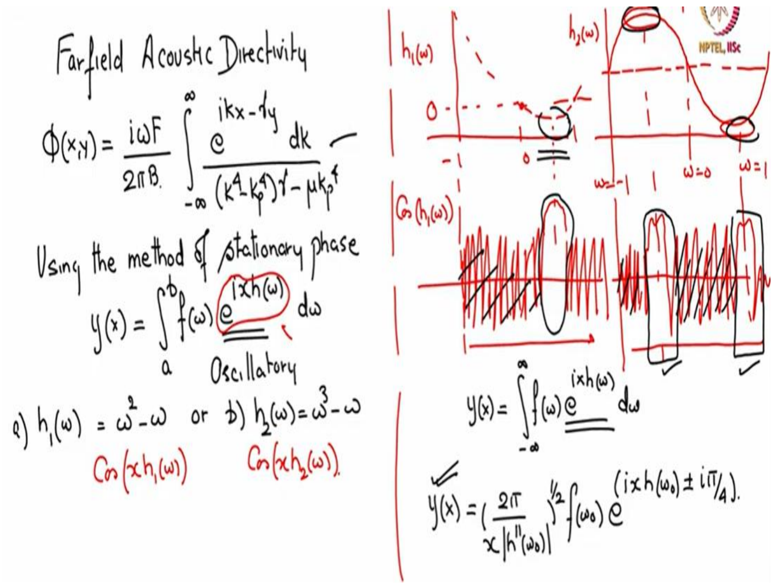

The next topic is Far field acoustic directivity. We have seen the uncoupled sound field, so we are going to look at the coupled sound field. Now if you recall

$$\phi(x,y) = \frac{i\omega F}{2\pi B}\int_{-\infty}^{\infty} \frac{e^{ikx-\gamma y}\,dk}{(k^4 - {k_p}^4)\gamma - \mu {k_p}^4}.$$

Now we are going to look at this or compute the integral using the method of stationary phase.

It is an amazing technique where a function oscillates in some places of the range and does not oscillate in some places of the range. So, this method of stationary phase is essentially I have an integral of this form

$$y(x) = \int_a^b f(\omega)\, e^{ixh(\omega)}\, d\omega \,.$$

Now this exponent this term can be either cosine or sine so it can be oscillatory can be heavily oscillatory.

For example, $h_1(\omega)$ I will take the first let us say equal to $\omega^2 - \omega$**.** Or another example I will take which is $h_2(\omega)$ is equal to $\omega^3 - \omega$**.** For the moment we are taking an example for method of stationary phase which we will apply here. So, for the moment this part this expression I will consider to be a cosine for making life easy.

So, it is $\cos(xh_1(\omega))$ in this case and $\cos(xh_2(\omega))$ here in this case which will go in here**.** Now these functions $h_1(\omega)$ and $h_2(\omega)$ how do they look? They look like this so $h_1(\omega)$ looks like this so – 1, 0, 1. So, it goes through 0 this is 0 some value whatever value is here it goes through 0 at 0 and it goes through 0 at 1. The next function $h_2(\omega)$ which is cubic in ω so it will go through 0 three places at $\omega = 0$, $\omega = -1$ and $\omega = 1$ so it looks like this.

Looks like this now correspondingly if I plot the $\cos(xh_1(\omega))$. It looks crazy it looks like this here and as we approach this minimum here it will actually let me mark it somewhere. So, it will come, and it will slow down, and it will start again going up. Similarly for this one here if we plot so here is a maximum here is a minimum. If we plot it will go oscillatory and then it will slow down**.**

So now the idea is if we integrate over this ω range here or here if we integrate then these regions are highly oscillatory, and they will cancel out the neighbouring regions cancel out positives and negatives. The contribution will come mainly from this region where this phase factor had a minimum or a maximum**.** So here also this phase has a maximum here it has a minimum over here.

And here the function slows down in its oscillations everywhere else the oscillations cancel each other. So, if you integrate the contribution will come from here and here that is the idea of the method of stationary phase. So now what is the statement if you have a function like this

$$y(x) = \int_{-\infty}^{\infty} f(\omega)\, e^{ixh(\omega)}\, d\omega\,,$$

$$y(x) \;=\; \left(\frac{2\pi}{x|h''(\omega_0)|}\right)^{1/2} f(\omega_0)\, e^{(ixh(\omega_0) \pm i\pi/4)}.$$

The plus or minus is chosen as whether it is a maximum or a minimum. So, this is the answer there is no more integral. So, I will show you an example using one of these functions that I have chosen.

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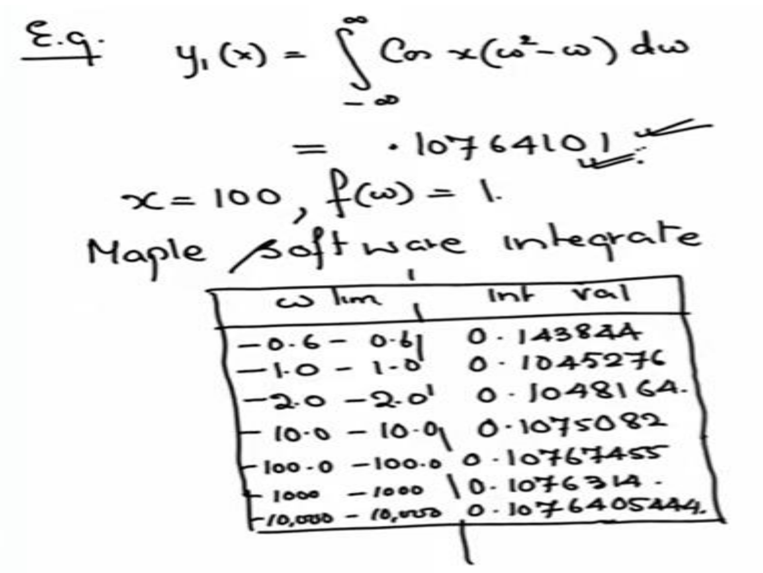

So, consider this function here which is

$$y_1(x) \;=\; \int_{-\infty}^{\infty} \cos x(\omega^2 - \omega)\; d\omega.$$

So, if I use the method of stationary phase I get an answer of 0.10764101 etc. And I choose x = 100 and $f(\omega)$ is 1 so this is method of stationary phase answer. Then I use some software say Maple software to integrate numerically so I will show you the numerical values.

So, the ω limits and then integral value so let us see

ω limits	Integral value
$-0.6\ -\ 0.6$	0.143844
$-$ 1 - 1	0.1045276
$-$ 2.0 - 2.0	0.1048164
$-\ 10\ -\ 10$	0.1075082
-100 - 100	0.10767455
$-$ 1000 -1000	0.1076314
-10000 - 10000	$0.1076405.$

So, you can see and see compare this with value so that is the method of stationary phase.

So, we are going to use this method to compute our acoustics integral we are run out of time so I will stop here and continue next class thanks.