Sound and Structural Vibration

Prof. Dr. Venkata Sonti
Department of Mechanical Engineering
Indian Institute of Science - Bengaluru

Lecture – 14
Light and Heavy Fluid Loading Continued

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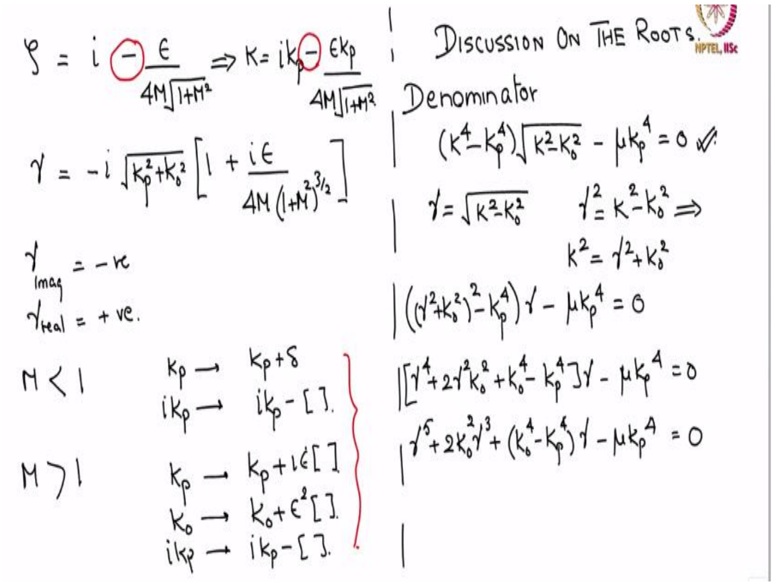

Good morning and welcome to this next lecture on sound and structural vibration. We had reached up till this point where we found coupled roots using asymptotics for M less than 1 and greater than 1. So, this is the picture we have now. Now, today I will give a brief explanation or a discussion on the roots. Now, the denominator in the dimensional form is given by $(k^4 - {k_p}^4)\sqrt{k^2 - {k_0}^2} - \mu {k_p}^4 = 0$.

And $\gamma = \sqrt{k^2 - {k_0}^2}$ and $\gamma^2 = k^2 - {k_0}^2$ which implies $k^2 = \gamma^2 + {k_0}^2$. So, if we write this denominator in terms of γ, so what we get is

$$\left((\gamma^2 + {k_0}^2)^2 - {k_p}^4\right)\gamma - \mu {k_p}^4 = 0,$$

$$[\gamma^4 + 2\gamma^2 {k_0}^2 + {k_0}^4 - {k_p}^4]\gamma - \mu {k_p}^4 = 0,$$

$$\gamma^5 + 2{k_0}^2\gamma^3 + ({k_0}^4 - {k_p}^4)\gamma - \mu {k_p}^4 = 0.$$

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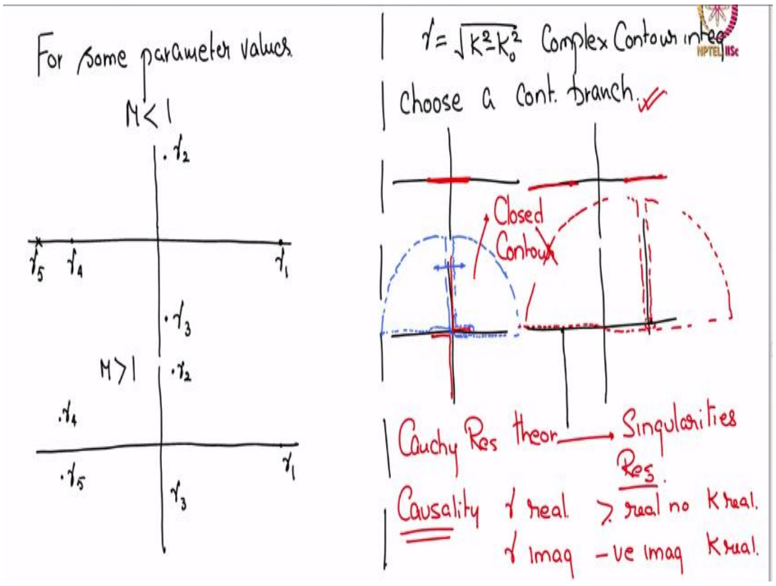

Now for some value of the parameters or some parameter values does not matter, so we have for M less than 1 a root at γ_1 and a root called γ_2, a root called at γ_3 and γ_5 and γ_4; we get these roots. And similarly for M greater than 1, we get a root at γ_1, then we get γ_2 and γ_3 and γ_4 and γ_5; these are the roots we get. Now, what is the thought process here?

If you recall we said that this γ is a $\sqrt{k^2 - {k_0}^2}$ and therefore if we do a complex contour integration then we have to choose a continuous branch for γ and so the branch cuts coming. So, if you recall I have shown you a cut where the cut is between k_0 and $-k_0$, I have shown you another where the cut extends from k_0 to ∞, $-k_0$ to $-\infty$. I have shown you a cut which is an L shaped cut.

So, I come from k_0 just about go vertical, I come from $-k_0$ just below and go down vertical this is an L shaped cut. And I also shown you a cut which goes from k_0 to $+i\infty$, $-k_0$ to $-i\infty$. I have shown you. So, these are first of all branches where the γ is continuous. That means what I just to show you I have to have the real line integral, I may take blue, for this case I come from $-\infty$ above get on to the cut, cross here, get down from the cut, go off to ∞, come over all the way at ∞, hit this line, go vertically down, go right, go around the cut, get down below, cross here, come here all the way and from ∞ I join. There is a sine change of π over here, so I cannot cross. Similarly on this picture here, the picture on the right here I take an integral, I will come all the way above, crossover, go below, go off to $+\infty$, come on an arc at ∞ and this is a cut ,so I cannot cross, I come down, I go down this branch here, go up at an arc of radius ∞ again come down and join. So this is a closed contour and this is a closed contour which we need for Cauchy residue theorem. We need this for the Cauchy residue theorem and after going round closed contours we look at isolated singularities and compute residues. So, finding a continuous branch is one point, the other was causality.

Causality had to be maintained that means if γ is a real number it should be a positive when k is real. Similarly γ if it is imaginary it should be negative imaginary, again when k is real. So, now with these restrictions what did we do?

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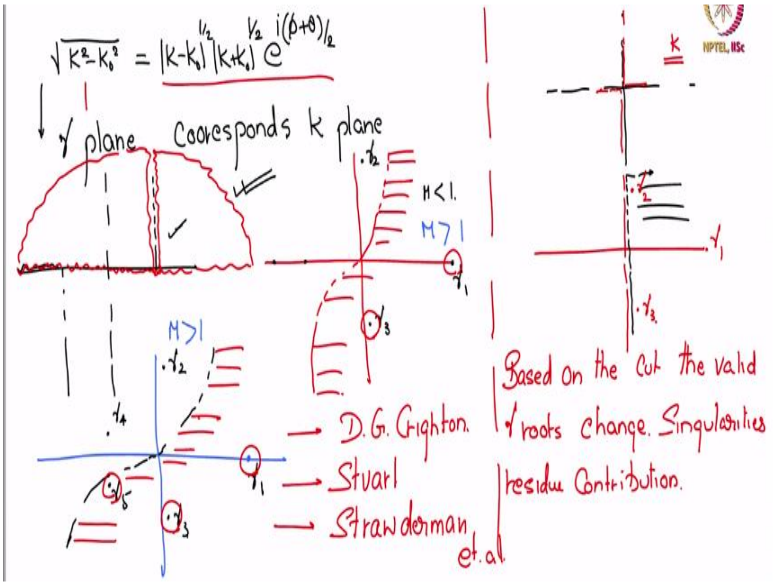

We looked at $\sqrt{k^2 - {k_0}^2}$ and we found it was magnitude $|k - k_0|^{1/2}|k + k_0|^{1/2}e^{i(\phi+\theta)/2}$. So, this gave a γ definition for this gave me a γ plane that corresponds to k plane. That means if I am moving let me just take the last cut, the last cut is here. So, if I am moving here, if I am moving here and crossing if I am moving here, if I am moving here.

And then if I go off at ∞ and come here and I am moving down the cut, going around the cut if I am moving in this manner, if I am moving in this manner, how will I move in the γ plane? Using this definition of k, how will I move in the γ plane? So, it turns out I will move in the γ plane in this fashion. This is my γ plane, I have to be on the right here to the γ plane.

So, now what roots did I get? I got a γ_1 root, I got a γ_2 root, I got a γ_3 root and I got γ_4 and γ_5 roots for M less than 1. For M greater than 1, let me do that, let me do it separately. For M greater than 1, I will get again γ_1, I get γ_2, I get γ_3, and let me write it closer I get a γ_4 and a γ_5. And again if I am moving in this fashion in the k plane with this cut I will get a region in the γ plane where I have to be again to the right here.

So that means now these are the γ roots that are valid whereas these are the γ roots that are valid because they obey the causality. So, now this is one story. The other story is that instead of this cut which has vertical portions if I take the L cut, I use this L cut over here and this is of course the k plane, right. This is k plane and move to the γ plane, then my γ plane happens to be to the right of this entire region which means what my γ_2 gets included, γ_1 gets included, γ_3 gets included.

So, now there is a slight difference based on the cut. Based on the cut the valid γ roots change. What does that do? That changes the singularities and therefore it changes the residue contribution. So now there has been a discussion on this if you look at the literature, the originator is of course D. G. Crighton, his work is the starting point. Then there has been some discussion by authors like Stuart and Strawderman and his coworkers, you can look at the literature.

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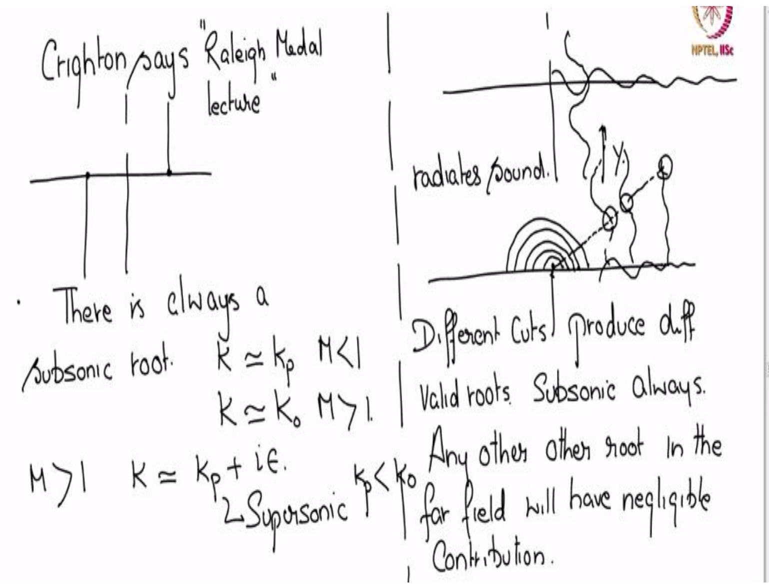

So, finally, we will go with what Crighton says in his Raleigh Medal lecture, it is interesting and one can read and follow that literature, but as a introductory course it is best avoided at this level. So finally we will go with what Crighton says and Crighton takes this cut, Crighton uses this vertical cut. He uses this vertical cut and what he says is that there is always a subsonic root k root and it is that k is near k_p for M less than 1 and that k is near k_0 for M greater than 1.

And then for M greater than 1 there is a root k which is near k_p + a small damping value. So this this root now as it moves on the plate it starts to decay away slowly and because this is supersonic now, this root is supersonic because k_p is less than k_0 it radiate sound and the sound radiation goes up in the y direction, it increases in the y direction, so little counterintuitive. However if this is my plate and this is my line forcing it may be increasing this way.

But as this happens to be the origin of the entire vibration and sound and the propagating wave along the plate is decreasing, so a little to the right the vibration has decreased and the sound field grows. A little further to the right, the vibrations further decrease and sound field grows. But if you look at a line originating from the forcing a radial line you will see that the sound field is decreasing.

So, we will follow and he says that different cuts produce different valid roots but you will find this root that is subsonic always and any other root you find based on your cut in the far field will have negligible contribution.

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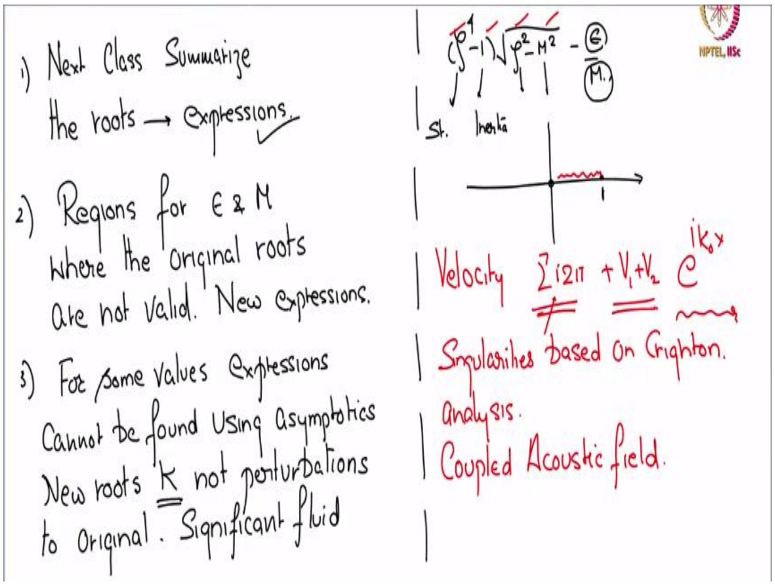

So now I will just give you what we are going to do. I am running out of time. So in the next class I will just summarize the roots that means their expressions that is one thing. Then there are regions for values of ϵ and M where the original roots are not valid, original expressions here are not valid, so we will find new expressions. Then for some values expressions cannot be found using asymptotics that means what?

New roots of k as a result of fluid loading are not small perturbations, they are not small perturbations to original roots, so they have moved far away and this region we call it as significant fluid loading. The fluid has loaded the structure enough that the original in vacuo roots are shifted far away and cannot be found using this perturbation method. So, we will find those regions.

Then we will examine this denominator which I said represents structural stiffness, structural inertia and so forth. So, we will change the values of ϵ and M and we examine which of these physical factors are dominant in which regions in frequency, some where structural inertia will be dominant, somewhere fluid inertia will be dominant along the frequency axis. We have the frequency axis going from 0, non-dimensional value 1 and above, right.

So, we will see which of these dominate in which regions in frequency as we move along frequency. Now, we have given the velocity field, coupled velocity field we have given if you recall, we gave it as the residues times I mean added with $v_1 + v_2$, so $v_1 + v_2$ we showed as a dominantly $e^{ik_0 x}$ with a decay in the x direction but we did not give the residue contribution.

The residue contribution will be straightaway given because we now know as the singularities based on Crighton's analysis. So, these singularities will contribute residues. So, what will be left then is the coupled acoustic field. So, with this I close this lecture. We will continue in next class. Thanks.